Mathematics-II – X20001

(2^{nd} Sem Civil PDDC 2013)

Assignment-7, Higher Order Differential Equation

SR NO.	SOLVE THE EXAMPLE:
1	$y'' + 4y = 2\sin 3x$ $16y'' - 8y' + 5y = 0$
2	$y'' - 2y' + 5y = 5x^3 - 6x^2 + 6x$. $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 6x + 3x^2 - 6x^3$ $x^2y'' - 4xy' + 6y = 0$ $y''' - y'' + 100y' - 100y = 0$ $\frac{d^4y}{dx^4} - 18\frac{d^2y}{dx^2} + 81y = 0$ $y = \frac{1}{(D+1)^2}\cosh x$, where $D = \frac{d}{dx}$ $y'' - 3y' + 2y = e^x$ $y'' + y = \sec x$ $y''' - 3y'' + 3y' - y = 4e^t$ Solve $(D^2 + a^2)y = \operatorname{cosec} ax$ $\left(D^4 + 2a^2D^2 + a^4\right)y = \cos ax$ $x^2y'' - 4xy' + 6y = 21x^{-4}$
3	$y' + 6x^2y = \frac{e^{-2x^3}}{x^2}$, where $y(1) = 0$ $y'' - 5y' + 6y = 0$ with initial condition $y(1) = e^2$ and $y'(1) = 3e^2$ $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x^2}$ $(x^2D^2 - 3xD + 4)y = 0$, $y(1) = 0$, $y'(1) = 3$ $y''' - y'' + 100y' - 100y = 0$, $y(0) = 4$, $y'(0) = 11$, $y''(0) = -299$
4	$y''' + 3y'' + 3y' + y = 30e^{-x}$, $y(0) = 3$, $y'(0) = -3$, $y''(0) = -47$.
5	$(x^2D^2 - 3xD + 3)y = 3\ln x - 4$
6	$x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$
7	Find general solution of $y'' + 9y = \sec 3x$ by method of variation of parameter.
8	Using the method of variation of parameters, solve $\frac{d^3y}{dx^3} + \frac{dy}{dx} = \operatorname{cosec} x$
9	Solve the nonhomogeneous Euler-Cauchy equation $x^3y''' - 3x^2y'' + 6xy' - 6y = x^4\log x$ by Variation of parameters method

①

M.O.D.:

Ex: ① ① $y'' + 4y = 2\sin 3x$

A.E: $D^2 + 4 = 0$

$\Rightarrow D^2 = -4 \Rightarrow D = \pm 2i$

$\therefore y_c = c_1 \cdot \cos 2x + c_2 \cdot \sin 2x$ — ① where c_1, c_2 are arbitrary constant

Now $y_p = \frac{1}{f(D)} x = \frac{1}{D^2+4}(2\sin 3x)$

$= 2 \cdot \frac{1}{-9+4} \sin 3x = \frac{2}{-5} \sin 3x$ — ②

$\therefore$ General soln. $y = y_c + y_p$

$= c_1 \cdot \cos 2x + c_2 \sin 2x - \frac{2}{5} \sin 3x$

② $16y'' - 8y' - 5y = 0$

A.E: $16D^2 - 8D - 5 = 0$

$\Rightarrow D = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(16)(5)}}{2(16)}$

$= \frac{8 \pm \sqrt{64 - 320}}{32}$

$= \frac{8 \pm \sqrt{-256}}{32} = \frac{8 \pm \sqrt{16^2 \cdot i^2}}{32}$

$= \frac{8 \pm 16i}{32} = \frac{1}{4} \pm \frac{1}{2} i$

$\therefore$ Gen. soln. $y = e^{\frac{1}{4}x}\left[c_1 \cdot \cos \frac{1}{2} x + c_2 \cdot \sin \frac{1}{2} x\right]$.

②. ① $y'' - 2y' + 5y = 5x^3 - 6x^2 + 6x$

A.E: $D^2 - 2D + 5 = 0$

$\Rightarrow$ ~~root~~ $D = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)}$

$= \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm \sqrt{4^2 \cdot i^2}}{2}$

②

Date :
Page :

$$= \frac{2 \pm 4i}{2} \quad 2\ 1 \pm 2i$$

$\therefore y_c = e^{1x}[c_1 \cos 2x + c_2 \sin 2x]$ — ①

where c_1, c_2 are arbitrary const.

→ Now $y_p = \frac{1}{f(D)} x$

$$= \frac{1}{D^2 - 2D + 5}(5x^3 - 6x^2 + 6x]$$

$$= \frac{1}{5[1 - \frac{2D}{5} + D^2/5]}(5x^3 - 6x^2 + 6x)$$

$$= \frac{1}{5}(1 - (\frac{2D}{5} - \frac{D^2}{5})]^{-1}(5x^3 - 6x^2 + 6x)$$

$$= \frac{1}{5}[1 + (\frac{2D}{5} - \frac{D^2}{5}) + (\frac{2D}{5} - \frac{D^2}{5})^2 + \cdots](5x^3 - 6x^2 + 6x)$$

$$= \frac{1}{5}[1 + \frac{2D}{5} - \frac{D^2}{5} + \frac{4D^2}{25} - \frac{4D^3}{25}](5x^3 - 6x^2 + 6x)$$

$$= \frac{1}{5}[1 + \frac{2D}{5} - \frac{D^2}{25} - \frac{4D^3}{25}](5x^3 - 6x^2 + 1)$$

$$= \frac{1}{5}[1(5x^3 - 6x^2 + 1) + \frac{2}{5}D(5x^3 - 6x^2 + 1)$$
$$- \frac{1}{25}D^2(5x^3 - 6x^2 + 1) - \frac{4}{25}D^3(5x^3 - 6x^2 + 1)]$$

$$= \frac{1}{5}[5x^3 - 6x^2 + 1 + \frac{2}{5}(15x^2 - 12x) - \frac{1}{25}(30x - 12)$$
$$- \frac{4}{25}(30)]$$

$$= \frac{1}{5}[125x^3 \cancel{-150x^2} - 150x^2 + 25 + 150x^2 - 120x$$
$$- 30x + 12 - 120]$$

$$= \frac{1}{5}\left[125x^3 - 150x - 108\right] \quad -②$$

$\therefore$ Gen. soln $y = y_c + y_p$

$$= e^x\left[c_1\cos 2x + c_2\sin 2x\right] + \frac{1}{5}\left[125x^3 - 150x - 108\right]$$

Q(2). $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 6x + 3x^2 - 6x^3$

A.E: $D^2 + D - 6 = 0$

$\Rightarrow (D+3)(D-2) = 0$

$\Rightarrow D = 2, -3$

$\therefore y_c = c_1 e^{2x} + c_2 e^{-3x} \quad -①$ c_1, c_2 arbitrary const.

Now $y_p = \frac{1}{f(D)}x = \frac{1}{D^2+D-6}(-6x^3+3x^2+6x)$

$$= \frac{-1}{6\left[1 + D/_{-6} + D^2/_{-6}\right]}(-6x^3+3x^2+6x)$$

$$= -\frac{1}{6}\left[1 - \left(\frac{D}{6} + \frac{D^2}{6}\right)\right]^{-1}(-6x^3+3x^2+6x)$$

$$= -\frac{1}{6}\left[1 + \left(\frac{D}{6}+\frac{D^2}{6}\right) + \left(\frac{D}{6}+\frac{D^2}{6}\right)^2 + \cdots\right](-6x^3+3x^2+6x)$$

$$= \cancel{-\frac{1}{6}\left[1(-6x^3+3x^2+6x)\right.}.$$

$$= -\frac{1}{6}\left[1 + \frac{D}{6} + \frac{D^2}{6} + \frac{D^2}{36} + \frac{2D^3}{36}\right](-6x^3+3x^2+6x).$$

$$= -\frac{1}{6}\left[1 + \frac{D}{6} + \frac{7D^2}{36} + \frac{1}{18}D^3\right](-6x^3+3x^2+6x).$$

$$= -\frac{1}{6}\left[1(-6x^3+3x^2+6x) + \frac{1}{6}D(-6x^3+3x^2+6x) + \frac{7}{36}D^2(-6x^3+3x^2+6x) + \frac{1}{18}D^3(-6x^3+3x^2+6x)\right]$$

$$= -\frac{1}{6}\left[-6x^3+3x^2+6x+\frac{1}{6}(-18x^2+6x+6)\right.$$

$$\left.+\frac{7}{36}(-36x+6)+\frac{1}{18}(-36)\right] \quad -(2)$$

$\therefore$ Gen. sol^n $y = y_c + y_p$

$$= c_1e^{2x}+c_2e^{-3x}-\frac{1}{6}\left[-6x^3+3x^2+6x-3x^2+x+1\right.$$
$$\left.-7x+\frac{7}{6}-2\right]$$

(*) (7) $x^2y'' - 4xy' + 6y = 0$.

Take $x = e^z$. So $z = \log x$.

& $x^2y'' = D(D-1)y$, $xy' = Dy$.

So given eq^n is

$[D(D-1) - 4D + 6]y = 0$

A.E.: $D^2 - D - 4D + 6 = 0$

$\Rightarrow D^2 - 5D + 6 = 0$

$\Rightarrow (D-3)(D-2) = 0$

$\therefore D = 3, 2$

$\therefore$ Gen. sol^n $y = c_1e^{3z} + c_2e^{2z}$ — ① where c_1, c_2 are arbitrary const.

$\Rightarrow \cancel{y = c_1.e^{3.e^z} + c_2.e^{3.e^z}}$

$\Rightarrow y = c_1.e^{3(\log x)} + c_2e^{2(\log x)}$.

$= \cancel{c_1e^{\log x}}$

(*) $y''' - y'' + 100y' - 100y = 0$

A.E is: $D^3 - D^2 + 100D - 100 = 0$

$\Rightarrow D^2(D-1) + 100(D-1) = 0$

$\Rightarrow (D^2+100)(D-1) = 0 \quad \Rightarrow D = -1 \quad \& \quad D^2 = -100$

$\Rightarrow D = 1, \quad D = \pm 10i \quad\quad = 100i^2$

$\therefore$ Gen sol^n: $y = c_1e^x + c_2\cos 10x + c_3\sin 10x$

5

Date :
Page :

where c_1, c_2, c_3 are arbitrary constants.

$$\frac{d^4y}{dx^4} - 18\frac{d^2y}{dx^2} + 81y = 0$$

A.E : $(D^4 - 18D^2 + 81) = 0$

$\Rightarrow (D^2 - 9)^2 = 0$

$\Rightarrow D^2 = 9, 9$

$\Rightarrow D = \pm 3, \pm 3$

$\Rightarrow$ Gen. soln. $y = (c_1 + c_2 x + c_3 x^2 + c_4 x^3) e^{3x}$

c_1, c_2, c_3, c_4 are arbitrary constants.

$$y = \frac{1}{(D+1)^2} \cosh x$$

i.e. $(D+1)^2 y = \cosh x = \left(\frac{e^x + e^{-x}}{2}\right)$.

~~A.E : $D^2+1=0 \Rightarrow D^2 = -1$~~

~~$\Rightarrow D = \pm i$~~

~~$\therefore y_c = c_1 \cdot \cos x + c_2 \sin x$ — ① c_1, c_2 are arbitrary constants~~

~~Now $y_p = \frac{1}{f(D)} X = \frac{1}{D+1}$~~

A.E : $(D+1)^2 = 0 \Rightarrow D = -1, -1$

$\therefore y_c = (c_1 + c_2 x) e^{-x}$ — ① c_1, c_2 arbitrary constants

Now $y_p = \frac{1}{f(D)} X = \frac{1}{(D+1)^2} \left[\frac{e^x + e^{-x}}{2}\right]$

$= \frac{1}{2}\left[\frac{1}{(D+1)^2} e^x + \frac{1}{(D+1)^2} e^{-x}\right]$

Date :
Page :

$$= \frac{1}{2}\left[\frac{1}{(1+1)^2}e^x + x\cdot\frac{1}{2(D+1)}\cdot e^{-x}\right]$$

$$= \frac{1}{2}\left[\frac{e^x}{4} + \frac{x}{2}\cdot x\cdot\frac{1}{1}e^{-x}\right]$$

$$= \frac{1}{2}\left[\frac{e^x}{4} + \frac{x^2 e^{-x}}{2}\right] \quad -②$$

$\therefore$ Gen. sol^n: $y = y_c + y_p$

$$= (c_1 + c_2 x)e^{-x} + \frac{e^x}{8} + \frac{x^2 e^{-x}}{4}$$

*. $y'' - 3y' + 2y = e^x$

A.E: $D^2 - 3D + 2 = 0$

$\Rightarrow (D-2)(D-1) = 0$

$\Rightarrow D = 1, 2$

$\therefore y_c = c_1 \cdot e^x + c_2 e^{2x}$ $\quad -①$ c_1, c_2 arbitrary const.

Now $y_p = \frac{1}{f(D)}x$

$$= \frac{1}{D^2 - 3D + 2}e^x = x\cdot\frac{1}{2D-3}e^x$$

$$= x\cdot\frac{1}{2(1)-3}e^x = -x\cdot e^x \quad -②$$

$\therefore$ Gen. sol^n $y = y_c + y_p$

$$= c_1 e^x + c_2 e^{2x} - xe^x$$

*. $y'' + y = \sec x$

A.E: $D^2 + 1 = 0$

$\Rightarrow D^2 = -1 = i^2$

$\Rightarrow D = \pm i$

$\therefore y_c = c_1\cos x + c_2 \sin x$ $\quad -①$ c_1, c_2 arbi. const.

→ Now take $y_1 = \cos x$, $y_2 = \sin x$

$\therefore W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$

$= \cos^2 x + \sin^2 x = 1 \neq 0$

So $y_p = -y_2 \int \frac{y_1 \cdot X}{W} dx + y_1 \int \frac{y_2 \cdot X}{W} dx$ Here $X = \sec x$.

$= -\sin x \int \frac{\cos x \cdot \sec x}{1} dx + \cos x \int \frac{\sin x \cdot \sec x}{1} dx$

$= -\sin x \int 1\, dx + \cos x \int \tan x\, dx$

$= -\sin x \cdot x + \cos x \cdot \log \sec x$ — ②

So Gen. soln $y = y_c + y_p$

$= c_1 \cos x + c_2 \sin x - x \sin x + \cos x \cdot \log \sec x$.

(*) $y''' - 3y'' + 3y' - y = 4e^x$.

A.E : $D^3 - 3D^2 + 3D - 1 = 0$

$\Rightarrow D^3 - D^2 - 2D^2 + 2D + D - 1 = 0$

$\Rightarrow D^2(D-1) - 2D(D-1) + 1(D-1) = 0$

$\Rightarrow (D-1)(D^2 - 2D + 1) = 0$

$\Rightarrow (D-1)^3 = 0$

$\Rightarrow D = 1, 1, 1$

$\therefore y_c = (c_1 + c_2 x + c_3 x^2) e^x$ — ① c_1, c_2, c_3 are arbitrary const.

Now $y_p = \frac{1}{f(D)} X = \frac{1}{(D-1)^3} 4e^x = 4 \cdot x \cdot \frac{1}{3(D-1)^2} e^x$

$= \frac{4x}{3} \cdot \frac{x}{2(D-1)} e^x = \frac{4x^2}{6} \cdot \frac{1}{1} e^x$

$= \frac{4x^2 e^x}{6} = \frac{2x^2 e^x}{3}$ — ②

$\therefore$ Gen. soln $y = y_c + y_p = (c_1 + c_2 x + c_3 x^2) e^x + \frac{2}{3} x^2 e^x$

(x) $\star$. $(D^2+a^2)\,y = \operatorname{cosec} ax$

A.E: $D^2+a^2=0$

$\Rightarrow D^2 = -a^2 = a^2 i^2$

$\Rightarrow D = \pm ai$

$\therefore y_c = c_1\cos ax + c_2 \sin ax$ — ① c_1, c_2 arbitrary const.

→ Take $y_1 = \cos ax$, $y_2 = \sin ax$

$\therefore W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos ax & \sin ax \\ -a\sin ax & a\cos ax \end{vmatrix}$

$= a[\cos^2 ax + \sin^2 ax]$

$= a \neq 0$.

so $y_p = -y_2\int \frac{y_1 \cdot X}{W}dx + y_1\int\frac{y_2 \cdot X}{W}dx$ where $X = \operatorname{cosec} ax$

$= -\sin ax \int \frac{\cos ax \cdot \operatorname{cosec} ax}{a}dx + \cos ax\int\frac{\sin ax \cdot \operatorname{cosec} ax}{a}dx$

$= -\frac{\sin ax}{a}\int \cot ax\, dx + \frac{\cos ax}{a}\int 1\, dx$

$= -\frac{\sin ax}{a}\cdot\frac{\log \sin ax}{a} + \frac{\cos ax}{a}\cdot x$ — ②

$\therefore$ Gen. soln $y = y_c + y_p$

$= c_1\cos ax + c_2 \sin ax + \frac{x\cos ax}{a} - \frac{\sin ax \cdot \log \sin ax}{a^2}$.

(x) $\star$: $(D^4 + 2a^2D^2 + a^4)\,y = \cos ax$.

A.E: $D^4 + 2a^2D^2 + a^4 = 0$

$\Rightarrow (D^2+a^2)^2 = 0$

$\Rightarrow D^2 = -a^2, -a^2$

$\Rightarrow D = \pm ia, \pm ia$

so $y_c = (c_1 + c_2 x)\cdot \cos ax + (c_3 + c_4 x)\sin ax$ — ①

where c_1, c_2, c_3, c_4 are arbitrary constants.

Date :
Page :

Now $\cancel{y_p=} \cancel{\frac{1}{f(D)}}$ $y_p = \frac{1}{f(D)} x$

$= \frac{1}{(D^2+a^2)^2} \cos ax \quad = \cancel{x \cdot \frac{1}{2(D^2+a^2) \cdot 2D} \cos ax}$

$= \frac{1}{D^4+2a^2D^2+a^4} \cos ax \qquad (\because \frac{1}{(-a^2)^2+2a^2(-a^2)+a^4} = \frac{1}{0}$

$= x \cdot \frac{1}{4D^3+4a^2D} \cos ax$

$= \frac{x}{4} \cdot \cancel{\frac{1}{-a^2D+a^2D}} \cos ax$

$= \frac{x^2}{4} \cdot \frac{1}{3D^2+a^2} \cos ax$

$= \frac{x^2}{4} \cdot \frac{1}{3(-a^2)+a^2} \cos ax = \frac{x^2}{4(-2a^2)} \cos ax$

$= -\frac{x^2}{8a^2} \cos ax \quad -(2)$

$\therefore y = y_c + y_p = (c_1+c_2x)\cos ax + (c_3+c_4x)\sin ax - \frac{x^2}{8a^2}\cos ax.$

(12) $x^2 y'' - 4xy' + 6y = 21x^{-4}$ — ①.

A.E. of above eq is.

$\cancel{(x^2D^2 - 4xD + 6)y = 21x^{-4}}$ let $x = e^z \Rightarrow z = \log x$

let $x\frac{dy}{dx} = Dy$ when $D = \frac{d}{dz}$.

$x^2\frac{d^2y}{dx^2} = D(D-1)y$.

above value put in eq. ①.

$D(D-1)y - 4Dy + 6y = 21(e^z)^{-4}$

$(D^2 - D)y - 4Dy + 6y = 21e^{-4z}$

∴ A.E = $(D^2 - D - 4D + 6)y = 0$.

∴ $D^2 - 5D + 6 = 0$.

∴ $(D-3)(D-2) = 0$.

∴ $D = 2, 3$

∴ C.F. = $c_1 e^{2z} + c_2 e^{3z}$.

now P.I. = $\frac{1}{D^2 - 5D + 6} 21e^{-4z}$

$= 21 \frac{1}{(-4)^2 - 5(-4) + 6} e^{-4z}$

$= \frac{21}{42} e^{-4z}$

$= \frac{1}{2} e^{-4z}$

∴ G.S. = C.F. + P.I.

$= (c_1 e^{2z} + c_2 e^{3z} + \frac{1}{2} e^{-4z})$

∴ General solution of eq ① is.

$\cancel{c_1 e^{2x} + c_2 e^{3x} + \frac{1}{2} e^{-4x}}$ $c_1 x^2 + c_2 x^3 + \frac{1}{2} x^{-4}$

Q : 3

(2) $y'' - 5y' + 6y = 0$ with initial condition $y(1) = e^2$ & $y'(1) = 3$

$\therefore$ A.E. $D^2 - 5D + 6 = 0$

$(D-3)(D-2) = 0$

$D = 3, 2$

$\therefore$ C.F. $y = C_1 e^{2x} + C_2 e^{3x}$ — (1)

Now $y' = 2C_1 e^{2x} + 3C_2 e^{3x}$ — (2)

Now $y(1) = e^2$

$\therefore x = 1$ then $y = e^2$

From eq (1) $e^2 = C_1 e^2 + C_2 e^3$

$\therefore C_1 + C_2 e = 1$ — (3)

also $y'(1) = 3e^2$

$\therefore x = 1$ then $y' = 3e^2$

From eq. (2) $3e^2 = 2C_1 e^2 + 3C_2 e^3$

$\therefore 2C_1 + 3C_2 e = 3$ — (4)

solving eq (3) and (4)

$$\begin{array}{r} 3C_1 + 3C_2 e = 3 \\ 2C_1 + 3C_2 e = 3 \\ - \quad - \quad\quad - \\ \hline C_1 = 0 \end{array}$$

From (3) $0 + C_2 e = 1$

$\therefore C_2 = 1/e$

From eq (1) $y = \frac{1}{e} e^{3x}$

Which is particular solution

(3) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x^2}$

$\therefore$ A.E. of above eq is.

$D^2 - 2D + 1 = 0$

$\therefore (D-1)(D-1) = 0$

$\therefore D = 1, 1.$

$\therefore C.F. = (C_1 + C_2 x) e^x.$

NOW = $P.I. = \frac{1}{D^2 - 2D + 1} e^x \cdot x^{-2}$

$= \frac{1}{(D^2 - 2D + 1)} e^x \cdot x^{-2}$

$= e^x \frac{1}{(D+1)^2 - 2(D+1) + 1} x^{-2}$

$= e^x \frac{1}{D^2 + \cancel{2D} + \cancel{1} - \cancel{2D} - \cancel{2} + \cancel{1}} x^{-2}$

$= e^x \frac{1}{D^2} x^{-2}$

$= e^x \frac{1}{(1 + (D^2 - 1))} x^{-2}$

$= e^x [1 + (D^2 - 1)]^{-1} x^{-2}$

$= e^x (1 - (D^2 - 1) + (D-1)^2 + \ldots) x^{-2}.$

$= e^x (1 - D^2 + 1 + D^2 - 2D + 1) x^{-2}$

$= e^x (x^{-2} - D^2 \cdot x^{-2} + x^{-2} + D^2 \cdot x^{-2} - 2D x^{-2} + x^{-2})$

$= e^x (x^{-2} - (\cancel{6}/x^2) + x^{-2} + (2/x^2) - 2(-2)\frac{1}{x} + x^{-2})$

$e^x \left(\frac{3}{x^2} + \frac{4}{x}\right)$ G.S. $= (C_1 + C_2 x) e^x + e^x \left(\frac{3}{x^2} + \frac{4}{x}\right).$

(4) $(x^2D^2 - 3xD + 4)y = 0$. $y(1) = 0$, $y'(1) = 3$.

Let $x = e^z$ $\therefore$ $z = \log x$

$\therefore$ and $x \cdot \frac{dy}{dx} = Dy$

$x^2 \frac{d^2y}{dx^2} = D(D-1)y$.

$\therefore$ when $\frac{d}{dz} = D$.

A.E. $(D(D-1) - 3D + 4) = 0$

$D^2 - D - 3D + 4 = 0$.

$\therefore D^2 - 4D + 4 = 0$.

$\therefore (D-2)(D-2) = 0$.

$\therefore D = 2, 2$.

$\therefore$ C.F. $\therefore y = (C_1 + C_2 x)e^{2x}$

$= C_1 e^{2x} + C_2 x e^{2x}$

$y' = 2C_1 e^{2x} + C_2(x 2 e^{2x} + e^{2x})$.

$= 2C_1 e^{2x} + 2C_2 x e^{2x} + C_2 e^{2x}$

Now $y(1) = 0$.

$\therefore x = 1 \Rightarrow y = 0$.

$0 = C_1 e^2 + C_2 e^2$ —— (1).

Now $y'(1) = 3$.

$x = 1 \Rightarrow y' = 3$.

~~$1 = 2C_1 e^6 + C_2 6 e^6 + e^6$~~

$\therefore 3 = 2C_1 e^2 + 2C_2 e^2 + C_2 e^2$

$3 = 2C_1 e^2 + 3C_2 e^2$ —— (2)

$\therefore$ From eq ① and ②

$$\cancel{0 = 2c_1 e^2 + 2c_2 e^2}$$

$$\underline{-3 = -2c_1 e^2 + 3c_2 e^2}$$

$$-3 = -c_2 e^2$$

$$\therefore 3 = c_2 e^2 \qquad \therefore c_2 = 3/e^2$$

From eq - ①

$$0 = c_1 e^2 + \frac{3}{e^2} e^2$$

$$\therefore c_1 = -3/e^2.$$

$\therefore$ particular solution of above eq is

$$p.s. = -\frac{3}{e^2} e^{2x} + \frac{3}{e^2} x e^{2x}.$$

(5) $y''' - y'' + 100y' - 100y = 0$ where $y(0) = 4$, $y'(0) = 11$, $y''(0) = -$

A.E of above eq is

$$(D^3 - D^2 + 100D - 100)y = 0.$$

$\therefore$ A.E. is $\quad D^3 - D^2 + 100D - 100 = 0.$

$$\therefore (D-1)(D^2 + 100) = 0.$$

$$\therefore (D-1) \text{ and } D = \frac{\pm\sqrt{0 - 4(1)100}}{2}$$

$$= \frac{\pm\sqrt{-400}}{2}$$

$$= \pm\, 200 i$$

$$\therefore (D-1)(D^2 + 100) = 0.$$

$$\therefore D = 1 \ \& \ D = \pm 10i$$

$$\therefore C.F.\ y = c_1 e^x + (c_2 \cos 10x + c_3 \sin 10x)$$

$$y = c_1 e^{x} + c_2 \cos 10x + c_3 \sin 10x \quad -(1)$$
$$\therefore y' = c_1 e^{x} - 10c_2 \sin 10x + 10 c_3 \cos 10x. \quad -(2)$$
$$y'' = c_1 e^{x} - 100c_2 \cos 10x - 100 c_3 \sin 10x \quad -(3)$$

Now. $y(0) = 4.$

$\therefore x = 0$ then $y = 4.$

From eq (1). $4 = c_1 + c_2 \quad -(4)$

Now $y'(0) = 11.$

$\therefore x = 0$ then $y' = 11$

From eq (2) $\therefore 11 = c_1 + 10c_3 \quad -(5)$

Now $y''(0) = -299.$

$x = 0$ then $y'' = -299.$

$-299 = c_1 - 100c_2 \quad -(6)$

From eq (4) and (6).

$$400 = 100c_1 + 100c_2$$
$$-299 = c_1 - 100c_2$$

$$101 = 101 c_1$$

$\therefore c_1 = 1.$

From eq (4) $c_2 = 3.$

From eq (5) $c_3 = 1.$

$\therefore$ put value in eq c_1, c_2, c_3 in eq (1).

$C.F = y = e^{x} + 3\cos 10x + \sin 10x$

Which is particular Soulution of above eq.

Q : 4 $y''' + 3y'' + 3y' + y = 30e^{-x}$, $y(0) = 3$, $y'(0) = -3$, $y''(0) = -47$.

$\therefore (D^3 + 3D^2 + 3D + 1) = 0.$

$\therefore (D+1)(D^2 + 2D + 1) = 0.$

$(D+1)(D+1)(D+1) = 0.$

$D = -1, -1, -1.$

$\therefore C.F. = (c_1 + c_2 x + c_3 x^2) e^{-x}.$

Now $P.I. = \frac{1}{(D^3 + 3D^2 + 3D + 1)} \cdot 30e^{-x}$

$= 30\left(\frac{x}{3D^2 + 6D + 3}\right) e^{-x}$

$\cancel{= 30\left(\frac{x}{3(-1)^2 + 6(-1) + 3}\right) e^{-x}}$

$= 30\left(\frac{x^2}{6D + 6}\right) e^{-x}$

$= 30\left(\frac{x^3}{6}\right) e^{-x}$

$= 5x^3 e^{-x}$

$\therefore G.S. = (c_1 e^{-x} + c_2 x e^{-x} + c_3 x^2 e^{-x} + 5x^3 e^{-x})$

$\therefore y = c_1 e^{-x} + c_2 x e^{-x} + c_3 x^2 e^{-x} + 5x^3 e^{-x}.$

$y' = c_1(-e^{-x}) + c_2(x(-e^{-x}) + e^{-x}) + c_3(x^2(-e^{-x}) + e^{-x} \cdot 2x) + 5(x^3(-e^{-x}) + e^{-x} 3$

$y' = -c_1 e^{-x} - c_2 x e^{-x} + c_2 e^{-x} - c_3 x^2 e^{-x} + c_3 2x e^{-x} - 5x^3 e^{-x} + 15x^2 e^{-x}$

$y'' = -c_1(-e^{-x}) - c_2(x(-e^{-x}) + e^{-x}) + c_2(-e^{-x}) - c_3(x^2(-e^{-x}) + e^{-x} 2x)$
$+ c_3 2(x(-e^{-x}) + e^{-x}) - 5(x^3(-e^{-x}) + e^{-x}(3x^2))$
$+ 15(x^2(-e^{-x}) + e^{-x} \cdot 2x)$

$= c_1 e^{-x} + c_2 x e^{-x} - c_2 e^{-x} - c_2 e^{-x} + c_3 x^2 e^{-x} - c_3 2x e^{-x} - c_3 2x e^{-x} + c_3 2 e^{-x}$
$+ 5x^3 e^{-x} - 15x^2 e^{-x} - 15x^2 e^{-x} + 30x e^{-x}$

$y'' = c_1 e^{-x} + c_2 x e^{-x} - c_2 2e^{-x} + c_3 x^2 e^{-x} - c_3 4x e^{-x} + c_3 2e^{-x} + 5x^3 e^{-x} - 30x^2 e^{-x} + 30x$

Now $y(0)=3. \Rightarrow x=0$ then $y=3$.

$$3=c_1 \quad \text{—①}$$

Now $y'(0)=-3 \Rightarrow x=0$ then $y'=-3$.

$$-3=-c_1+c_2 \quad \text{—②}$$

Now $y''(0)=-47. \Rightarrow x=0$ then $y''=-47$.

$$-47=c_1-2c_2+2c_3 \quad \text{—③}$$

$\therefore c_1=3$ and $c_2=0$. $c_3=-25$.

$\therefore P.I.=y=3e^{-x}+-25x^2e^{-x}+5xe^{-x}$.

Q.6. $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$. —①.

Let $x=e^z \quad \therefore z=\log x$

$\cancel{x\frac{dy}{dx}=Dy}$

$x^2\frac{d^2y}{dx^2}=D(D-1)y$.

$x^3\frac{d^3y}{dx^3}=D(D-1)(D-2)y$

When $\frac{d}{dz}=D$.

above value put in eq—①.

$\cancel{(D(D-1)(D-2)y)+2(D(D-1))y+2y=10\left(e^z+\frac{1}{e^z}\right)}$

$\therefore (D^2-D)(D^2-2D)+2D^2-2+2=0$

$\therefore D^4-2D^3-D^3+2D^2+2D^2-2+2=0$.

$D^4-3D^3+4D^2=0$.

$\therefore \cancel{(D^2-D)(D-2)+2(D^2-D)+2)y=0}$

$\therefore D^3-2D^2-D^2+2D+2D^2-2D+2=0$.

$\therefore D$

$\therefore (D^2-D)(D-2)y + 2(D^2-D)y + 2y = 10\left(e^{2} + \frac{1}{e^2}\right)$.

$\therefore (D^3 - 2D^2 - D^2 + \cancel{2D} + 2D^2 - 2D + 2)y = 10\left(e^2 + \frac{1}{e^2}\right)$.

A.E. eq. is.

$D^3 - D^2 + 2 = 0$.

$\cancel{(D+1)(D^2-2D+2) = 0}$

$\cancel{(D+1)(D-1)(D-1) = 0}$.

$(D+1)$ & $D = \frac{+2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2}$

$= \frac{+2 \pm \sqrt{-4}}{2}$

$D = -1$ & $D = 1 \pm 1i$

$\therefore$ C.F. $= c_1 e^{-2} + (c_2 \cos 2 + c_3 \sin 2) e^{2}$

Now = P.I. $= \frac{1}{D^3 - D^2 + 2} 10\left(e^2 + \frac{1}{e^2}\right)$.

$= 10\left[\left(\frac{1}{D^3 - D^2 + 2}\right) e^{2} + \frac{1}{D^3 - D^2 + 2} e^{-2}\right]$.

$= 10\left[\frac{1}{2} e^{2} + \frac{2}{3D^2 - 2D} e^{-2}\right]$.

$= 10\left[\frac{1}{2} e^{2} + \frac{2}{5} e^{-2}\right]$.

$\therefore$ G.S. = C.F. + P.I.

$c_1 e^{-2} + e^{2}(c_2 \cos 2 + c_3 \sin 2) + 5e^{2} + 2 \cdot 2e^{-2}$.

~~Now putting value of 2.~~

$c_1 x^{-1} + x(c_2 \cos(\log x) + c_3 \sin(\log x)) + 5x + 2\log x$.

Q:7 Find general solution of $y''+9y = \sec 3x$ by method of variation of parameter.

$$y''+9y = \sec 3x$$

$$= (D^2+9)y = \sec 3x.$$

$$D^2+9 = 0.$$

$$\therefore D = \pm 3i$$

$$C.F. = c_1 \cos 3x + c_2 \sin 3x.$$

Where $y_1 = \cos 3x$ and $y_2 = \sin 3x$.

and $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$.

$$= \begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix}.$$

$$= 3\cos^2 3x - (-3\sin^2 3x)$$

$$= 3(\cos^2 3x + \sin^2 3x)$$

$$= 3.$$

Now $p.I. = -y_1 \int \frac{y_2 X}{W} dx + y_2 \int \frac{y_1 X}{W} dx$

$$= -\cos 3x \int \frac{\sin 3x \cdot \sec 3x}{3} dx + \sin 3x \int \frac{\cos 3x \cdot \sec 3x}{3} dx$$

$$= -\cos 3x \frac{1}{3} \int \sin 3x \cdot \frac{1}{\cos 3x} dx + \sin 3x \frac{1}{3} \int \frac{\cos 3x}{\cos 3x} dx$$

$$= -\cos 3x \frac{1}{3} \int \tan 3x \, dx + \sin 3x \cdot \frac{1}{3} \int 1 \, dx.$$

$$= +\frac{\cos 3x}{9} \log \cos 3x + \frac{x}{3} \sin 3x.$$

$$\therefore G.S. = (c_1 \cos 3x + c_2 \sin 3x) + \frac{\cos 3x}{9} + \frac{x}{3} \sin 3x.$$

Q:8 Using the method of variation of parameters, solve $\frac{d^3y}{dx^3} + \frac{dy}{dx} = \operatorname{cosec} x$.

her $(D^3 + D)y = \operatorname{cosec} x$

Q:9 Solve the nonhomogeneous Euler-Cauchy equation $x^3y''' - 3x^2y'' + 6xy' - 6y = x^4 \log x$

Let $x = e^z \Rightarrow z = \log x$.

$$x\frac{dy}{dx} = Dy.$$

$$x^2\frac{d^2y}{dx^2} = D(D-1)y.$$

$$x^3\frac{d^3y}{dx^3} = D(D-1)(D-2)y.$$

When $D = \frac{d}{dz}$.

Now $D(D-1)(D-2)y - 3D(D-1)y + 6Dy - 6y = e^{4z}\cdot z$

$$\therefore ((D^2-D)(D-2) - 3D^2 + 3D + 6D - 6)y = e^{4z}\cdot z.$$

$$\therefore D^3 - 2D^2 - D^2 + 2D - 3D^2 + 3D + 6D - 6 = 0.$$

$$\therefore D^3 - 6D^2 + 11D - 6 = 0.$$

$$(D-1)(D^2 - 5D + 6) = 0.$$

$$(D-1)(D-3)(D-2) = 0.$$

$$\therefore D = 1, 2, 3.$$

$$\therefore C.F. = c_1e^{z} + c_2e^{2z} + c_3e^{3z}$$

Now. $$P.I. = \frac{1}{(D-1)(D-2)(D-3)}e^{4z}\cdot z.$$

$$= e^{4z}\frac{1}{((D+4)-1)((D+4)-2)((D+4)-3)}z.$$

$$= e^{4z} \frac{1}{(D+3)(D+2)(D+1)} z.$$

$$= e^{4z} \frac{1}{D^3+6D^2+11D+6} z.$$

$$= \frac{e^{4z}}{6} \left[\left(\frac{D^3}{6} + D^2 + \frac{11D}{6} \right) + 1 \right]^{-1} \cdot z.$$

$$= \frac{e^{4z}}{6} \left[1 + \left(\frac{D^3}{6} + D^2 + \frac{11}{6} D \right) \right] z.$$

$$= \frac{e^{4z}}{6} \left[1 - \left(\frac{D^3}{6} + D^2 + \frac{11}{6} D \right) + \left(\frac{D^3}{6} + D^2 + \frac{11}{6} D \right)^2 \ldots \right] z.$$

$$= \frac{e^{4z}}{6} \left[1 - \frac{1}{6} D^3 \cdot z + D^2 \cdot z + \frac{11}{6} D z \right].$$

$$= \frac{e^{4z}}{6} \left[1 - \frac{1}{6}(0) + (6) + \frac{11}{6} \right]$$

$$= \frac{e^{4z}}{6} \left[1 + \frac{11}{6} \right].$$

$$= \frac{17 e^{4z}}{36}$$

$$\therefore G.S. = c_1 e^{z} + c_2 e^{2z} + c_3 e^{3z} + \frac{17}{36} e^{4z}.$$

$$= c_1 x + c_2 x^2 + c_3 x^3 + \frac{17}{36} x^4$$

Assignment-8, Higher Order Differential Equation

SR NO.	EXAMPLES
1	Solve $(D^2+3D+2)y=x^2+e^{-x}$. Solve : $(D^2+6D+9)y=0$
2	Solve $y''+9y=3x^2$. $(D^2-3D+2)y=\cos 3x$ $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+4y=e^x\cos x$ $D^2y-a^2y=0$ $(D^2+1)y=\sin x$ $\frac{d^2y}{dx^2}-\frac{dy}{dx}-6y=e^{3x}$ $\frac{d^2y}{dx^2}+\frac{dy}{dx}=x^2+2x+4.$ $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=4\cos^2 x.$ $(D^2+5D+6)y=e^x$ $(D^2-5D+6)y=\sin 3x$. $(D^2+D)y=x^2+2x+4$ $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-3y=x^2$
3	Solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x.$ $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}+\frac{dy}{dx}=e^{-x}+\sin 2x.$ $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=e^{-2x}\sin 2x$ $(D^2-2D+4)y=e^x\cos x$.
4	Solve $x^2y''-xy'+4y=\cos(\log x)+x\sin(\log x)$.

5	$(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=2\sin(\log(1+x))$ $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\log x$ Solve : $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\cos(\log x)$. $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\log x$. $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=2\sin(\log(1+x))$.
6	Using the method of variation of parameter, solve $y''+y=\operatorname{cosec} x$.
7	Using method of variation of parameters, solve the differential equation : $y''-6y'+9y=e^{3x}/x^2$
8	Using variation of parameter solve $\frac{d^2y}{dx^2}+4y=\tan 2x$
9	Using method of variation of parameters, solve the differential equation: $(D^2+4)y=\tan 2x$.
10	Using the method of variation of parameter solve the differential equation $y''+y=\sec x$.
11	In an L-C-R circuit, the charge q on a plate of a condenser is given by $L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{c}=E\sin pt$. The circuit is tuned to resonance so that p^2 = 1/LC. If initially the current i and the charge q be zero, show that, for small values of R/L, the current in the circuit at time t is given by (Et/2L) sin(pt).
12	The differential equation for a circuit in which self-inductance and capacitance neutralize each other is $L\frac{d^2i}{dt^2}+\frac{i}{C}=0$. Find the current i as a function of t given that I is the maximum current and $i=0$ when $t=0$.
13	The deflection of a strut of length l with one end ($x=0$) built-in and the other supported and subjected to end thrust P satisfies the equation $\frac{d^2y}{dx^2}+a^2y=\frac{a^2R}{P}(l-x)$. Prove that the deflection curve is $y=\frac{R}{P}\left(\frac{\sin ax}{a}-l\cos ax+l-x\right)$, where $al=\tan al$.

Q:1.(a) solve $(D^2+3D+2)y = x^2+e^{-x}$

A.E.

$$(D^2+3D+2)=0.$$

$$\therefore (D+1)(D+2)=0.$$

$$\therefore D=-1,\ -2.$$

$$\therefore C.F. = c_1e^{-x}+c_2e^{-2x}$$

now. $$P.I. = \frac{1}{(D^2+3D+2)}(x^2+e^{-x}).$$

$$= \frac{1}{D^2+3D+2}x^2 + \frac{1}{D^2+3D+2}e^{-x}$$

$$= \frac{1}{2\left(\frac{D^2+3D}{2}+1\right)}x^2 + \frac{x}{2D+3}e^{-x}$$

$$= \frac{1}{2}\left[1+\left(\frac{D^2+3D}{2}\right)\right]^{-1}x^2 + xe^{-x}$$

$$= \frac{1}{2}\left[1-\left(\frac{D^2+3D}{2}\right)+\left(\frac{D^2+3D}{2}\right)^2+\dots\right]x^2+xe^{-x}$$

$$= \frac{1}{2}\left[x^2-\frac{1}{2}D^2(x^2)-\frac{3}{2}D(x^2)+\frac{1}{4}D^4(x^2)+6D^3(x^2)+\frac{9}{4}D^2(x^2)\right] + xe^{-x}$$

$$= \frac{1}{2}\left[x^2-\frac{1}{2}(2)-\frac{3}{2}(2x)+\frac{1}{4}(0)+6D^3(0)+\frac{9}{42}(2)\right]+xe^{-x}$$

$$= \frac{1}{2}\left[x^2-1-3x+\frac{9}{2}\right]+xe^{-x}$$

$$= \frac{1}{2}\left[x^2-3x+\frac{7}{2}\right]+xe^{-x}$$

$$\therefore G.S. = c_1e^{-x}+c_2e^{-2x}+\frac{1}{2}\left[x^2-3x+\frac{7}{2}\right]+xe^{-x}.$$

(b) solve $(D^2+6D+9)y=0.$

$$\therefore A.E.\quad D^2+6D+9=0$$

$$(D+3)(D+3)=0.$$

$D = -3, -3.$

$\therefore C.F. = (C_1 + C_2 x) e^{-3x}$

$G.S. = (C_1 + C_2 x) e^{-3x}.$

Q: 2 (a) solve $y'' + 9y = 3x^2$

$\therefore (D^2 + 9) y = 3x^2.$

$\therefore$ A.E is $D^2 + 9 = 0.$

$\therefore D = \pm 3i$

$C.F. = (C_1 \cos 3x + C_2 \sin 3x).$

Now $P.I. = \dfrac{1}{D^2+9} \cdot 3x^2$

$= \dfrac{3}{9} \cdot \left(\dfrac{1}{1 + D^2/9}\right) x^2.$

$= \dfrac{3}{9} [1 + D^2/9]^{-1} \cdot x^2$

$= \cancel{\dfrac{3}{3} \left[1 - (1 + D^2/9) + (1 + D^2/9)^2 - \ldots\right] x^2}$

$= 3 \left[1 - (D^2/9) + (D^2/9)^2\right] x^2$

$= 3 \left(x^2 - \frac{1}{9} D^2 \cdot x^2 + \frac{1}{81} D^4 \cdot x^2\right]$

$= 3 \left[x^2 - \frac{2}{9} + \frac{1}{81} (0)\right].$

$= 3x^2 - 2/3$

$\therefore G.S. = C_1 \cos 3x + C_2 \sin 3x + 3x^2 - 2/3.$

(b) $(D^2-3D+2)y = \cos 3x$.

$\therefore$ A.E. eq. is $D^2-3D+2=0$.

$$(D-1)(D-2)=0.$$

$$D=1,\ 2.$$

$\therefore$ C.F. $= c_1e^{x}+c_2e^{2x}$

Now P.I. $= \dfrac{1}{D^2-3D+2}\cos 3x$

$$= \frac{1}{-9-3D+2}\cos 3x.$$

$$= -\frac{1}{3D+7}\cos 3x.$$

$$= -\frac{1}{(3D+7)}\frac{(3D-7)}{(3D-7)}\cos 3x.$$

$$= -\frac{(3D-7)}{(9D^2-49)}\cos 3x$$

$$= -\frac{(3D-7)}{-81-49}\cos 3x.$$

$$= \frac{1}{130}(3D-7)\cos 3x.$$

$$= \frac{1}{130}(3\cdot D\cos 3x - 7\cos x).$$

$$= \frac{1}{130}(3(-3\sin 3x - 7\cos x).$$

$$= -\frac{1}{130}(9\sin 3x + 7\cos x).$$

$\therefore$ G.S. $= c_1e^{x}+c_2e^{2x}-\dfrac{9}{130}\sin 3x-\dfrac{7}{130}\cos x.$

(c) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 4y = e^{x}\cdot \cos x.$

$\therefore (D^2 - 2D + 4)y = e^{x}\cos x.$

$A.E. = D^2 - 2D + 4$

$\therefore D = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4.(1)(4)}}{2}$

$= \dfrac{2 \pm \sqrt{-12}}{2}$

$= \dfrac{2 \pm 2\sqrt{3}i}{2}$

$= 1 \pm \sqrt{3}i$

$\therefore C.F. = e^{x}(c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x).$

NOW $P.I. = \dfrac{1}{D^2-2D+4}\cdot e^{x}\cos x$

$= e^{x}\dfrac{1}{(D+1)^2-2(D+1)+4}\cos x$

$= e^{x}\dfrac{1}{D^2+2D+1-2D-2+4}\cos x.$

$= e^{x}\dfrac{1}{D^2+3}\cos x.$

$= e^{x}\cdot\dfrac{1}{2}\cos x.$

$G.S. = e^{x}(c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x) + \dfrac{e^{x}\cdot\cos x}{2}$

(d) $D^2y - a^2y = 0.$

$\therefore (D^2 - a^2)y = 0.$

$\therefore D^2 - a^2 = 0$

$D^2 = a^2$

$\therefore D = \pm ai$

$\therefore C.F. = (c_1 \cos ax + c_2 \sin ax)$.

$\therefore G.S. = c_1 \cos ax + c_2 \sin ax$.

(e) $(D^2+1)y = \sin x$.

A.E. $D^2+1 = 0$.

$\therefore D = \pm 1i$

$\therefore C.F. = c_1 \cos x + c_2 \sin x$

now $P.I. = \dfrac{1}{D^2+1} \sin x$.

$= \dfrac{x}{2D} \sin x$.

$= \dfrac{x \cdot 2D}{4D^2} \sin x$

$= \dfrac{2x \cdot D}{-4} \sin x$.

$= -\frac{1}{2} x \cos x$

$\therefore G.S. = c_1 \cos x + c_2 \sin x - \frac{1}{2} x \cos x$

(f) $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 6y = e^{3x}$.

$(D^2 - D - 6)y = e^{3x}$.

$\therefore D^2 - D - 6 = 0$.

$(D+2)(D-3) = 0$.

$\therefore D = -2, 3$.

$\therefore C.F. = c_1 e^{-2x} + c_2 e^{3x}$

Now $P.I. = \frac{1}{(D^2 - D - 6)} e^{3x}$

$$= \frac{x}{(2D-1)} e^{3x}$$

$$= \frac{x}{5} e^{3x}$$

$\therefore G.S. = C_1 e^{-2x} + C_2 e^{3x} + \frac{x}{5} e^{3x}$

(g). $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4.$

$\therefore (D^2 + D) y = x^2 + 2x + 4.$

$\therefore D^2 + D = 0.$

$\therefore D(D+1) = 0.$

$D = 0, \; D = -1.$

$\therefore C.F. = C_1 + C_2 e^{-x}$

Now $P.I. = \frac{1}{D^2 + D} (x^2 + 2x + 4).$

$$= \frac{1}{D} [1 + D]^{-1} (x^2 + 2x + 4).$$

$$= \frac{1}{D} (1 - D + D^2] (x^2 + 2x + 4)$$

$$= \frac{1}{D} \left[(x^2 + 2x + 4) - D(x^2 + 2x + 4) + D^2 (x^2 + 2x + 4) \right]$$

$$= \frac{1}{D} [x^2 + \cancel{2x} + 4 - \cancel{2x} - \cancel{2} + 2]$$

$$= \frac{1}{D} [x^2 + 4].$$

$$= \frac{x^3}{3} + 4x$$

$$\boxed{G.S. = C_1 + C_2 e^{-x} + x^3/3 + 4x}$$

(h). $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y = 4\cos^2 x$

$(D^2+3D+2)y = 4\cos^2 x$

$\therefore D^2+3D+2 = 0$

$(D+1)(D+2) = 0.$

$\therefore D = -1,\ D = -2$

$\therefore C.F. = C_1 e^{-x} + C_2 e^{-2x}$

Now $P.I. = \frac{1}{D^2+3D+2} 4\cos^2 x$

$= 4\frac{1}{D^2+3D+2}\left(\frac{1+\cos 2x}{2}\right).$

$= 4\left[\frac{1}{D^2+3D+2}\cdot\frac{1}{2} + \frac{1}{D^2+3D+2}\cos 2x\right].$

$= 4\left[\frac{1}{4} + \frac{1}{3D-2}\cos 2x\right].$

$= 4\left[\frac{1}{4} + \frac{3D+2}{9D^2-4}\cos 2x\right].$

$= 4\left[\frac{1}{4} + \frac{3D+2}{-40}\cos 2x\right].$

$= 4\left[\frac{1}{4} - \frac{1}{40}\left[3D\cos 2x + 2\cos 2x\right]\right].$

$= 4\left[\frac{1}{4} - \frac{1}{40}\left[-6\sin 2x + 2\cos 2x\right]\right].$

$= 1 - \frac{1}{10}\left[-6\sin 2x + 2\cos 2x\right].$

$= 1 + \frac{3}{5}\sin 2x - \frac{1}{5}\cos 2x$

$\therefore G.S. = C_1 e^{-x} + C_2 e^{-2x} + 1 + \frac{3}{5}\sin 2x - \frac{1}{5}\cos 2x$

(i) $(D^2+5D+6)y = e^{x}.$

A.E. : $D^2+5D+6=0.$

$\therefore (D+2)(D+3)=0.$

$\therefore D=-2,-3.$

$\therefore$ C.F. $= c_1 e^{-2x} + c_2 e^{-3x}.$

Now P.I. $= \dfrac{1}{D^2+5D+6} e^{x}$

$= \cancel{\dfrac{x}{2D+5} e^{x}} = \dfrac{1}{12} e^{x}.$

$= \cancel{\dfrac{x}{8} e^{x}}$

$\therefore$ G.S. $= c_1 e^{-2x} + c_2 e^{-3x} + \frac{1}{12} e^{x}.$

(j) $(D^2-5D+6)y = \sin 3x$

A.E. : $D^2-5D+6=0.$

$\therefore (D-3)(D-2)=0.$

$\therefore D=3,2$

$\therefore$ C.F. $= c_1 e^{2x} + c_2 e^{3x}$

Now P.I. $= \dfrac{1}{D^2-5D+6} \sin 3x.$

$= -\dfrac{1}{(5D+3)} \sin 3x$

$= -\dfrac{(5D-3)}{(25D^2-9)} \sin 3x$

$= \dfrac{1}{234}\left[5D \sin 3x - 3\sin 3x\right].$

$$= \frac{1}{234}\left[15\cos 3x - 3\sin 3x\right]$$

$$\therefore G.S. = C_1 e^{2x} + C_2 e^{3x} + \frac{1}{234}\left[15\cos 3x - 3\sin 3x\right]$$

(k) This is same as per (g)

(l). $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2$

Let $x = e^z$ $\quad \therefore z = \log x$.

and $x\frac{dy}{dx} = Dy$.

$x^2 \frac{d^2y}{dx^2} = D(D-1)y$

where $\frac{d}{dz} = D$.

$\therefore D(D-1)y - Dy - 3y = e^{2z}$

$(D^2 - D - D - 3)y = e^{2z}$.

A.E. $\therefore (D^2 - 2D - 3) = 0$.

$\therefore (D-3)(D+1) = 0$

$\therefore D = -1, 3$

$\therefore C.F. = C_1 e^{-z} + C_2 e^{3z}$

Now $P.I. = \frac{1}{(D^2 - 2D - 3)} e^{2z}$

$= -\frac{1}{3} e^{2z}$.

$\therefore G.S. = C_1 e^{-z} + C_2 e^{3z} - \frac{1}{3} e^{2z}$.

$G.S = C_1 \frac{1}{x} + C_2 x^3 - \frac{1}{3} x^2$

Q:3. (b) $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}+\frac{dy}{dx}=e^{-x}+\sin 2x.$

A.E. $(D^3+2D^2+D)y=e^{-x}+\sin 2x.$

A.E. $D(D^2+2D+1)=0.$

$\therefore D(D+1)(D+1)=0.$

$\therefore D=0,-1,-1.$

$\therefore$ C.F. $= c_1+(c_2+c_3x)e^{-x}.$

Now P.I. $=\frac{1}{(D^3+2D^2+D)}e^{-x}+\frac{1}{(D^3+2D^2+D)}\sin 2x$

$=\frac{x}{(3D^2+4D+1)}e^{-x}+\frac{1}{-(3D+8)}\sin 2x$

$=\frac{x^2}{(6D+4)}e^{-x}-\frac{(3D-8)}{(9D^2-64)}\sin 2x.$

$=-\frac{x^2}{2}e^{-x}-\frac{(3D-8)}{-100}\sin 2x$

$=-\frac{x^2}{2}e^{-x}+\frac{1}{100}(3D\sin 2x-8\sin 2x)$

$=-x^2/2\, e^{-x}+\frac{1}{100}(6\cos 2x-8\sin 2x).$

$\therefore$ G.S. $= c_1+(c_2+c_3x)e^{-x}-\frac{x^2}{2}e^{-x}+\frac{1}{100}(6\cos 2x-8\sin 2x).$

(c) $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=e^{-2x}\sin 2x.$

A.E. $(D^2+5D+6)y=e^{-2x}\sin 2x$

A.E. $D^2+5D+6=0.$

$\therefore (D+3)(D+2)=0.$

$\therefore D=-3,-2$

$\therefore C.F. = c_1 e^{-2x} + c_2 e^{-3x}.$

Now $P.I. = \frac{1}{D^2+5D+6} e^{-2x} \sin 2x.$

$= e^{-2x} \frac{1}{(D-2)^2+5(D-2)+6} \sin 2x$

$= e^{-2x} \frac{1}{D^2-4D+4+5D-10+6} \sin 2x$

$= e^{-2x} \frac{1}{D^2+D} \sin 2x.$

$= e^{-2x} \frac{1}{D-4} \sin 2x$

$= e^{-2x} \frac{(D+4)}{D^2-16} \sin 2x.$

$= e^{-2x} \frac{(D+4)}{-4-16} \sin 2x.$

$= -\frac{e^{-2x}}{20} (D \sin 2x + 4 \sin 2x).$

$= -\frac{e^{-2x}}{20} (2\cos 2x + 4 \sin 2x)$

$\therefore G.S. = c_1 e^{-2x} + c_2 e^{-3x} - \frac{1}{20} e^{-2x} (2\cos 2x + 4\sin 2x).$

(d) $(D^2-2D+4)y = e^{x} \cos x.$ $\quad (D+1)(D+1)$ $\quad D^2+D+D+1$

A.E. $D^2-2D+4=0.$

$\therefore D = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(4)}}{2}$

$= \frac{2 \pm \sqrt{-12}}{2}$

$= 1 \pm \sqrt{3} i$

$\therefore C.F = e^{x} (c_1 \cos \sqrt{3} x + c_2 \sin \sqrt{3} x).$

$$\therefore C.F. = e^{x}\left(c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x\right)$$

Now. $P.I. = \dfrac{1}{D^2-2D+4} e^{x}\cos x.$

$$= e^{x}\frac{1}{(D+1)^2-2(D+1)+4}\cos x.$$

$$= e^{x}\frac{1}{D^2+2D+1-2D-2+4}\cos x$$

$$= e^{x}\frac{1}{D^2+3}\cos x.$$

$$= e^{x}\frac{1}{2}\cos x$$

$$\therefore G.S. = e^{x}\left(c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x\right) + \frac{e^{x}}{2}\cos x$$

Q:4. $x^2y'' - xy' + 4y = \cos(\log x) + x\sin(\log x).$

$\cancel{\therefore x^2D^2 - xD + 4y = \cos(\log x) + x\sin(\log x)}$

Let $x = e^{z}$ $\therefore$ $z = \log x$

$$x\frac{dy}{dx} = Dy.$$

$$x^2\frac{d^2y}{dx^2} = D(D-1)y.$$

where $D = \dfrac{d}{dz}.$

So, $\cancel{D(D-1)y - Dy + 4y = \cos z + e^{z}\sin z}$

$$\therefore (D^2 - D - D + 4)y = \cos z + e^{z}\sin z.$$

A.E. : $D^2 - 2D + 4 = 0.$

$$\therefore D = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2}$$

$$= 1 \pm \sqrt{3}i$$

$$\therefore C.F. = e^{z}\left(c_1 \cos\sqrt{3}z + c_2 \sin\sqrt{3}z\right).$$

now. $P.I. = \frac{1}{D^2-2D+4}\cos z + e^z \sin z$

$= \frac{1}{D^2-2D+4}\cos z + \frac{1}{D^2-2D+4} e^z \sin z$

$= \frac{1}{2D-3}\cos z + e^z \frac{1}{(D+1)^2-2(D+1)+4}\sin z$

$= \frac{2D+3}{4D^2-9}\cos z + e^z \frac{1}{D^2+2D+1-2D-2+4}\sin z$

$= -\frac{1}{13}(2D\cos z + 3\cos z) + e^z \frac{1}{D^2+3}\sin z$

$= -\frac{1}{13}(2(-\sin z)+3\cos z) + \frac{e^z}{2}\sin z$

$= \frac{1}{13}(2\sin z - 3\cos z) + \frac{e^z}{2}\sin z$

$\therefore G.S. = e^z(C_1\cos\sqrt{3}z + C_2\sin\sqrt{3}z) + \frac{1}{13}(2\sin z - 3\cos z) + \frac{e^z}{2}\sin z$

~~now z and putting z value.~~

$\therefore G.S. = x(C_1\cos(\sqrt{3}\log x) + C_2\sin(\sqrt{3}\log x)) + \frac{1}{13}[2\sin(\log x) - 3\cos(\log x)] + \frac{x}{2}\sin(\log x)$

Q:5 (a) $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin(\log(1+x))$

Take $(x+1) = e^z \quad \therefore z = \log(x+1)$.

$\therefore (ax+b)\frac{dy}{dz} = aDy$.

$\therefore (ax+b)^2\frac{d^2y}{dx^2} = a^2D(D-1)y$.

Wher $D = \frac{d}{dz}$.

$\therefore D(D-1)y + Dy + y = 2\sin z$

$\therefore (D^2 - D)y + Dy + y = 2\sin z.$

A.E : $(D^2 - D + D + 1)y = 0.$

$\therefore (D^2+1) = 0$

$\therefore D = \pm 1i$

$\therefore$ C.F. $= (C_1\cos z + C_2\sin z)$

Now P.I. $= \frac{1}{D^2+1} 2\sin z.$

$= 2 \cdot \frac{z}{2D} \sin z.$

$= 2 \cdot \frac{z(2D)}{4D^2} \sin z.$

$= -z D\sin z.$

$= -z\cos z$

$\therefore$ G.S $= (C_1\cos z + C_2\sin z) - z\cos z$

Now puttin z value

~~Cos = (C1 cos(log~~

G.S. $= C_1\cos(\log(x+1)) + C_2\sin(\log(x+1)) - \log(x+1)\cos(\log(x+1)).$

(b) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$

take $x = e^z \Rightarrow z = \log x$

$x\frac{dy}{dx} = Dy.$

$x^2\frac{d^2y}{dx^2} = D(D-1)y.$ Wher $D = \frac{d}{dz}$

$\therefore D(D-1)y - Dy + y = z$

$\therefore (D^2 - D)y - Dy + y = z.$

A.E. $\therefore (D^2 - D - D + 1)y = z.$

$\therefore D^2 - 2D + 1 = 0.$

$(D-1)(D-1) = 0.$

$D = 1, 1.$

C.F. $= (c_1 + c_2 z)e^z$

Now $P.I. = \dfrac{1}{(D^2 - 2D + 1)} z.$

$= [1 + (D^2 - 2D)]^{-1} \cdot z.$

$= (1 - (D^2 - 2D) + (D^2 - 2D)^2 \ldots\ldots] z.$

$= (z - D^2 \cdot z - 2Dz + D^4 z - 4D^3 z + 4D^2 z).$

$= (z - (0) - 2(1) + (0) - 4(0) + 4(0)).$

$= z - 2$

$\therefore G.S. = (c_1 + c_2 z)e^z + z - 2.$

$G.S. = (c_1 + c_2 \log x)x + \log x - 2.$

(c) $x^3 \dfrac{d^3y}{dx^3} + 3x^2 \dfrac{d^2y}{dx^2} + x \dfrac{dy}{dx} + 8y = 65 \cos(\log x).$

Let $x = e^z$ $\therefore \log x = z.$

$x \dfrac{dy}{dx} = Dy$ $\qquad x^3 \dfrac{d^3y}{dx^3} = D(D-1)(D-2)y.$

$x^2 \dfrac{d^2y}{dx^2} = D(D-1)y.$ $\qquad$ and here $D = \dfrac{d}{dz}.$

$\therefore D(D-1)(D-2)y + 3D(D-1)y + Dy + 8y = 65\cos z$

$\therefore (D^2-D)(D-2)y + (3D^2-3D)y + Dy + 8y = 65\cos z.$

$\therefore (D^3-2D^2-D^2+2D)y + (3D^2-3D)y + Dy + 8y = 65\cos z.$

A.E. $\therefore (D^3-2D^2-D^2+2D+3D^2-3D+D+8)y = 65\cos z.$

$\therefore (D^3+8) = 0.$

$\therefore (D+2)(D^2-2D+4) = 0.$

$\therefore (D+2) \quad \therefore D = \dfrac{-(-2) \pm \sqrt{(-2)^2-4(1)(4)}}{2}$

$= \dfrac{2 \pm \sqrt{-12}}{2}$

$= 1 \pm \sqrt{3}i$

$\therefore D = -2,\ 1 \pm \sqrt{3}i$

$\therefore C.F. = c_1 e^{-2z} + e^{z}(c_2\cos\sqrt{3}z + c_3\sin\sqrt{3}z)$

Now $=$ P.I $= \dfrac{1}{(D^3+8)} 65\cos z.$

$= 65\left(\dfrac{1}{D(-1)+8}\right)\cos z.$

$= 65\left(-\dfrac{1}{D-8}\right)\cos z.$

$= -65\left(\dfrac{D+8}{D^2-64}\cos z\right).$

$= -65\left(-\dfrac{(D+8)\cos z}{65}\right).$

$= D\cos z + 8\cos z.$

$= (-\sin z) + 8\cos z.$

$= 8\cos z - \sin z.$

$\therefore G.S. = c_1 e^{-2z} + e^{z}(c_2\cos\sqrt{3}z + c_3\sin\sqrt{3}z) + 8\cos z - \sin z$

putting z val

$= c_1 x^{-2} + x(c_2\cos(\sqrt{3}\log x) + c_3\sin(\sqrt{3}\log x) + 8\cos(\log x) - \sin(\log$

Q: 6. Using the method of variation of parameters solve

$y'' + y = \operatorname{cosec} x$

$\therefore$ ~~D^2+y~~ $D^2y + y = \operatorname{cosec} x.$

A.E. $(D^2+1) = 0.$

$\therefore D = \pm i.$

$\therefore$ C.F. $= (c_1 \cos x + c_2 \sin x).$

$\therefore y_1 = \cos x,\ \&\ y_2 = \sin x$

$$\therefore W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}.$$

$$= \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$= \cos^2 x + \sin^2 x$

$= 1.$

now $$P.I = -y_1 \int \frac{y_2 X}{W} + y_2 \int \frac{y_1 X}{W}$$

$$= -\cos x \int \sin x \cdot \operatorname{cosec} x\, dx + \sin x \int \cos x \cdot \operatorname{cosec} x\, dx$$

$$= -\cos x \int \sin x \cdot \frac{1}{\sin x} dx + \sin x \int \cos x \cdot \frac{1}{\sin x} dx$$

$$= -\cos x \int dx + \sin \int \cot x\, dx$$

$$= -x \cos x + \sin x \cdot \log \sin x.$$

$\therefore$ G.S. $= (c_1 \cos x + c_2 \sin x) - x \cos x + \sin x \cdot \log \sin x.$

Q.7. Using method of variation of parameters, solve the diff. eq. $y'' - 6y' + 9y = e^{3x}/x^2$

$\therefore \quad D^2y - 6D + 9y = e^{3x}/x^2$

A.E. $(D^2 - 6D + 9)y = e^{3x}/x^2$

$\therefore D^2 - 6D + 9 = 0$

$\therefore D = 3, 3$

$\therefore C.F. = (C_1 + C_2 x) e^{3x}$

Now P.I. $y_1 = e^{3x}$ and $y_2 = xe^{3x}$

$$\therefore \quad W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & x \cdot 3e^{3x} + e^{3x} \end{vmatrix}$$

$$= e^{3x}(3xe^{3x} + e^{3x}) - 3xe^{6x}$$

$$= 3xe^{6x} + e^{6x} - 3xe^{6x}$$

$$= e^{6x}$$

Now $P.I. = -y_1 \int \frac{y_2 X}{W} + y_2 \int \frac{y_1 X}{W}$

$$= -e^{3x} \int \frac{x e^{3x} \cdot e^{3x}}{e^{6x} x^2} dx + xe^{3x} \int \frac{e^{3x} \cdot e^{3x}}{e^{6x} \cdot x^2} dx$$

$$= -e^{3x} \int \frac{1}{x} dx + xe^{3x} \int \frac{1}{x^2} dx$$

$$= -e^{3x} \log x + xe^{3x} \left[\frac{x^{-2+1}}{-1} \right].$$

$$= -e^{3x} \log x - xe^{3x} \, 1/x$$

$= \cancel{e^{3x} \log x}$

$= -e^{3x} \log x - e^{3x}$

$= -e^{3x} (\log x + 1).$

$\therefore$ G.S. $= (c_1 + c_2 x) e^{3x} - e^{3x} (\log x + 1).$

Q:8 Using variation of parameter solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$

$D^2 y + 4y = \tan 2x$

$\therefore$ A.E $(D^2 + 4) = 0.$

$\therefore D = \pm 2i$

$\therefore$ C.F. $= (c_1 \cos 2x + c_2 \sin 2x).$

$\therefore y_1 = \cos 2x$ and $y_2 = \sin 2x.$

$$\therefore W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}.$$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}.$$

$= 2\cos^2 2x + 2\sin^2 2x$

$= 2.$

$$P.I. = -y_1 \int \frac{y_2 X}{W} dx + y_2 \int \frac{y_1 X}{W} dx$$

$$= -\cos 2x \int \frac{\sin 2x \cdot \tan 2x}{2} dx + \sin 2x \int \frac{\cos 2x \cdot \tan 2x}{2} dx$$

$$= -\cos 2x \, \frac{1}{2} \int \sin 2x \cdot \frac{\sin 2x}{\cos 2x} + \sin 2x \, \frac{1}{2} \int \cos 2x \cdot \frac{\sin 2x}{\cos 2x} dx$$

$$= -\frac{\cos 2x}{2} \int \frac{\sin^2 2x}{\cos 2x} dx + \frac{\sin 2x}{2} \int \sin 2x \, dx.$$

$$= -\frac{\cos 2x}{2}\int \frac{(1-\cos^2 2x)}{\cos 2x}\,dx + \frac{\sin 2x}{2}\int \sin 2x\, dx$$

$$= -\frac{\cos 2x}{2}\left[\int \frac{1}{\cos 2x}\,dx - \int \cos 2x\,dx\right] + \frac{\sin 2x}{2}\int \sin 2x\, dx.$$

$$= -\frac{\cos 2x}{2}\left[\int \sec 2x\,dx - \int \cos 2x\,dx\right] + \frac{\sin 2x}{2}\int \sin 2x\, dx.$$

$$= -\frac{\cos 2x}{2}\left[\frac{\log(\sec 2x + \tan 2x)}{2} - \frac{\sin 2x}{2}\right] + \frac{\sin 2x}{2}\left[-\frac{\cos 2x}{2}\right.$$

$$= -\frac{1}{4}\cos 2x\left[\log(\sec 2x + \tan 2x)\right] + \cancel{\frac{1}{4}\cos 2x \sin 2x}$$

$$\cancel{-\frac{1}{4}\sin 2x \cos 2x}.$$

$$= -\frac{1}{4}\cos 2x \log(\sec 2x + \tan 2x).$$

$$\therefore G.S. = C_1\cos 2x + C_2\sin 2x - \frac{1}{4}\cos 2x \log(\sec 2x + \tan 2x)$$

Q:10. Using the method of variation of parameter solve the differential equation $y'' + y = \sec x$.

$$\therefore D^2 y + y = \sec x.$$

A.E. $(D^2+1)\,y = \sec x$

$$\therefore D = \pm i.$$

$$C.F. = (C_1\cos x + C_2 \sin x).$$

$$\therefore y_1 = \cos x \text{ and } y_2 = \sin x$$

$$\therefore W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}.$$

$$= \cos^2 x + \sin^2 x$$

$$= 1.$$

$$P.I. = -y_1 \int \frac{y_2 X}{W} dx + y_2 \int \frac{y_1 X}{W} dx$$

$$= -\cos x \int \sin x \cdot \sec x \, dx + \sin x \int \cos x \sec x \, dx$$

$$= -\cos x \int \tan x \, dx + \sin x \int dx$$

$$= -\cos x(-\log \cos x) + \sin x \cdot x$$

$$= \cos x \log \cos x + x \sin x.$$

$$\therefore G.S. = C_1 \cos x + C_2 \sin x + \cos x \log \cos x + x \sin x.$$