

# GUJARAT TECHNOLOGICAL UNIVERSITY

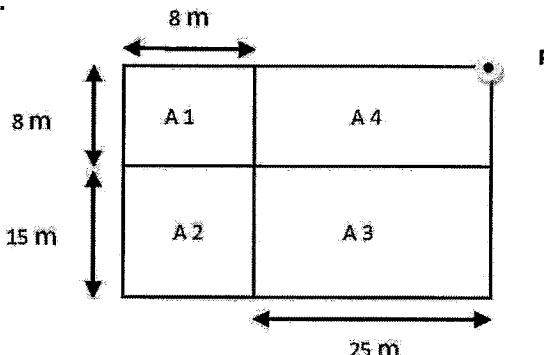
4<sup>th</sup> Semester Civil Engineering – PDDC

**Subject Code & Name :** X40603 - Soil Engineering

## Tutorial – 2

**Date : 05-04-2015**

1. Determine the equation for concentrated load by Boussinesq Equation. (With assumptions).
2. Determine the equation for concentrated load by Westergaard's Analysis. (With assumptions).
3. What is pressure Bulb (Isobar)? Explain the Newmark's influence chart.
4. A water tank supported by a ring foundation having outer diameter of 12 m and inner diameter of 10 m. The ring foundation transmits uniform load intensity of 160 kN/m<sup>2</sup>. Compute the vertical stress induced at a depth of 4 m, below the centre of ring foundation, using (a) Boussinesq Equation and (b) Westergaard's Equation, taking  $\mu = 0$
5. A building in plan exerts a pressure of 140 kN/m<sup>2</sup> on soil. Determine the vertical stress at a depth of 5 m below the outer corner P.



6. What is Earth pressure? Explain the active and passive earth state of plastic equilibrium.
7. Explain the Rankine's theory for active earth pressure for dry backfill with no surcharge.
8. Compute the intensities of active and passive earth pressure at depth of 8 m in dry cohesionless sand with angle of internal friction of 30° and unit weight of 18kN/m<sup>3</sup>. What will be the intensities of active and passive earth pressure if the water level rises to the ground level? Take saturated unit weight of sand as 22kN/m<sup>3</sup>.
9. A retaining wall 4 m high, has a smooth vertical back. The backfill has a horizontal surface in level with the top of the wall. There is uniformly distributed surcharge load of 36kN/m<sup>2</sup> intensity over the backfill. The unit weight of the backfill is 18kN/m<sup>3</sup>; its angle of internal friction is 30° and cohesion is zero. Determine the magnitude and point of application of active pressure per meter length of the wall.
10. Explain the Swedish slip circle method for slope stability.
11. Explain the Friction circle method for slope stability.

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1. Determine the equation for concentrated load by Boussinesq Equation.

Boussinesq [1885] gave the theoretical solutions for the stress distribution in an elastic medium subjected to a concentrated load on its surface.

The following assumptions are made:

- 1) The soil mass is an elastic medium for which elasticity  $E$  is constant.
- 2) The soil is homogeneous, i.e. all its constituents parts or elements are similar and it has identical properties at different points.
- 3) The soil is isotropic, i.e. has identical properties in all directions.
- 4) The soil mass is semi-infinite, i.e. it extends to infinity in the down-ward direction and lateral directions.
- 5) The self weight of soil is neglected.
- 6) The soil is initially stress free.
- 7) The change in volume of the soil upon application of the load on it is neglected.



- 8) The top surface of the medium is free of shear stress.
- 9) The continuity of stress is considered to exist in the medium.

[Note: The stresses due to self weight are computed separately as explained in the preceding section.]

Let a vertical point load  $\sigma$  be acting at the soil surface at a point O which is taken as the origin of the x, y and z axes as shown in Fig 3.2.

Let P (x, y, z) be the point in the soil mass where vertical and horizontal stresses are to be determined due to applied load  $\sigma$  on the ground surface.

Boussinesq proved that polar radial stress ( $\sigma_r$ ) at point P (x, y, z) is given by

$$\text{Ans} \quad \sigma_r = \frac{3}{2} \frac{\sigma}{\pi} \frac{\cos \beta}{r^2}$$

where,

$r$  = Polar distance between the origin O and point P



$\beta$  = angle which the line of makes with the vertical

Obviously,

$$R = \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2}$$

where,

$$r^2 = x^2 + y^2$$

$$\sin \beta = \frac{r}{R} \quad \text{and} \quad \cos \beta = \frac{z}{R}$$

The vertical stress ( $\sigma_z$ ) at point P is given by

$$\sigma_z = \sigma_R \cos^2 \beta$$

$$\text{or } \sigma_z = \frac{3Q}{2\pi} \cdot \frac{\cos^3 \beta}{R^2}$$

$$\text{or } \sigma_z = \frac{3Q}{2\pi} \cdot \frac{(z/R)^3}{R^2} = \frac{3Q}{2\pi} \cdot \frac{z^3}{R^5}$$

$$\text{or } \sigma_z = \frac{3Q}{2\pi} \cdot \frac{1}{z^2} \cdot \frac{z^5}{R^5}$$

$$\sigma_z = \frac{3Q}{2\pi} \cdot \frac{1}{z^2} \cdot \frac{z^5}{(r^2 + z^2)^{1/2 \cdot 5}}$$

$$\therefore R = \sqrt{r^2 + z^2}^{1/2}$$



$$\sigma_z = \frac{3Q}{2\pi} \cdot \frac{1}{z^2} \cdot \left[ \frac{1}{1 + (\sigma/z)^2} \right] \quad S_{12}$$

$$\text{Our } \sigma_z = I_B = \frac{\alpha}{z^2}$$

where,

$$I_B = \frac{3}{2\pi} \left[ \frac{1}{1 + (\sigma/z)^2} \right] \quad S_{12}$$

Where  $I_B$  is called the Boussinesq influence coefficient for the vertical stress.



2 Determine the equation for concentrated load by Westergaard's Analysis

Westergaard (1938) also solved the problem of pressure distribution in soil under point load. He assumed that there are thin sheets of rigid materials sand-wiched in a homogeneous soil mass. The thin sheets are closely spaced and are of negligible thickness and infinite rigidity, which permits only downward displacement of the soil mass as a whole without allowing it to undergo any lateral strain.

Boussinesq solution assumes that the soil mass is isotropic, but, there are generally thin layers of sand embedded in homogeneous clay strata which accentuates the non-isotropic condition. Therefore, Westergaard's solution represents more closely the actual field conditions.

According to Westergaard, the vertical stress  $\sigma_z$  at a point P at a depth  $z$  below the point load Q is given by



$$\sigma_z = \frac{1}{2\pi} \sqrt{\frac{1-2\mu}{2-2\mu}} \cdot \frac{\alpha}{z^2} \left[ \left( \frac{1-2\mu}{2-2\mu} \right) + \left( \frac{\sigma_z}{\alpha} \right)^2 \right]^{3/2}$$

For elastic materials the value of Poisson's ratio  $\mu$  ranges from 0 to 0.5. However, for large lateral restraint the value of  $\mu$  may be taken as zero.

Thus for the case of  $\mu = 0$ ,  $\sigma_z$  is given by

$$\sigma_z = \frac{1}{\pi} \left[ \left( \frac{1}{1+2(\sigma_{12})^2} \right)^{3/2} \right] \cdot \frac{\alpha}{z^2}$$

$$\therefore \sigma_z = I_w \cdot \frac{\alpha}{z^2}$$

Where,

$$I_w = \frac{1}{\pi} \left[ \frac{1}{1+2(\sigma_{12})^2} \right]^{3/2}$$

Where,

$I_w$  is known as Westergaard's influence factor. Similar to Boussinesq's influence factor, Westergaard's influence factor is also dimensionless and is a function of the ratio.



3. What is pressure bulb (Isobar)? Explain the Newmark's influence chart.

As Isobar is a curve or contours connecting all the points below the ground surface of equal vertical pressure. In other words, an Isobar is a contour of equal vertical stresses. As Isobar is a spatial curved surface of the shape of an electric bulb. The curved surface is symmetrical about the vertical axis passing through the load point.

The vertical pressure on a horizontal plane is the same in all directions at points located at equal radial distance around the axis of loading. The zone in a loaded soil mass bounded by an Isobar of given vertical pressure intensity is called pressure bulb. The vertical pressure at every point on the surface pressure bulb is the same.

A vertical normal stresses at points inside the pressure bulb are greater than that at a point on the surface of the pressure bulb and those at points outside the pressure bulb are smaller than that value. A number of pressure bulbs or Isobars



may be drawn for any applied load. A system of isobars indicate the decrease in stress intensity from the inner to the outer ones.

The procedure for plotting an isobar is as follows:

Suppose an isobar of  $\sigma_z = 0.20 \text{ Q}$  ( $20\%$  of  $\text{Q}$ ) per unit area is to be plotted from equation (3.5),

$$\sigma_z = I_B \cdot \frac{\text{Q}}{z^2}$$

$$\therefore I_B = \frac{\sigma_z \cdot z^2}{\text{Q}} = \frac{0.20 \times z^2}{\text{Q}} = 0.2 z^2$$

Assuming various value of  $z$ , the corresponding values of  $I_B$  are computed from the above equation. For these values of  $I_B$ , the corresponding  $\sigma_z$  values are obtained from Table 3.1 and for assumed values of  $z$ , the values of  $\sigma_z$  are computed. Thus the co-ordinates  $(\sigma_z, z)$  of a number of points where  $\sigma_z = 0.2 \text{ Q}$  per unit area are obtained. The calculations may be performed in a tabular form as shown in Table 3.2



The Newmark's Influence Chart is useful for the determination of Vertical Stress ( $\sigma_z$ ) at any point below the uniformly loaded area of any shape. This method is based on the concept of the vertical stress at a point below the centre of a uniformly loaded circular area.

A chart, consisting of number of circles and radiating lines, is so prepared that the influence of each area unit (formed in the shape of a sector between two concentric circles and two adjacent radial lines) is the same at the centre of the circle, i.e. each area unit causes the equal vertical stress at the centre of the circle.

Consider a uniformly loaded circular area of radius  $r_1$ , divide it into 20 equal sectors (area units) as shown in Fig. If  $q$  is the intensity of loading and  $\sigma_z$  is the vertical stress at any point  $P$  at a depth  $z$  below the centre of the loaded area, then due to each area unit such as  $OAB$ , the vertical stress at point  $P$  is given by equation 3.19 as,

$$\frac{\sigma_z}{20} = \frac{q}{20} \left[ 1 - \left\{ \frac{1}{1 + \left( \frac{r_1}{l_2} \right)^2} \right\} \right]$$



If right hand side of equation 3.30 is given an arbitrary fixed value, say  $0.005q$ , equation 3.30 becomes

$$0.005q = \frac{q}{20} \left[ 1 - \left\{ \frac{1}{1 + (\sigma_1/z)^2} \right\}^{3/2} \right]$$

Solving this equation, we get

$$\frac{\sigma_1}{z} = 0.270$$

Thus every one-twentieth sector of the circle with a radius  $\sigma_1 = 0.270z$ , would give a vertical stress equal to  $0.005q$  at its centre. The arbitrary fixed function  $0.005$  is called the influence value.



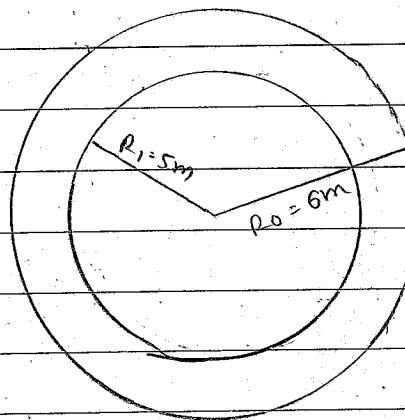
4) A water tank supported by a ring foundation having outer diameter of 12m and inner diameter of 10m. The ring foundation transmits uniform load intensity of 160 kN/m<sup>2</sup>. Compute the vertical stress induced at a depth of 4m, below the centre of ring foundation using (a) Boussineq's equation and (b) Westergaard's equation, taking  $\mu = 0$

A Boussineq equation theory

$$\text{inner radius } R_1 = 10/2 = 5\text{m}$$

$$\text{outer radius } R_2 = 12/2 = 6\text{m}$$

$$z = 4\text{m}$$



$$\therefore 6z = q \left[ 1 - \left\{ \frac{1}{1 + (R/z)^2} \right\}^{3/2} \right]$$

$$= 160 \left[ 1 - \left[ \frac{1}{1 + (6/4)^2} \right]^{3/2} \right] - 160 \left[ 1 - \left[ \frac{1}{1 + (5/4)^2} \right]^{3/2} \right]$$

$$= 160 [1 - 0.170] - 160 [1 - 0.243]$$



$$= 160 \times 0.829 - 160 \times 0.756 \\ = 11.68 \text{ KN/m}^2$$

[b] westergaard's analysis.

$$G_0Z = q = \left[ 1 - \left[ \frac{1}{1 + 2\sigma \left( \frac{R}{z} \right)^2} \right]^{1/2} \right]$$

$$G_Z = 160 \left[ 1 - \left[ \frac{1}{1 + Z \times \left( \frac{6}{14} \right)^2} \right]^{1/2} \right] - 160 \left[ 1 - \frac{1}{1 + 2 \left( \frac{6}{14} \right)^2} \right]$$

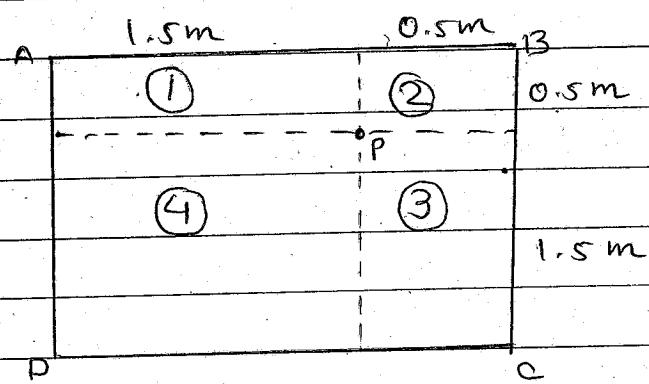
$$= 160 [1 - 0.426] - 160 [1 - 0.492]$$

$$= 91.77 - 81.28$$

$$= 10.49 \text{ KN/m}^2$$



5) A square footing  $2m \times 2m$  carries a uniformly distributed load of  $314 \text{ kN/m}^2$ . Find the intensity of vertical pressure at a depth of  $6\text{m}$  below a point  $0.5\text{ m}$  inside each of the two adjacent sides of footing.



The loaded area is divided into four parts as shown in the figure, such that point p forms a corner of each part.

using Boussinesq theory,

$$\sigma_z = q \left[ \frac{2mn\sqrt{m^2+n^2+1}}{4\pi(m^2+n^2+1+m^2n^2)} \right] \times \frac{m^2+n^2+2}{m^2+n^2+1}$$

$$+ \tan^{-1} \left[ \frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1-m^2n^2} \right]$$

$$= 9 = 314 \text{ kN/m}^2$$

$$z = 6\text{m}$$



For Area (i)

$$m = \frac{B}{Z} = \frac{1.5}{6} = 0.25$$

$$n = \frac{L}{Z} = \frac{0.5}{6} = 0.083$$

$$\therefore \sigma_{z1} = \frac{314}{4\pi} \left[ \frac{2 \times 0.25 \times 0.083 \sqrt{0.25^2 + 0.083^2 + 1}}{0.25^2 + 0.083^2 + 1 + 0.25^2 \times 0.083^2} \right]$$

$$\times \frac{0.25^2 + 0.083^2 + 2}{0.25^2 + 0.083^2 + 1}$$

$$+ \tan^{-1} \left[ \frac{2 \times 0.25 \times 0.083 \sqrt{0.25^2 + 0.083^2 + 1}}{0.25^2 + 0.083^2 + 1 - 0.25^2 \times 0.083^2} \right]$$

$$= \frac{314}{4\pi} \left[ \frac{0.0415 \sqrt{1.069}}{1.069 + 4.3 \times 10^{-4}} \times \frac{2.069}{1.069} \right]$$

$$+ \tan^{-1} \left[ \frac{0.0415 \sqrt{1.069}}{1.069 - 4.3 \times 10^{-4}} \right]$$

$$= \frac{314}{4\pi} \left[ 0.0776 + 2.29 \times \frac{\pi}{180} \right]$$

$$= 2.94 \text{ kN/m}^2$$



Four cuted (c2):

$$m = \frac{B}{2} = \frac{0.5}{6} = 0.083$$

$$n = \frac{L}{2} = \frac{0.5}{6} = 0.083$$

$$\sigma_{z2} = \frac{314}{4\pi} \left[ \frac{2 \times 0.083 \times 0.083 \sqrt{0.083^2 + 0.083^2 + 1}}{0.083^2 + 0.083^2 + 1 + 0.083^2 \times 0.083^2} \right]$$

$$\times \frac{0.083^2 + 0.083^2 + 2}{0.083^2 + 0.083^2 + 1} + \tan^{-1}$$

$$\left\{ \frac{2 \times 0.083 \times 0.083 \sqrt{0.083^2 + 0.083^2 + 1}}{0.083^2 + 0.083^2 + 1 - 0.083^2 \times 0.083^2} \right\}$$

$$= \frac{314}{4\pi} \left[ \frac{0.0138 \sqrt{1.014} \times 2.014}{1.014 + 0.0138} \right] \times \frac{1.014}{1.014 - 0.0138}$$

$$+ \tan^{-1} \left[ \frac{0.0138 \sqrt{1.014}}{1.014 - 0.0138} \right]$$

$$= \frac{314}{4\pi} \left[ \frac{0.0268 + 0.795 \times \frac{\pi}{180}}{1.014 - 0.0138} \right]$$

$$= 1.02 \text{ kN/m}^2$$



Four areas (3)

$$\therefore G_{z3} = 2.94 \text{ kN/m}^2 \quad \text{--- same as area 1)}$$

Four areas (4)

$$m = \frac{B}{Z} = \frac{1.5}{6} = 0.25$$

$$m = \frac{L}{Z} = \frac{1.5}{6} = 0.25$$

$$\therefore G_{z4} = \frac{314}{4\pi} \left[ \frac{2 \times 0.25 \times 0.25 \sqrt{0.25^2 + 0.25^2 + 1}}{0.25^2 + 0.25^2 + 1 + 0.25^2 \times 0.25^2} \right]$$

$$\times \frac{0.25^2 + 0.25^2 + 2}{0.25^2 + 0.25^2 + 1}$$

$$+ \tan^{-1} \left\{ \frac{2 \times 0.25 \times 0.25 \sqrt{0.25^2 + 0.25^2 + 1}}{0.25^2 + 0.25^2 + 1 - 0.25^2 \times 0.25^2} \right\}$$

$$= \frac{314}{4\pi} \left[ \frac{0.125 \sqrt{1.125}}{1.125 + 0.0039} \times \frac{2.215}{1.125} \right]$$

$$+ \tan^{-1} \left\{ \frac{0.125 \sqrt{1.125}}{1.125 - 0.0039} \right\}$$



$$= \frac{314}{4\pi} \left[ 0.2218 + 6.744 \times \frac{\pi}{180} \right]$$

$$= 8.48 \text{ KN/m}^2$$

$$\therefore \sigma_z = \sigma_{z_1} + \sigma_{z_2} + \sigma_{z_3} + \sigma_{z_4}$$

$$= 2.94 + 1.02 + 2.94 + 8.48$$

$$= 15.38 \text{ KN/m}^2$$



6. What is earth pressure? Explain the active and passive earth state of plastic equilibrium.

The pressure exerted by earth backfill on the back of the wall is called lateral earth pressure.

At the top of the wall earth pressure is zero.

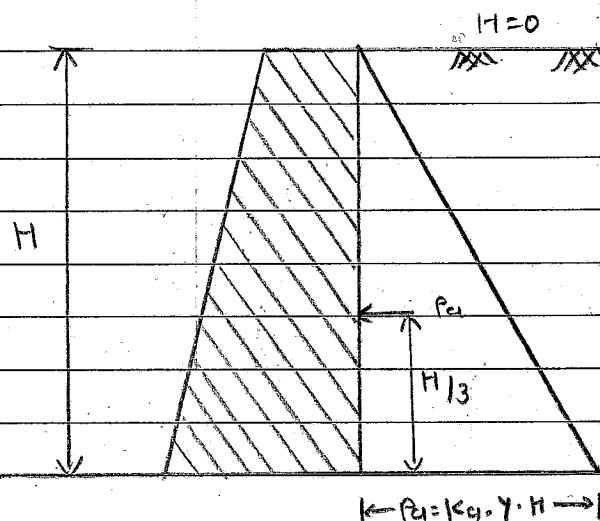
At the base of the wall pressure acting on the wall is,

$$P_{c1} = K_a \cdot v \cdot H$$

Total force acting on the back of the wall is

$$P_u = 1/2 \cdot K_a \cdot v \cdot H^2$$

This force is equal to the area of the pressure triangle. This total force acts at height  $H/3$  from the base.





As per Rankine theory (1860), the lateral earth pressure depends upon the following forces:

- mode of movement of the wall
- The flexibility of the wall
- The properties of soil
- the drainage conditions.

### Plastic equilibrium in soils:

A soil mass is said to be in a state of plastic equilibrium if every point of it is on the verge of failure. Rankine theory considers the stress in a soil mass when it reaches a state of plastic equilibrium, that is, when shear failure is imminent at every point within a soil mass.

### Active earth pressure:

When the retaining wall moves away from the back fill, earth pressure on the back of the wall decreases and becomes minimum. With further movement of wall the pressure does not decrease. This minimum pressure is known as active earth pressure.



A state of active earth pressure occurs when the soil mass yields in such a way that it tends to stretch horizontally. It is a state of plastic equilibrium as the entire soil mass is on the verge of failure.

### Passive earth pressure:

When the retaining wall moves towards the backfill, earth pressure on the back of the wall increases and becomes maximum. With further movement of the wall the pressure does not increases. The maximum pressure is known as passive earth pressure.

A state of passive earth pressure exists when the movement of the wall is such that the soil tends to compress horizontally.



- Q-7 Explain the Rankine's theory for active earth pressure for dry backfill with no surcharge.

Rankine (1857) considered the equilibrium of a soil element within a soil mass bounded by a plane surface. The following assumptions were made by Rankine for the derivation of earth pressure.

- 1> The soil mass is homogeneous and semi-infinite
- 2> The soil is dry and cohesionless
- 3> The ground surface is plane, which may be horizontal or inclined
- 4> The back of the retaining wall is smooth and vertical
- 5> The soil element is in a state of plastic equilibrium, i.e. at the verge of failure.

→ Active Earth pressure by Rankine Theory (For cohesionless soils)

Now shall consider the following cases of cohesionless backfill:

- 1> Dry or moist backfill with no surcharge.



- 2) Submerged backfill
- 3) Backfill with uniform surcharge
- 4) Backfill with sloping surcharge
- 5) Inclined back and surcharge.
6. Dry or moist backfill with no surcharge:

Pressure at the base of the wall,

$$\begin{aligned}
 P_u &= k_a \cdot r \cdot H & k_a &= \frac{1 - \sin \phi}{1 + \sin \phi} \\
 &&&= \tan^2 \left[ 45 - \frac{\phi}{2} \right] \\
 &&&= \cot^2 \alpha
 \end{aligned}$$

$$\therefore \text{cost } d = N k_a$$

Total pressure acting on the wall  
= Area of pressure triangle

$$P_u = \frac{1}{2} k_a \cdot r \cdot H^2$$

This pressure acts at  $H/3$  above the base of the wall.

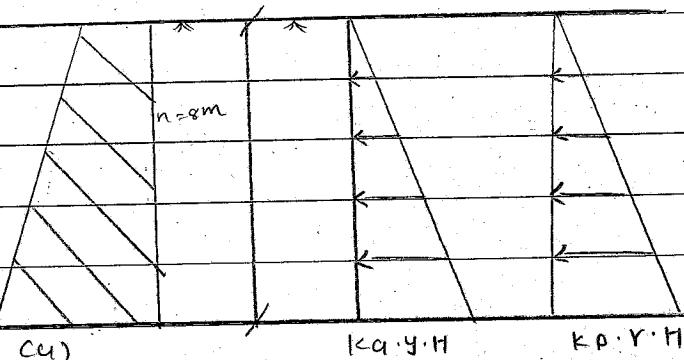
If the soil is dry,  $r$  is the dry weight of soil.

If the soil is moist,  $r$  is the moist weight of soil.



2) 8. Compute the intensities of active and passive earth pressure at depth of 8m in dry cohesionless sand with angle of internal friction of  $30^\circ$  and unit weight of  $18 \text{ kN/m}^3$ . What will be the intensities of active and passive earth pressure if the water level rises to the ground level? Take saturated unit weight of sand as  $22 \text{ kN/m}^3$

(a) When soil is dry



$$\gamma = 18 \text{ kN/m}^3 \quad \textcircled{a} \text{ active case}$$

$$H = 8 \text{ m} \quad \phi = 30^\circ$$

(c) Passive case

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = \frac{1}{3}$$

$$\therefore K_p = \frac{1}{K_a} = 3$$

$$P_a = K_a \cdot \gamma \cdot H = \frac{1}{3} \times 18 \times 8 = 48 \text{ kN/m}^2$$



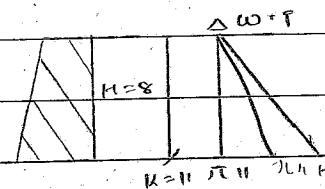
$$P_p = k_p \cdot Y \cdot H = 3 \times 18 \times 8 = 432 \text{ kN/m}^2$$

(b) When backfill is submerged (w-t at ground level)

$$Y = Y_{sat} - \gamma_w = 22 - 9.81 \\ = 12.19 \text{ kN/m}^3$$

$$\therefore P_a = k_a \cdot Y \cdot H + \gamma_w \cdot H = \frac{1}{3} \times 12.19 \times 8 + 9.81 \times 8 \\ = 111 \text{ kN/m}^2$$

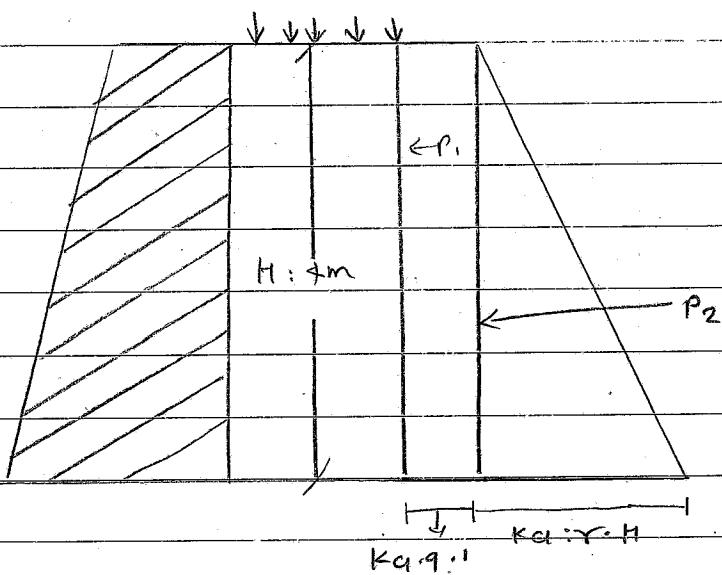
$$P_p = k_p \cdot Y \cdot H + \gamma_w \cdot H = (3 \times 12 - 19 \times 8) + \\ (9.81 \times 8) \\ = 371 \text{ kN/m}^2$$





9) A retaining wall 4 m high, has a smooth vertical back. The backfill has a horizontal surface in level with the top of the wall. There is uniformly distributed surcharge load of  $36 \text{ kN/m}^2$  intensity over the backfill. The unit weight of the backfill is  $18 \text{ kN/m}^3$ ; its angle of internal friction is  $30^\circ$  and cohesion is zero. Determine the magnitude and point of application of active pressure per meter length of the wall.

$$q = 36 \text{ kN/m}^2$$



Data

$$H = 4\text{m}$$

$$q = 36 \text{ kN/m}^2$$

$$\gamma = 18 \text{ kN/m}^3$$

$$\phi = 30^\circ$$

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1 - \frac{1}{12}}{1 + \frac{1}{12}}$$



$$\frac{0.5}{1.5} = 3.33$$

①  $P_1$  = pressure due to surcharge

$$P_1 = K_a \cdot q = 3.33 \times 36 \\ = 119.88 \text{ kN/m}^2$$

$$\therefore P_1 = 119.88 \times a = 479.52 \text{ kN/m} @ 412 = 2 \text{ m}$$

from the base

②  $P_2$  = pressure due to backfill

$$= K_a \cdot Y \cdot H \\ = 3.33 \times 18 \times 6 \\ = 359.64 \text{ kN/m}^2$$

$$\therefore P_2 = \frac{1}{2} \times 359.64 \times 6 \\ = 1078.92 \text{ kN/m}^2$$

$$\therefore 1078.92 \text{ kN/m}^2 \text{ acting at } 413 \\ = 1.33 \text{ m from the base}$$

③ Total pressure  $p = P_1 + P_2$

$$= 479.52 + 1078.92 \\ = 1558.44 \text{ kN/m}$$



$$\therefore \text{point of application} = \frac{P_1 Z_1 + P_2 Z_2}{P}$$

$$= \frac{(479.52 \times 2) + (1078.92 \times 1.33)}{1558.44}$$

$$= \frac{959.04 + 1434.96}{1558.44}$$

= 1.53m above the base



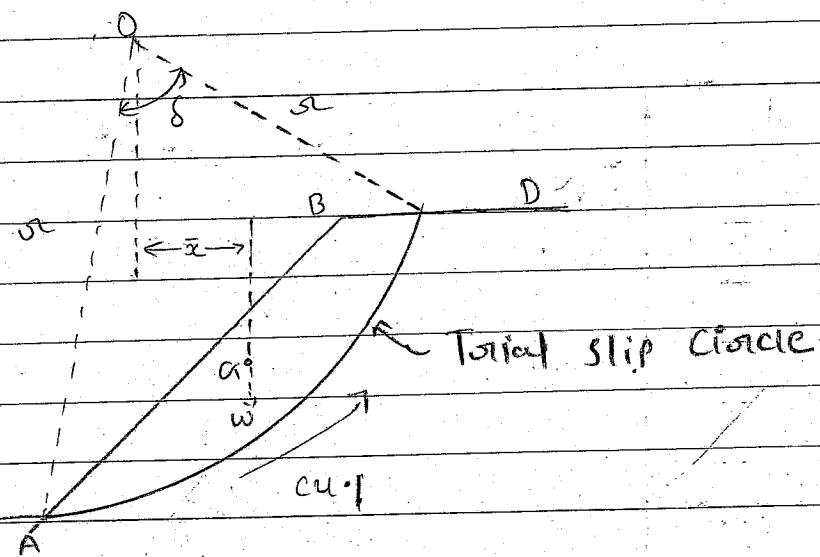
- 10) Explain the swedish slip circle method for slope stability.

The method developed by Swedish engineers, assumes that the sliding surface or the slipping surface is an arc of a circle.

The following two cases are considered:

- 1) Analysis of purely cohesive soil ( $c \neq 0$ ,  $\phi = 0$  soil)
- 2) Analysis of a cohesive frictional soil ( $c \neq 0$ ,  $\phi \neq 0$  soil)

- (i)  $\phi = 0$  Analysis (purely cohesive soil):



Shows a finite slope AB, the stability of which is to be analysed. The method consists of assuming a number



of trial slip circles, and finding the factor of safety of each. The circle corresponding to the minimum factor of safety is the critical slip circle.

Let AD be a trial slip circle, with  $r$  as the radius and  $O$  as the centre of rotation. Let  $w$  be the weight of the soil of the wedge ABDA of unit thickness acting through the centroid O. The driving moment  $M_D$  will be equal to  $w \bar{r}c$ , where  $\bar{r}c$  is the distance of line of action of  $w$  from the vertical line passing through the centre of rotation O.

If  $c_u$  is the unit cohesion, and  $l$  is the length of the slip circle AD, the shear resistance developed along the slip surface will be equal to  $c_u \cdot l$ , which act at a radial distance  $r$  from the centre of rotation O.

Hence the resisting moment  $M_R$  will be equal to  $c_u \cdot l \cdot r$ .

The length of the slip surface AD is given by.



$$l = \frac{2\pi r\delta}{360^\circ}$$

where,

$\delta$  = angle subtended by the arc of slip circle at the centre of rotation  $O$ .

Driving moment,  $M_D = W \bar{sc}$

Resisting moment  $M_R = C_u \cdot l \cdot r$

The factor of safety  $F$  is then given by

$$F = \frac{M_R}{M_D} = \frac{C_u \cdot l \cdot r}{W \bar{sc}}$$

Alternatively :

Let,  $C_m$  = mobilised shear resistance of soil ( $\phi = 0$ ) necessary for equilibrium

$$\text{Then } W \bar{sc} = C_m \cdot l \cdot r$$

$$\therefore C_m = \frac{W \bar{sc}}{l \cdot r}$$

$$\text{Hence, } F = \frac{C_u}{C_m} = \frac{C_u \cdot l \cdot r}{W \bar{sc}}$$



The distance  $\bar{x}$  of the centroid of the wedge from the centre of rotation O can be determined by dividing the wedge into a number of vertical slices and dividing the algebraic sum of moment of weight of each slice about the vertical line passing through O, by the weight of the wedge.



ii. Explain the friction circle method for slope stability.

The Friction circle method is useful for the stability analysis of slopes made of homogeneous soils. This method also assumes the failure surface as the circumference of a circle. Fig 1.21. shows a failure circle AD of radius  $r$  with O as the centre of rotation. If a small concentric circle is drawn with O as the centre and  $r \sin \phi$  as the radius, any line EF tangential to this smaller circle will cut the failure circle AD at an obliquity  $\phi$ . Conversely, any reactor representing reaction AR cut an obliquity  $\phi$  to the failure circle ~~AD~~ AD will be tangential to the small circle. This small circle of radius  $r \sin \phi$  is called Friction circle or  $\phi$ -circle.

The forces acting on the sliding wedge ABDA are:

i) weight W of the wedge

ii) total frictional resistance R

iii) Total cohesive resistance  $C_m L$  mobilised along the slip surface.



In the slip circle, the slip circle is divided into elementary arcs of length  $\Delta l$ , the elementary reaction  $\Delta R$  of each arc, acting at an obliquity  $\phi$  to the normal to the arc, will be tangential to the friction circle. Thus, as shown in Fig. the elementary reaction  $\Delta R$  at points E and E<sub>1</sub> are tangential to the friction circle at point F and F<sub>1</sub> respectively. Also, the point O of intersection of EF and E<sub>1</sub>F<sub>1</sub> will thus pass through O<sub>1</sub> and hence will not be tangential to the friction circle. If R is the reaction (Fig. 1.21-b), at an obliquity  $\phi$  to the normal direction, it will miss the friction circle by a small margin; in fact it will be tangential to another circle shown dotted, concentric to the friction circle, and of radius  $k \sin \phi$ , where k is a factor depending upon the central angle  $\delta$  of the slip circle.