

Assignment-1, Fourier Integral and Fourier Transform

(Based on Fourier Integral)

1. Find the (a) Fourier Cosine integral and (b) Fourier sine integral representation of $f(x) = \begin{cases} \sin x, & 0 \leq x, \leq \pi \\ 0, & x > \pi \end{cases}$.
2. Using Fourier integral representation, show that $\int_0^\infty \frac{\cos x\lambda + \lambda \sin x\lambda}{1 + \lambda^2} d\lambda = \begin{cases} 0, & x < 0 \\ \frac{\pi}{2}, & x = 0 \\ \pi e^{-x}, & x > 0 \end{cases}$.
3. Using Fourier sine integral show that $\int_0^\infty \frac{1 - \cos \pi\lambda}{\lambda} \sin x\lambda d\lambda = \begin{cases} \frac{\pi}{2}, & 0 < x < \pi \\ 0, & x > 0 \end{cases}$.
4. Find the Fourier integral representation of the function $f(x) = \begin{cases} 2, & |x| < 2 \\ 0, & |x| > 2 \end{cases}$.
5. Prove that $\int_0^\infty \frac{\cos x\lambda}{1 + \lambda^2} d\lambda = \frac{\pi}{2} e^{-x}; \quad x \geq 0$.
6. Find the (a) Fourier Cosine integral and (b) Fourier sine integral representation of $f(x) = \begin{cases} e^x; & 0 < x < 1 \\ 0, & x > 1 \end{cases}$.
7. Find the Fourier integral representation of $f(x) = \begin{cases} \cos x; & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0, & |x| > \frac{\pi}{2} \end{cases}$.
8. Find the Fourier cosine integral of $f(x) = \begin{cases} x; & 0 < x < a \\ 0, & |x| > a \end{cases}$.

(Based on Fourier Transform)

1. Find the Fourier Transform of $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$.
2. Find the Fourier Transform of $f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \frac{x \cos x - \sin x}{x^3} \cos \frac{x}{2} dx$.
3. Find the Fourier transform of $e^{-a^2 x^2}$, $a > 0$. deduce that $e^{-x^2/2}$ self reciprocal in respect of Fourier transform.
4. Find the Fourier cosine transform of e^{-x^2} .
5. Find the Fourier sine transform of $e^{|x|}$. Hence show that $\int_0^\infty \frac{x \sin mx}{1 + x^2} dx = \frac{\pi e^{-m}}{2}$, $m > 0$.
6. Find the Fourier cosine transform of $f(x) = \begin{cases} x; & 0 < x < 1 \\ 2 - x; & 1 < x < 2 \\ 0; & x > 2 \end{cases}$.
7. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$.
8. Find the Fourier sine and cosine transform of $f(x) = \begin{cases} 1; & 0 < x < a \\ 0, & x > a \end{cases}$.
9. Find the Fourier transform of $f(x) = \begin{cases} x^2; & |x| < a \\ 0, & |x| > a \end{cases}$.

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Assignment-5, Beta and Gamma Functions)

1. Prove that $\Gamma(m+1) = m\Gamma(m)$. Also prove $\Gamma(m+1) = m!$ when m is positive integer.
2. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.
3. Prove that $\int_0^\infty e^{-x^2} dx = \sqrt{\pi} = \Gamma\left(\frac{1}{2}\right)$.
4. Evaluate following integrals using Beta and Gamma functions
 - (a) $\int_0^1 \left(\log \frac{1}{y}\right)^{n-1} dy$.
 - (b) $\int_0^\infty \frac{y^{q-1}}{(1+y)^{p+q}} dy$.
 - (c) $\int_0^1 \frac{x^{p-1} + x^{q-1}}{(1+x)^{p+q}} dx$.
 - (d) $\int_0^1 x^5 \left[\log \frac{1}{x}\right]^3 dx$.
 - (e) $\int_0^1 \frac{x}{\sqrt{1-x^5}} dx$.
 - (f) $\int_0^1 \frac{1}{\sqrt{1+x^4}} dx$.
 - (g) $\int_0^1 x^3 (1-\sqrt{x})^5 dx$.
 - (h) $\int_0^1 \frac{dx}{\sqrt{1-x^n}}$.
5. Prove that
 - (a) $\frac{\beta(m+1, n)}{m} = \frac{\beta(m, n+1)}{n} = \frac{\beta(m, n)}{m+n}$.
 - (b) $\beta(m+1, n) + \beta(m, n+1) = \beta(m, n)$.
 - (c) $\int_b^a (x-a)^m (b-x)^n dx = (b-a)^{m+n+1} \beta(m+1, n+1)$.
6. Compute $\beta(1.5, 2.5)$, $\beta\left(\frac{9}{2}, \frac{7}{2}\right)$.
7. Show that $\int_0^{\pi/2} \sqrt{\sin \theta} d\theta \times \int_0^{\pi/2} \frac{1}{\sqrt{\cos \theta}} d\theta = \pi$.

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Assignment-6, Z-Transform

1. Write Down all formula of Z -transform.
2. Find the Z -Transform of the following
 - (a) $3n - 4 \sin \frac{n\pi}{4} + 5a$.
 - (b) $(n + 1)^2$.
 - (c) $\sin(3n + 5)$.
 - (d) e^{an} .
 - (e) ne^{an} .
 - (f) $n^2 e^{an}$.
 - (g) $e^t \sin 2t$.
 - (h) $\cosh n\theta$.
 - (i) $a^n \cosh n\theta$.
 - (j) $\cosh \left(\frac{n\pi}{2} + \theta\right)$.
 - (k) $n \sin n\theta$.
3. If $U(z) = \frac{2z^2 + 5z + 14}{(z - 1)^4}$, evaluate u_2 and u_3 .
4. If $U(z) = \frac{2z^2 + 3z + 12}{(z - 1)^4}$, evaluate u_2 and u_3 .
5. Show that $Z\left(\frac{1}{n}\right) = z \log \frac{z}{z - 1}$.
6. Find $Z\left\{\frac{1}{n(1 + n)}\right\}$.

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Assignment-7, Higher Order Differential Equation

SR NO.	SOLVE THE EXAMPLE:
1	$y'' + 4y = 2\sin 3x$ $16y'' - 8y' + 5y = 0$
2	$y'' - 2y' + 5y = 5x^3 - 6x^2 + 6x$. $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 6x + 3x^2 - 6x^3$ $x^2 y'' - 4xy' + 6y = 0$ $y''' - y'' + 100y' - 100y = 0$ $\frac{d^4y}{dx^4} - 18\frac{d^2y}{dx^2} + 81y = 0$ $y = \frac{1}{(D+1)^2} \cosh x$, where $D = \frac{d}{dx}$ $y'' - 3y' + 2y = e^x$ $y'' + y = \sec x$ $y''' - 3y'' + 3y' - y = 4e^t$ Solve $(D^2 + a^2)y = \cos ec ax$ $(D^4 + 2a^2 D^2 + a^4)y = \cos ax$ $x^2 y'' - 4xy' + 6y = 21x^{-4}$
3	$y' + 6x^2 y = \frac{e^{-2x^3}}{x^2}$, where $y(1) = 0$ $y'' - 5y' + 6y = 0$ with initial condition $y(1) = e^2$ and $y'(1) = 3e^2$ $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x^2}$ $(x^2 D^2 - 3xD + 4)y = 0$, $y(1) = 0$, $y'(1) = 3$ $y''' - y'' + 100y' - 100y = 0$, $y(0) = 4$, $y'(0) = 11$, $y''(0) = -299$
4	$y'' + 3y'' + 3y' + y = 30e^{-x}$, $y(0) = 3$, $y'(0) = -3$, $y''(0) = -47$
5	$(x^2 D^2 - 3xD + 3)y = 3\ln x - 4$
6	$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$
7	Find general solution of $y'' + 9y = \sec 3x$ by method of variation of parameter.
8	Using the method of variation of parameters, solve $\frac{d^3y}{dx^3} + \frac{dy}{dx} = \cosec x$
9	Solve the nonhomogeneous Euler-Cauchy equation $x^3 y'' - 3x^2 y' + 6xy' - 6y = x^4 \log x$ by Variation of parameters method

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Assignment-8, Higher Order Differential Equation

SR NO.	EXAMPLES
1	<p>Solve $(D^2 + 3D + 2)y = x^2 + e^{-x}$.</p> <p>Solve : $(D^2 + 6D + 9)y = 0$</p>
2	<p>Solve $y'' + 9y = 3x^2$.</p> $(D^2 - 3D + 2)y = \cos 3x$ $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x$ $D^2y - a^2y = 0$ $(D^2 + 1)y = \sin x$ $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = e^{3x}$ $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4.$ $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4 \cos^2 x.$ $(D^2 + 5D + 6)y = e^x$ $(D^2 - 5D + 6)y = \sin 3x.$ $(D^2 + D)y = x^2 + 2x + 4$ $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 3y = x^2$
3	<p>Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$.</p> $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{-x} + \sin 2x.$ $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x} \sin 2x$ $(D^2 - 2D + 4)y = e^x \cos x.$
4	<p>Solve $x^2y'' - xy' + 4y = \cos(\log x) + x \sin(\log x)$.</p>

5	$(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 2 \sin(\log(1+x))$ $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \log x$ <p>Solve : $x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + 8y = 65 \cos(\log x).$</p> $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \log x .$ $(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 2 \sin(\log(1+x)) .$
6	Using the method of variation of parameter, solve $y'' + y = \cos ex.$
7	Using method of variation of parameters, solve the differential equation : $y'' - 6y' + 9y = e^{3x} / x^2$
8	Using variation of parameter solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$
9	Using method of variation of parameters, solve the differential equation: $(D^2 + 4)y = \tan 2x .$
10	Using the method of variation of parameter solve the differential equation $y'' + y = \sec x .$
11	In an L-C-R circuit, the charge q on a plate of a condenser is given by $L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = E \sin pt .$ The circuit is tuned to resonance so that $p^2 = 1/LC .$ If initially the current i and the charge q be zero, show that, for small values of $R/L ,$ the current in the circuit at time t is given by $(Et/2L) \sin(pt).$
12	The differential equation for a circuit in which self-inductance and capacitance neutralize each other is $L \frac{d^2i}{dt^2} + \frac{i}{C} = 0 .$ Find the current i as a function of t given that I is the maximum current and $i = 0$ when $t = 0 .$
13	The deflection of a strut of length l with one end ($x = 0$) built-in and the other supported and subjected to end thrust P satisfies the equation $\frac{d^2y}{dx^2} + a^2y = \frac{a^2R}{P}(l-x) .$ Prove that the deflection curve is $y = \frac{R}{P} \left(\frac{\sin ax}{a} - l \cos ax + l - x \right) ,$ where $al = \tan al .$

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Assignment-9, Partial Differential Equation

SR NO.	EXAMPLES
1	SOLVE BY SEPERATION OF VARIABLES: $\frac{\partial^2 z}{\partial x^2} = \cos x,$ $\frac{\partial^2 z}{\partial x \partial y} = x^2 + y^2$
2	
3	Form the partial differential equation from $z = f(x^2 + y^2),$ $f(xy + z^2, x + y + z) = 0,$ $(x - a)^2 + (y - b)^2 + z^2 = c^2.$ $z = f(x^2 - y^2)$ $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ $z = f\left(\frac{xy}{z}\right).$ $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$
4	Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ by using the method of separation of variables.
5	Using the method of separation of variables, solve $u_{xx} = 25u_{yy}$.
6	Solve the equation $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$, given that $u(0, y) = 8e^{-3y}$ by the method of separation of variable.
7	Solve by the method of separation of variable $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$.
8	Solve by the method of separation of variable $\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$
9	Obtain the complete solution of the equations: $p(1+q) = qz$ $p + q = \sin x + \sin y$ $p \tan x + q \tan y = \tan z$

	$(z - y)p + (x - z)q = y - x$ $p^2 + q^2 = 2$ $z = px + qy + 2\sqrt{pq}$ $p^2 + q^2 = x + y$ $(mz - ny)p + (nx - lz)q = ly - mx$ $x(y - z)p + y(z - x)q = z(x - y).$ $pq + p + q = 0$ $(y + z)p - (z + x)q = x - y.$ $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ $x^2 p^2 + y^2 q^2 = z^2.$ $\sqrt{p} + \sqrt{q} = 1$ $((x^2 - y^2 - z^2)p + 2xyq) = 2xz$
10	<p>A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = \sin \frac{\pi x}{l}$ from which it is released at time $t = 0$. Show that the displacement of any point at distance x from one end at time t is given by $y(x, t) = a \sin \frac{\pi x}{l} \cos \frac{\pi c t}{l}$.</p>