# GUJARAT TECHNOLOGICAL UNIVERSITY

3rd Semester Civil Engineering – PDDC

**Subject Code & Name :** X30604 - Advanced Fluid Mechanics

## Assignment - 4 (Turbulent Flow)

**Date : 16-10-2014**

**Theory :**

1. Explain hydro dynamically smooth and rough pipe
2. Obtain an expression for the velocity distribution for turbulent flow in smooth pipe.
3. Explain Prandtle's mixing length theory.

**Examples :**

1. A smooth cast iron pipe 0.4 m in diameter conveys crude oil at a velocity of 3 m/s. calculates the loss of head per km length of pipe. Kinematic viscosity of oil 0.42 stokes and S=0.9

2. Water flowing through a rough pipe of diameter 600 mm at the rate of 550 litres/second. The wall roughness is 3 mm. Find the power lost for 1.2 km length of pipe

Q-1 Explain hydrodynamically smooth & rough pipe.

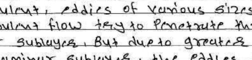
→ In turbulent flow, there is a thin layer very close to the boundary in which the flow is laminar. This layer is called laminar sublayer

In laminar sub-layer viscous shear stress predominates, while the shear stress due to turbulence is negligible.

let,
K = average height of the irregularities projecting the surface of a boundary
$\delta'$ = thickness of ~~a~~ the laminar sub-layer

If the average height K ~~o~~of the irregularities projecting the surface of a boundary is much less then $\delta'$, the boundary is called smooth boundary.

As the flow outside the laminar sub-layer is turbulent, eddies of various sizes present in turbulent flow try to penetrate through a laminar sublayer, But due to greater thickness of the laminar sublayer, the eddies can't reach the surface irregularities, and thus the boundary acts as a smooth boundary. This type of boundary is called hydrodynamically smooth Boundary.




Smooth Boundary.

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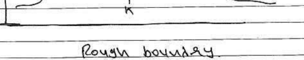
With the increase in Reynolds number the thickness of the laminar sublayer decreases. If the thickness of the laminar sublayer becomes much smaller than the average height K of irregularities the boundary will act as rough boundary. The irregularities will then project through the laminar sublayer and the laminar sublayer is completely destroyed. The eddies will come in contact with the surface irregularities and large amount of energy loss will take place. This type of boundary is called hydrodynamically rough boundary.




Rough boundary.

- It has been found by Nikuradse's experiment that:

a) if $\frac{K}{\delta'} < 0.25$ ___ Smooth boundary

b) if $\frac{K}{\delta'} > 6.0$ ___ Rough boundary

c) if $0.25 < \frac{K}{\delta'} < 6.0$ ___ transition boundary.

In terms of roughness Reynold number :

a) If $\frac{V_* k}{\nu} < 4$ --- smooth boundary

b) if $\frac{V_* k}{\nu} > 100$ ---- rough boundary

c) if $4 < \frac{V_* k}{\nu} < 100$ ---- Transition boundary

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Q-2 Obtain an expression for the velocity distribution for turbulent flow in smooth pipe.

→ The Prandtl's universal velocity distribution for turbulent flow in pipe is given by

$$u = \frac{V_*}{k} \log_e y + C \quad \text{---- (1)}$$

at the boundary (i.e at $y=0$), this velocity distribution gives $u$ equal to $-\infty$ (minus infinity)

Hence at some finite distance from wall, the velocity will be equal to zero.

let, the velocity will be zero at distance $y'$ from the pipe wall.

To find the ~~velocity~~ ~~will~~ constant $C$, applying boundary condition at $y = y'$, $u = 0$

$$\therefore \; 0 = \frac{V_*}{k} \log_e y' + C$$

$$\therefore \; C = -\frac{V_*}{k} \log_e y'$$

Substitute value of $C$ in equation (1) above)

$$\therefore \; u = \frac{V_*}{k} \log_e y - \frac{V_*}{k} \log_e y'$$

$$= \frac{V_*}{k} \log_e \left(\frac{y}{y'}\right)$$

Substitute $k = 0.4$ we get

$$u = \frac{V_*}{0.4} \log_e\left(\frac{y}{y'}\right)$$

$$\therefore u = 2.5\, V_* \log_e\left(\frac{y}{y'}\right)$$

$$\therefore \frac{u}{V^*} = 2.5 \log_e\left(\frac{y}{y'}\right)$$

$$\therefore \frac{u}{V_*} = 2.5 \times 2.3 \log_{10}\left(\frac{y}{y'}\right)$$

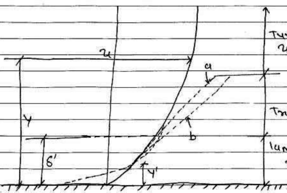
$$\therefore \frac{u}{V_*} = 5.75 \log_{10}\left(\frac{y}{y'}\right) \quad \text{——— (A)}$$

Near the smooth boundary there exists a laminar sublayer. The flow in the laminar sub-layer being laminar has a parabolic velocity distribution. Since thickness of the laminar sub-layer is very small. the velocity distribution in this region may be considered as straight line.

Above the zone of laminar motion there exists a transition zone where the flow changes from laminar to turbulent. Beyond the transition zone the flow is turbulent and the velocity distribution is logarithmic.

For laminar flow, $u \sim y$
For turbulent flow $u \sim \log(y)$

The intersection of parabolic and logarithmic velocity distribution curves, as shown in fig. is arbitrarily chosen as nominal border line between the two types of flow. Moreover the same point is taken as the limit of laminar sublayer.




Velocity distribution for turbulent flow near a smooth boundary.

in the sublayer (laminar)

$$\tau = \tau_0 = \mu \frac{du}{dy} = \mu \cdot \frac{u}{y}$$

( $\because$ assuming linear velocity distribution $\frac{du}{dy} = \frac{u}{y}$ )

dividing both sides by mass density ($\rho$)

$$\frac{\tau_0}{\rho} = \frac{\mu}{\rho} \cdot \frac{u}{y}$$

(Shear velocity)

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$\therefore V_*^2 = \nu \cdot \frac{u}{y}$ | $V_* = \sqrt{\frac{\tau_0}{\rho}}$

$\frac{u}{V_*} = \frac{V_* \cdot y}{\nu}$ --- (*) | $\nu = \frac{\mu}{\rho}$

eqn (*) gives velocity distribution within laminar sublayer that is from $y = 0$ to $y = \delta'$

The term $\frac{V_* \cdot y}{\nu}$ is an equivalent ~~within laminar sublayer~~ ~~that~~ ~~is from~~ ~~$y=0$~~. form of Reynolds number.

Nikuradse's experiment's for turbulent flow in smooth pipes has shown that.

for $y = \delta'$ $\frac{V_* \cdot y}{\nu} = \frac{V_* \delta'}{\nu} = 11.6$ —(a)

($y'$ = distance from pipe wall where velocity is zero)

for $y = y'$, $\frac{V_* \cdot y}{\nu} = \frac{V_* \cdot y'}{\nu} = 0.108$ (b)

$\therefore y' = \frac{0.108 \nu}{V_*}$

Dividing (b) by (a)

$\frac{y'}{\delta'} = \frac{0.108}{11.6}$ $\therefore y' = \frac{\delta'}{107}$

Substitute $y' = \frac{0.108 \nu}{V_*}$ in (A)

$$\frac{u}{V_*} = 5.75 \log_{10} \left[ \frac{y}{\frac{0.108\nu}{V_*}} \right]$$

$$= 5.75 \log_{10} \left[ \frac{V_* y}{\nu} \right] + 5.75 \log_{10} 9.259$$

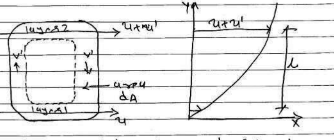
$$= 5.75 \log_{10} \left[ \frac{V_* y}{\nu} \right] + 5.50$$

The above $eq^n$ is known as Karman-Prandtl equation for hydrodynamically smooth boundaries.

Q-3 Explain Prandtle's mixing length theory.

-> The Prandtl's hypothesis is that lumps of fluid particles move bodily from a layer of one velocity to another layer of diffrent velocity. This displacement of fluid lumps results in a momentum exchange between the two layer. The distance $l$ that lump of fluid travels before losing it's own momentum and acquiring the momentum of new layer is called the mixing length.

let $u$ and $(u+u')$ be the flow velocities at two layers seperated by distance $l$ as shown in fig.




Prandtl mixing length hypothesis.

The velocity fluctuation in x direction $(u')$ is related to the mixing length as

$$u' = l \cdot \frac{du}{dy}$$

The velocity fluctuation in the y-direction $(v')$ is related to the mixing ~~point~~ length as

$$v' = l \frac{du}{dy}$$

$$\therefore \overline{u' \times v'} = \overline{u'v'} = \left(l \frac{du}{dy}\right) \cdot \left(l \frac{du}{dy}\right)$$

$$= l^2 \left(\frac{du}{dy}\right)^2$$

We know the Reynold's eqn for turbulent Shear stress as

$$\overline{\tau_t} = \overline{\rho u'v'}$$

$$\overline{\tau_t} = \rho \cdot l^2 \left(\frac{du}{dy}\right)^2$$

~~Thus.~~ Thus, Total Stress (shear) at any Point in a turbulent flow is given by

$$\tau = \tau_v + \overline{\tau_t}$$

$$= \mu \cdot \frac{du}{dy} + \rho l^2 \left(\frac{du}{dy}\right)^2$$

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## Example

Ex-1 A smooth cast iron pipe 0.4 m in dia conveys crude oil at a velocity of 3 m/s. Calculates the loss of head per km length of pipe. Kinematic viscocity of oil 0.42 stoke and $S = 0.9$

→ Dia of pipe $= 0.4\,m$
Velocity $V_{av} = 3\,m/s$

Kinematic viscocity $\nu = 0.42$ stoke
Specific gravity $S = 0.9$

Reynolds number $R_e = \dfrac{V_{av} D}{\nu}$

$$= \frac{3 \times 0.4}{0.42 \times 10^{-4}} \quad \left[\because 1\,\text{stoke} = 1 \times 10^{-4}\,m^2/s\right]$$

$$\therefore R_e = 28571.4$$

Since $R_e$ is between $4 \times 10^3$ to $4 \times 10^5$, hence equation 3 can be used to compute friction factor $f$

$$\therefore f = \frac{0.316}{(R_e)^{1/4}}$$

$$= \frac{0.316}{(28571.4)^{1/4}}$$

$$= 0.0243$$

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loss of head is given as

$$h_f = \frac{f L V_{av}^2}{2 g D}$$

$$= \frac{0.0243 \times 1000 \times 3^2}{2 \times 9.81 \times 0.4} \quad (L = 1 \text{ km})$$

$\therefore h_f = 27.86 \text{m}$

Power required $= \gamma Q h_f$

$= 0.9 \times 9810 \times (\text{Area} \times \text{velocity}) \times 27.86$

$= 0.9 \times 9810 \times \left[\frac{\pi}{4} \times 0.4^2 \times 3\right] \times 27.86$

$\therefore$ Power $= 92730.96$ watts

$= 92.73$ kW

Q-2 water flowing through a rough pipe of dia 600 mm at the rate of 550 lit/s. The wall roughness is 3mm. find the power lost of 1.2 km length of pipe.

→ $D = 600\,mm = 0.6\,m$
$Q = 550\,lit/s = 0.55\,m^3/s$
$k = 3\,mm = 0.003\,m$
$L = 1.2\,km = 1200\,m$

friction factor (f) for turbulent flow in rough pipe is

$$\frac{1}{\sqrt{f}} = 2.0 \log_{10}\left(\frac{R}{k}\right) + 1.74$$

$$\frac{1}{\sqrt{f}} = 2.0 \log_{10}\left(\frac{0.3}{0.003}\right) + 1.74$$

$$= 5.74$$

$$\therefore f = 0.0303$$

head loss due to friction

$$h_f = \frac{fLV^2}{2gD}$$

$$\text{velocity} = V = \frac{Q}{A}$$

$$= \frac{0.55}{0.2827}$$

$$\because A = \frac{\pi}{4} \times 0.6^2 = 0.2827\,m^3$$

$$= 1.945\,m/s$$

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$$\therefore h_f = \frac{0.0303 \times 1200 \times (1.945)^2}{2 \times 9.81 \times 0.6}$$

$$= 11.68 \text{ m}$$

Power lost $= w Q h_f$

$= 9810 \times 0.55 \times 11.68$

$\left( \because \text{For water } w = 1000 \times 9.81 = 9810 \right)$

$= 63019$ watt

$\therefore$ Power lost $= 63.02$ kW