GUJARAT TECHNOLOGICAL UNIVERSITY

3rd Semester Civil Engineering – PDDC

Subject Code & Name : X30604 - Advanced Fluid Mechanics

Assignment – 2 (Viscous Flow)

Date : 14-10-2014

Theory :

1. Describe Reynolds's Experiment and discuss.
2. Derive an expression for the velocity distribution for viscous flow through circular pipe. Also sketch the velocity distribution and shear stress distribution across a section of the pipe.
3. Derive the relation for laminar flow between two parallel plates the mean velocity is equal to two-third of the maximum velocity.

Examples :

1. An oil of viscosity 1 poise and specific gravity 0.85 is flowing through a circular pipe of diameter 10 cm at a rate of 6 litre/s.
   Calculate i). Pressure drop in a length of 400 m
   ii). Shear stress at the pipe wall.

2. Oil of specific gravity 0.82 is pumped through a horizontal pipe line 15 cm in diameter and 3 km long at the rate of 900 litres per minute. Pump has efficiency of 68% and requires 7.35kw to pump the oil. Determine the dynamic viscosity of oil and verify whether the flow is laminar or not.

3. Calculate the rate of flow of oil (μ=0.8 poise) flowing between two fixed plates kept at a distance of 20 mm apart. The drop of pressure in a length of 4 m is 4×10^4 N/m^2.The width of the plates is 150mm.

Q-1 Describe Reynold's Experiment and discuss

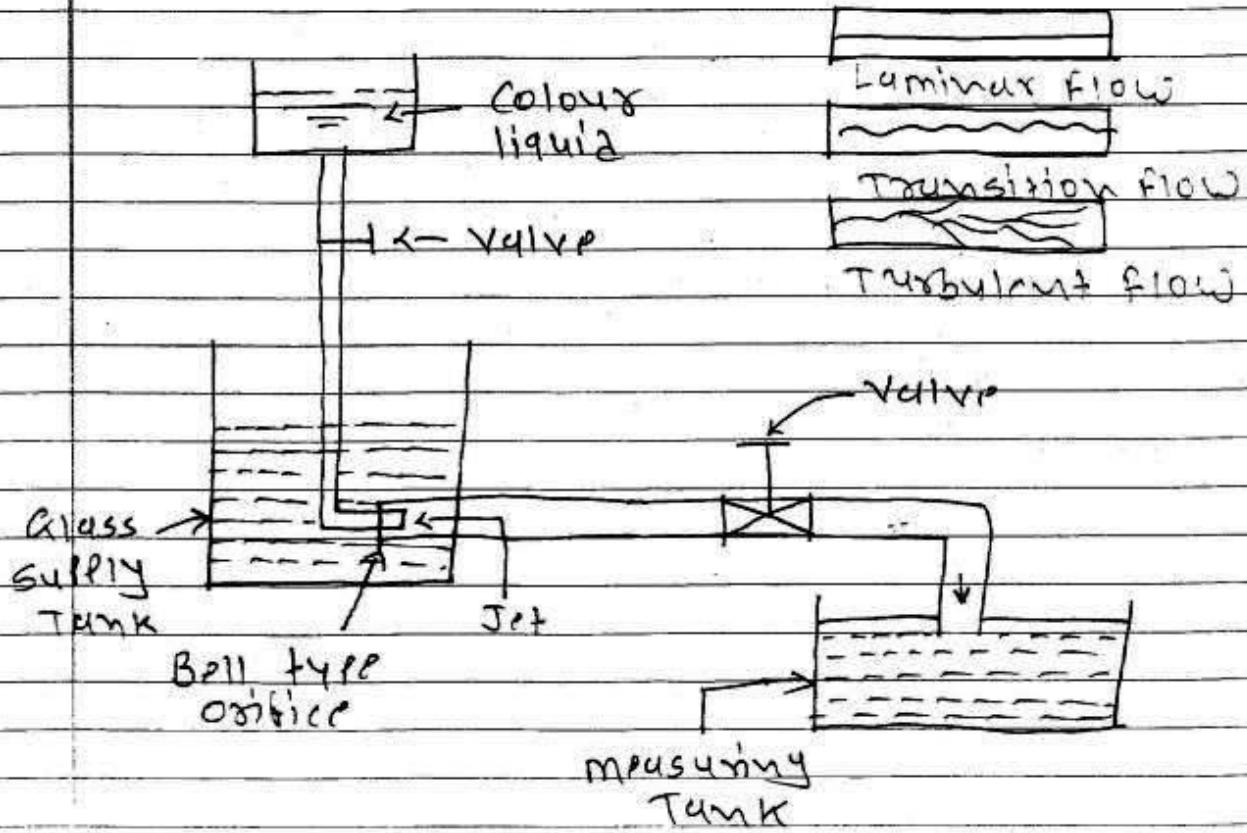
→ Reynold's Experiment

Osborne Reynold was first to show the existance of two types of flow He show that types of flow is determined by a dimensionless number

$$\left(\frac{\rho V d}{\mu}\right)$$

which is called as Reynold's number

$$Re = \frac{\rho V d}{\mu} \text{ or } \frac{V \times d}{\nu}$$



Reynold's experiment

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Reynold's apparatus consis of a tank containing water and a small container containing coloured water (dye). A glass tube of 1.5 m length and 50 mm diameter is fitted to the tank, through which water can flow. The flow of water through the glass tube can be regulated by adjusting the regulating ~~water~~ valve.

Water in the tank is first allowed to stand for several hours to allow it to come completely at rest. The outlet valve of glass tube is then slightly opened. Then a jet of dye having same specific gravity as that of water is also allowed to enter in the centre of the glass tube. It will be seen that a fine thread will move so steadily that it will be hardly seen to be in motion such flow is known as laminar flow.

If we open the regulating valve slowly, the velocity of flow through the glass tube will increase. A stage will come, when the dye thread will start becoming irregular and then breaking up as shown in figure. Such a velocity at which the dye thread starts becoming irregular is known as lower critical velocity.

Beyond the upper velocity critical the flow will be fully disturbed and such a flow is known as turbulent flow.

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Q-2 Derive an expression for the velocity distribution for viscous flow through circular pipe. Also sketch the velocity distribution and shear stress distribution across a section of the pipe.

→ To achieve velocity distribution across a section of pipe. the newton's law of viscosity is utilized. According to that law $\tau = \mu \left(\frac{dy}{dy}\right)$; in which u is the velocity at a distance y from the pipe wall. As per fig $y = R - r$ therefore $dy = -dr$ and the expression for τ then becomes

$$\tau = -\mu \frac{du}{dr} \quad \text{—— (1)}$$

Substituting the value of τ from eqn we get

$$-\mu \frac{du}{dr} = -\frac{\partial p}{\partial x} \frac{r}{2}$$

$$\text{or} \quad \frac{du}{dr} = \frac{1}{\mu} \frac{\partial p}{\partial x} \frac{r}{2} \quad \text{—— (2)}$$

Since $\frac{\partial p}{\partial x}$ is not a f^n of r. so intrgration of equation 2.4 with respect to r gives the following expression for velocity distribution

$$u = \frac{1}{4\mu} \frac{\partial p}{\partial x} r^2 + C \quad \text{—— (3)}$$

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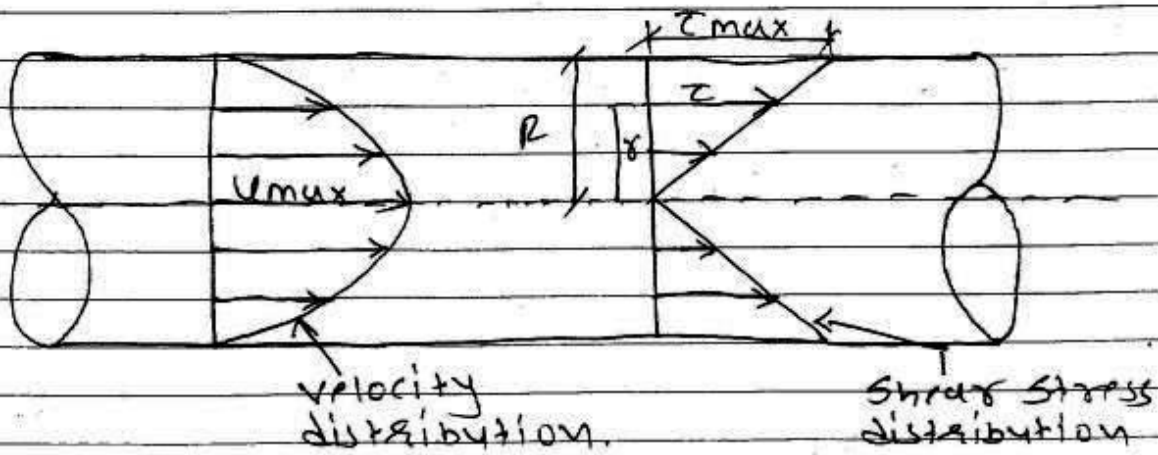
Here C is the constant of integration which can be evaluated using boundary condition that

$r = R, u = 0$ [At the pipe wall velocity is zero]

$$C = -\frac{1}{4\mu}\frac{\partial P}{\partial x} R^2$$

Now eq^n 3 becomes

$$u = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right)(R^2 - r^2) \quad \text{— (4)}$$



Velocity and Shear stress distribution across the section of pipe.

The maximum velocity u_{max} will occur when $r = 0$ i.e at the centre of pipe The magnitude of maximum velocity will be

$$u_{max} = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right) R^2$$

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Example :-

Ex :-1 An oil of viscosity 1 Poise and specific gravity 0.85 is flowing through a circular pipe of diameter 10 cm at a rate of 6 lit/s
Calculate :- (i) Pressure drop in length of 400m (ii) Shear stress at the pipe wall.

→ Viscosity of oil, $\mu = 1 \text{ Poise} = 0.1 \text{ N.S/m}^2$

$D = 10 \text{ cm} = 0.10 \text{ m}$
$Q = 6 \text{ lit/s} = 0.006 \text{ m}^3/\text{s}$
$L = 400 \text{ m}$

w = Unit weight of oil (weight density)

$= 0.85 \times 9810$
$= 8338.5 \text{ N/m}^3$

(i) Pressure drop in a length of 400m.

$$(P_1 - P_2) = \frac{128 \mu \cdot Q \cdot L}{\pi D^4}$$

$$= \frac{128 \times 0.1 \times 0.006 \times 400}{\pi \times (0.10)^4}$$

$$= 97784.80 \text{ N/m}^2$$

loss of Pressure head (h_f)

$$h_f = \frac{P_1 - P_2}{w} = \frac{97784.80}{8338.5}$$

$$= 11.73 \text{ m}$$

(ii) Shear stress at the pipe wall:

$$\tau_{max} = \left(-\frac{\partial p}{\partial x}\right) \cdot \frac{R}{2}$$

$$V_{ave} = \frac{Q}{A} = \frac{0.006}{\frac{\pi}{4} \times (0.1)^2} = 0.764 \text{ m/s}$$

$$-\frac{\partial p}{\partial x} = \frac{8 \mu V_{ave}}{R^2} = \frac{32 \mu V_{ave}}{D^2}$$

$$= \frac{32 \times 0.1 \times 0.764}{(0.1)^2}$$

$$= 244.48 \text{ N/m}^2 \text{ per m}$$

$$\therefore \tau_{max} = \left(-\frac{\partial p}{\partial x}\right) \cdot \frac{R}{2}$$

$$= 244.48 \times \frac{0.05}{2}$$

$$= 6.11 \text{ N/m}^2$$

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Ex:-2 Oil of specific gravity 0.82 pumped through a horizontal pipe line 15 cm in diameter & 3 km long at the rate of 900 litres per minute. The pump has an efficiency of 68%. & requires 7.35 kW to pump the oil. Determine the Dynamic viscosity of oil & verify whether the flow is laminar.

→ $Q = 900 \text{ litres/minute} = \frac{900}{1000 \times 60} = 0.015 \text{ m}^3/\text{s}$

Specific gravity = 0.82

$D = 15 \text{ cm} = 0.15 \text{ m}$

$L = 3 \text{ km} = 3000 \text{ m}$

Efficiency of pump = $\eta = 68\%$.

Power, $P = 7.35 \text{ kW} = 7350 \text{ watt}$

Power required,

$$P = \frac{W Q . h_f}{\eta}$$

$$7350 = \frac{(0.82 \times 9810) \times 0.015 \times h_f}{0.68}$$

$\therefore h_f = 41.42 \text{ m}$

Power = W.hf

= W.Q.hf

W = weight of oil flowing per second

= WQ

$$\text{Vave} = \frac{Q}{A} = \frac{0.015}{\frac{\pi}{4}(0.15)^2}$$

$= 0.848 \text{ m/s}$

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Now,

$$h_f = \frac{32 \mu V_{avg} \cdot L}{\omega D^2}$$

$$11.47 = \frac{32 \times \mu \times 0.842 \times 3000}{(0.82 \times 9810) \times (0.15)^2}$$

$\mu = 0.092$ N.s/m^2 dynamic viscosity

Reynold no.

$$R_N = \frac{\rho \cdot V_{avg} \cdot D}{\mu}$$

$$= \frac{(0.82 \times 1000) \times 0.842 \times 0.15}{0.092}$$

$$= 1133.73 < 2000$$

Since R_N is less than 2000, the flow is laminar

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Ex-3 Calculate the rate of flow of oil ($\mu = 0.8$ Poise) flowing between two fixed plate kept at a distance of 20 mm apart. The drop of pressure in a length of 4 m is 4×10^4 N/m^2. The width of the plates is 150 mm.

⇒ Data given

$\mu = 0.8$ Poise $= 0.08$ N.S/m^2

Spacing between plates

$B = 20\text{mm} = 0.02\text{m}$

Pressure drop $= P_1 - P_2 = 4 \times 10^4$ N/m^2

Length $= L = 4.0$ m

Width of Plate $= 0.15$ m

The difference of pressure is given by equation

$$(P_1 - P_2) = \frac{12 \mu \cdot V_{av} \cdot L}{B^2}$$

$$4 \times 10^4 = \frac{12 \times 0.08 \times V_{av} \times 4}{(0.02)^2}$$

$$V_{av} = 4.167 \text{ m/s}$$

Rate of flow = V_{av} x Area of flow

$= 4.167 \times (0.15 \times 0.02)$

$= 0.0125$ m^3/s

$= 12.5$ litre/s