

# GUJARAT TECHNOLOGICAL UNIVERSITY

3<sup>rd</sup> Semester Civil Engineering – PDDC

**Subject Code & Name :** X30604 - Advanced Fluid Mechanics

## Assignment - 1 (Kinematics and Dynamics)

**Date : 18-08-2014**

### **Theory :**

1. Describe various types of fluid flow.
2. Derive an equation of continuity for three dimensional flow.
3. Discuss velocity potential function and stream function and also state how they differ.
4. Derive & Explain Euler's Equation of motion.
5. Explain "Flow Net". Write its uses and limitations.

### **Examples :**

1. A 25cm diameter pipe carries oil of sp.gravity 0.9 at a velocity of 3 m/s. At another section the diameter is 20 cm. find the velocity at this section and also find mass rate of flow of oil.
2. The velocity in x y and z directions are given by
$$u = 2x - yt$$
$$v = y - zt$$
$$w = x - 3z + t$$
Determine the acceleration and velocity at point (1, 1, 2) and  $t = 1$ .
3. In a two dimensional incompressible flow, the fluid velocity components are given by  $U = x - 4y$  and  $V = -y - 4x$ . Show that velocity potential exists and determine its form. Find also the stream function.
4. Water is flowing through a pipe having dia 30 cm and 15 cm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 29.43 N/cm<sup>2</sup> and the pressure at the upper end is 14.715 N/cm<sup>2</sup>. Determine the difference in datum head if the rate of flow through pipe is 50lit/s



Q-1 Describe various type of fluid flow.

- The fluid flow is classified as:
- 1) steady and unsteady flows
  - 2) Uniform and non-uniform flow
  - 3) Laminar and turbulent flow
  - 4) Compressible and incompressible flow
  - 5) Rotational and irrotational flow
  - 6) one, two and three dimensional flow.

Type 1, 2 and 3 are described in Assignment-3

4) Compressible and Incompressible flow.

Compressible flow is that type of flow in which density of the fluid changes from point to point or in other words the density ( $\rho$ ) is not constant for the fluid. Thus, mathematically, for compressible flow

$$\rho \neq \text{Constant}$$

Incompressible flow is that flow in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible. mathematically, for incompressible flow

$$\rho = \text{Constant}$$

5) Rotational and Irrotational flows:

Rotational flow is that type of flow in which the fluid particles while flowing along streamlines, also rotate about their own axis. And if the fluid particles while flowing along streamlines, do not rotate about their own axis then that type of flow called irrotational flow.



### 6) - a) - One dimensional flow.

One dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only. Say  $x$ . For a steady one-dimensional flow, the velocity is a function of one-space-co-ordinate only. The variation of velocities in other two mutually perpendicular directions is assumed negligible. Hence mathematically for one-d-flow

$$u = f(x), \quad v = 0 \quad \text{and} \quad w = 0$$

### b) - Two-dimensional flow

Two dimensional flow is that type of flow in which the velocity is a function of time and two rectangular space co-ordinates say  $x$  and  $y$ . For a steady two dimensional flow is velocity is a function of two space coordinates only. The variation of velocity in the third direction is negligible. Thus, mathematically for two-d-flow

$$u = f_1(x, y), \quad v = f_2(x, y) \quad \text{and} \quad w = 0$$

### c) - Three dimensional flow

Three dimensional flow is that type of flow in which the velocity is a function of time and three mutually perpendicular directions. But for a steady three-d-flow the fluid parameters are functions of three space co-ordinates ( $x, y, z$ ) only. Thus, mathematically, for three-d-flow

$$u = f_1(x, y, z), \quad v = f_2(x, y, z) \quad \text{and}$$

$$w = f_3(x, y, z)$$



Q-2 Derive an equation of continuity for three dimensional flow.

→ Consider a fluid element of length  $dx, dy$  and  $dz$  in the direction of  $x, y$  and  $z$ . Let  $u, v$  and  $w$  are the inlet velocity components in  $x, y$  and  $z$  direction respectively. mass of fluid entering the face ABCD per second

$$\begin{aligned} &= \rho \times \text{Velocity in } x\text{-direction} \times \text{Area of ABCD} \\ &= \rho \times u \times (dy \times dz) \end{aligned}$$

Then mass of fluid leaving in the face EFGH per second

$$= \rho u dy dz + \frac{\partial}{\partial x} (\rho u dy dz) dx$$

∴ Gain of mass in  $x$ -direction

$$\begin{aligned} &= \text{mass through ABCD} - \text{mass through EFGH per second} \\ &= \rho u dy dz - \rho u dy dz - \frac{\partial}{\partial x} (\rho u dy dz) dx \end{aligned}$$

$$= - \frac{\partial}{\partial x} (\rho u dy dz) dx$$

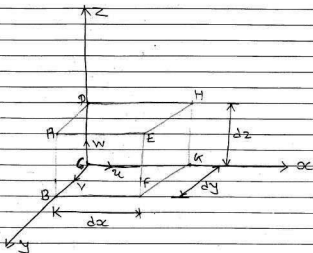
$$= - \frac{\partial}{\partial x} (\rho u) dx dy dz \quad (\because dy dz \text{ is constant})$$

Similarly, the net gain of mass in  $y$ -direction

$$= - \frac{\partial}{\partial y} (\rho v) dx dy dz$$



$$= - \frac{\partial}{\partial z} (\rho w) dx dy dz \text{ (in } z \text{ direction)}$$



and in  $z$  direction  $= - \frac{\partial}{\partial z} (\rho w) dx dy dz$

$\therefore$  net gain of masses

$$= - \left[ \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) \right] dx dy dz$$

Since the mass is neither created nor destroyed in the fluid element, the net increase of mass per unit time in the fluid element must be equal to the rate of increase of mass of fluid in the element. But mass of fluid in the element is  $\rho \cdot dx \cdot dy \cdot dz$  and its rate of increase with time is  $\frac{\partial}{\partial t} (\rho \cdot dx \cdot dy \cdot dz)$ , or  $\frac{\partial \rho}{\partial t} \cdot dx \cdot dy \cdot dz$ .



Equating the two expressions,

$$\text{or } - \left[ \frac{\partial}{\partial x} (P_u) + \frac{\partial}{\partial y} (P_v) + \frac{\partial}{\partial z} (P_w) \right] dx dy dz$$
$$= \frac{\partial P}{\partial t} dx dy dz$$

$$\text{or } \frac{\partial P}{\partial t} + \frac{\partial}{\partial x} (P_u) + \frac{\partial}{\partial y} (P_v) + \frac{\partial}{\partial z} (P_w) = 0$$

[Cancelling  $dx, dy, dz$  from both sides]



Q-3 Discuss velocity potential function and stream function and also state how they differ.



\* Velocity potential function

It is defined as a scalar function of space and time such that its negative derivative with res. to any direction gives velocity in that direction. It is denoted by  $\phi$  (Phi). It is defined as  $\phi = \phi(x, y, z)$  for steady flow such that

$$u = - \frac{\partial \phi}{\partial x}$$

$$v = - \frac{\partial \phi}{\partial y}$$

$$w = - \frac{\partial \phi}{\partial z}$$

} - (1)

Where  $u$ ,  $v$  and  $w$  are the components of velocity in  $x$ ,  $y$  and  $z$  direction respectively.

The velocity components in cylindrical polar co-ordinates in terms of velocity potential function are given by

$$\left. \begin{aligned} u_r &= \frac{\partial \phi}{\partial r} & u_\theta &= \frac{1}{r} \frac{\partial \phi}{\partial \theta} \end{aligned} \right\} - (2)$$

Where  $u_r$  = Velocity component in radial direction (in  $r$  direction)

$u_\theta$  = velocity component in tangential direction (in  $\theta$  direction)



The Continuity equation for an incompressible fluid flow is  $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

Substituting the values of  $u, v$  and  $w$  from eqn ① we get

$$\frac{\partial}{\partial x} \left( -\frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( -\frac{\partial \phi}{\partial y} \right) + \frac{\partial}{\partial z} \left( -\frac{\partial \phi}{\partial z} \right) = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad \text{--- ③}$$

This eqn is a Laplace equation

For two dimension case, eqn ③ reduces

$$\text{to } \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

If any value of  $\phi$  that satisfies the Laplace eqn will correspond to some case of fluid flow.

Properties of the Potential function, the rotational components are given by

$$\omega_z = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$\omega_y = \frac{1}{2} \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)$$

$$\omega_x = \frac{1}{2} \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$$





Substituting the values of  $u$ ,  $v$  and  $w$  from eq<sup>n</sup> in the above rotational components we get.

$$\begin{aligned}w_z &= \frac{1}{2} \left[ \frac{\partial}{\partial x} \left( -\frac{\partial \phi}{\partial y} \right) - \frac{\partial}{\partial y} \left( -\frac{\partial \phi}{\partial x} \right) \right] \\&= \frac{1}{2} \left[ -\frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial y \partial x} \right]\end{aligned}$$

Similarly

$$w_y = \frac{1}{2} \left[ -\frac{\partial^2 \phi}{\partial x \partial z} + \frac{\partial^2 \phi}{\partial z \partial x} \right]$$

$$w_x = \frac{1}{2} \left[ -\frac{\partial^2 \phi}{\partial y \partial z} + \frac{\partial^2 \phi}{\partial z \partial y} \right]$$

If  $\phi$  is a Continuous function, then

$$\frac{\partial^2 \phi}{\partial x \partial y} = \frac{\partial^2 \phi}{\partial y \partial x} ; \frac{\partial^2 \phi}{\partial z \partial x} = \frac{\partial^2 \phi}{\partial x \partial z} ; \text{ etc}$$

$$\therefore w_z = w_y = w_x = 0$$

$\therefore$  When rotational components are zero the flow is called irrotational. Hence the properties of the potential  $\phi$  are

- 1) If velocity potential ( $\phi$ ) exists, the flow should be irrotational.
- 2) If velocity potential ( $\phi$ ) satisfies the above eq<sup>n</sup>, it represents the possible steady incompressible irrotational flow.



\* **Stream function** - It is defined as the scalar function of space and time such that its partial derivative with res. to any direction gives the velocity component at right angles to that direction. It is denoted by  $\psi$  (Psi) and defined only for two-dimensional flow. It is denoted as  $\psi = f(x, y)$  such that,

$$\left. \begin{aligned} \frac{\partial \psi}{\partial x} &= v \\ \frac{\partial \psi}{\partial y} &= -u \end{aligned} \right\} \text{--- ①}$$

and

The velocity components in cylindrical polar co-ordinates in term of stream  $\psi$  are given as

$$u_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta} \quad \text{and} \quad u_\theta = -\frac{\partial \psi}{\partial r} \text{--- ②}$$

where  $u_r$  = radial velocity

$u_\theta$  = tangential velocity

The Continuity eq<sup>n</sup> for two-dimensional flow is  $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

Substituting the values of  $u$  and  $v$  from eq<sup>n</sup> ① we get

$$\frac{\partial}{\partial x} \left( -\frac{\partial \psi}{\partial y} \right) + \frac{\partial}{\partial y} \left( \frac{\partial \psi}{\partial x} \right) = 0$$



$$\text{or } -\frac{\partial \psi}{\partial x \partial y} + \frac{\partial \psi}{\partial y \partial x} = 0$$

Hence existence of  $\psi$  means a possible case of fluid flow. The flow may be rotational or irrotational.

The rotational component  $\omega_z$  is given by

$$\omega_z = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

Substituting the values of  $u$  and  $v$  from eqn (1) in the above rotational component,

$$\begin{aligned} \text{we get } \omega_z &= \frac{1}{2} \left[ \frac{\partial}{\partial x} \left( \frac{\partial \psi}{\partial x} \right) - \frac{\partial}{\partial y} \left( -\frac{\partial \psi}{\partial y} \right) \right] \\ &= \frac{1}{2} \left[ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right] = 0 \end{aligned}$$

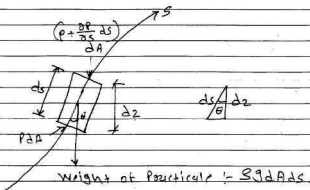
The properties of stream  $f^n(\psi)$  are

- 1) if stream  $f^n(\psi)$  exists, it is a possible case of fluid flow which may be rotational or irrotational.
- 2) if stream function  $(\psi)$  satisfies the Laplace eqn, it is a possible case of an irrotational flow.



Q-4 Derive & Explain Euler's Equation of motion.

→ This is the eq<sup>n</sup> of motion in which the forces due to gravity and pressure are considered. It is derived by considering the motion of a fluid element ~~at~~ along a stream line.



Consider a fluid particle along a stream line in the S-direction as shown in fig. Let  $dA$  is the area of cross-section and  $ds$  is the length of particle. The forces acting on fluid particles are

- (i) Weight of particle  $Sg dA ds$
  - (ii) Pressure force in the direction of flow  $p dA$  (upstream face)
  - (iii) Pressure force in the down stream face  $(p + \frac{\partial p}{\partial s} ds) dA$  in the direction opposite to direction of flow
- opposite to direction of flow
- As per Newton's second law of motion.



The resultant force on the fluid element must be equal to the product of mass and acceleration in  $s$ -direction.

Hence,

$$\Sigma F_s = m a_s$$

$$\rho dA \left( p + \frac{\partial p}{\partial s} ds \right) dA - \rho g ds dA \cos \theta \\ = \rho dA ds a_s$$

Here  $a_s$  is the acceleration of fluid element along the stream line dividing whole eq<sup>n</sup> by  $\rho ds dA$  and simplifying.

$$\frac{1}{\rho} \frac{\partial p}{\partial s} + g \cos \theta + a_s = 0$$

The acceleration  $a_s$  is  $\frac{dv}{dt}$ , where  $v$  is  $f^m$  of space and time both.

$$\therefore v = f(s, t)$$

Total derivation of  $v$  can be written as

$$\frac{dv}{dt} = \frac{\partial v}{\partial s} \frac{ds}{dt} + \frac{\partial v}{\partial t} \frac{dt}{dt}$$

$$a_s = \frac{\partial v}{\partial s} \frac{ds}{dt} + \frac{\partial v}{\partial t}$$

$$\text{or } a_s = v \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}$$

In case of steady flow  $\frac{\partial v}{\partial t} = 0$

$$\Rightarrow a_s = v = \frac{\partial v}{\partial s}$$



Now, eq<sup>n</sup> becomes

$$\frac{1}{\rho} \frac{\partial p}{\partial s} + g \frac{dz}{ds} + v \frac{\partial v}{\partial s} = 0$$

Since  $p$ ,  $z$  and  $v$  now are the function of  $s$  only, hence Partial derivatives may be replaced by total derivative.

○ Hence  $\frac{1}{\rho} \frac{dp}{ds} + g \frac{dz}{ds} + v \frac{dv}{ds} = 0$

$$\text{or } \frac{dp}{\rho} + g dz + v dv = 0$$

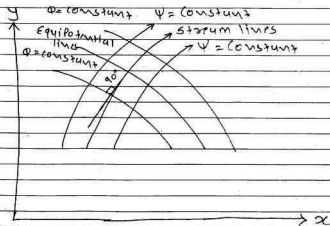
∴ eq<sup>n</sup> is known as Euler's eq<sup>n</sup> of motion and applicable under following assumptions.

- 
- (i) The motion of fluid is along a stream line
  - (ii) The flow of fluid is steady
  - (iii) The fluid is frictionless.



Q-5 Explain "Flow net". Write it's uses and limitations.

→ A grid obtained by drawing a series of ~~as~~ equipotential lines and ~~to~~ stream lines is called a flow net. The flow net is an important tool in ~~analyzing~~ analysing two-dimensional irrotational flow problems.



Element of flow net.



## Examples

Ex-1 A 25 cm dia pipe carries oil of sp. gr 0.9 at a velocity of 3 m/s. At another section the dia is 20 cm find the velocity and also mass rate of flow of oil.

→ at Section ①

$$D_1 = 25 \text{ cm}$$

$$= 0.25 \text{ m}$$

$$A_1 = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 0.25^2$$

$$= 0.049 \text{ m}^2$$

$$V_1 = 3 \text{ m/s}$$

at Section ②

$$D_2 = 20 \text{ cm}$$

$$= 0.20 \text{ m}$$

$$A_2 = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 0.20^2$$

$$= 0.0314 \text{ m}^2$$

$$V_2 = ?$$

mass rate of flow of oil = ?

Applying Continuity eq<sup>n</sup> at Section 1 & 2

$$A_1 V_1 = A_2 V_2$$

$$\therefore 0.049 \times 3 = 0.0314 \times V_2$$

$$\therefore V_2 = \frac{0.049 \times 3}{0.0314} = 4.68$$

$$\therefore V_2 = 4.68 \text{ m/s}$$





Mass rate of flow of oil

$$= \text{mass density} \times Q \\ = \rho \times A \times V$$

$$\text{Sp. gr. of oil} = \frac{\text{Density of oil}}{\text{Density of water}}$$

$$\begin{aligned} \text{Density of oil} &= \text{Sp. gr. of oil} \times \text{Density of water} \\ &= 0.9 \times 1000 \text{ kg/m}^3 \\ &= 900 \text{ kg/m}^3 \end{aligned}$$

$\therefore$  mass rate of flow =

$$= 900 \times 0.049 \times 3.0 \text{ kg/s}$$

$$= 132.23 \text{ kg/s}$$



Ex-2 The velocity in  $x$ ,  $y$  and  $z$  direction are given by

$$u = 2x - 4t$$

$$v = y - 2t$$

$$w = x - 3z + t$$

Determine the acceleration and velocity at Point  $(1, 1, 2)$  and  $t = 1$

○  $\rightarrow$  velocity component at  $(1, 1, 2)$  at  $x = 1, y = 1, z = 2$  &  $t = 1$

$$u = 2(1) - 4(1)$$

$$= 1 - 4$$

$$\therefore u = -3$$

$$v = y - 2t$$

$$= 1 - 2(1)$$

$$= 1 - 2$$

$$\therefore v = -1$$

$$w = x - 3z + t$$

$$= 1 - 3(2) + 1$$

$$= 1 - 6 + 1$$

$\therefore$  velocity vector  $V$  at  $(1, 1, 2)$

$$= 1\hat{i} - 1\hat{j} - 4\hat{k}$$

$\therefore$  Resultant velocity

$$= \sqrt{u^2 + v^2 + w^2}$$

$$= \sqrt{1^2 + (-1)^2 + (-4)^2}$$

$$= 4.24 \text{ units.}$$



\* Acceleration is given by eq<sup>n</sup>

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

○ Now velocity components we have.

$$\frac{\partial u}{\partial x} = 2 \quad \frac{\partial u}{\partial y} = -t \quad \frac{\partial u}{\partial z} = 0 \quad \frac{\partial u}{\partial t} = -y$$

$$\frac{\partial v}{\partial x} = 0 \quad \frac{\partial v}{\partial y} = 1 \quad \frac{\partial v}{\partial z} = -t \quad \frac{\partial v}{\partial t} = -z$$

$$\frac{\partial w}{\partial x} = 1 \quad \frac{\partial w}{\partial y} = 0 \quad \frac{\partial w}{\partial z} = -3 \quad \frac{\partial w}{\partial t} = 1$$

Substituting this value

$$\begin{aligned} a_x &= (2x - yt) \cdot 2 + (y - 2t) \cdot (-t) + 0 - y \\ &= 2 + 3 - 1 = 4 \end{aligned}$$

$$\begin{aligned} a_y &= 0 + (y - 2t) \cdot 1 + (x - 3z + t) \cdot (-t) + (-z) \\ &= -1 + 4 - 2 = 1 \end{aligned}$$



$$a_z = 1 + 0 + 12 + 1$$
$$= 14$$

acceleration is

$$A = a_x i + a_y j + a_z k$$
$$= 2i + 1j + 14k$$

○

or Resultant

$$A = \sqrt{2^2 + 1^2 + 14^2}$$
$$= 14.177 \text{ Unit}$$

○



Ex-3 In a two-dimensional incompressible flow the fluid velocity components are given by  $u = x - 4y$  and  $v = -y - 4x$ . Show that velocity potential exists and determine its form. Find also the stream function.

$$\rightarrow \quad u = x - 4y \quad \frac{\partial u}{\partial x} = 1 \quad \text{and} \quad \frac{\partial v}{\partial y} = -1$$
$$v = -y - 4x$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 - 1 = 0$$

Hence flow is continuous and velocity potential exists.

• let  $\phi =$  velocity potential

let velocity components in terms of velocity potential is given by

$$\frac{\partial \phi}{\partial x} = -u = -(x - 4y) = -x + 4y \quad \text{--- (i)}$$

$$\frac{\partial \phi}{\partial y} = -v = -(-y - 4x) = y + 4x \quad \text{--- (ii)}$$

and

integrating eq<sup>n</sup> (i) we get  $\phi = -\frac{x^2}{2} + 4xy + C$  (iii)

where  $C$  is a constant of integration, which is independent of  $x$ .

This constant can be a f<sup>n</sup> of  $y$

Differentiating the above eq<sup>n</sup>. i.e. eq<sup>n</sup> (iii)

with  $x$ s. to  $y$  we get



$$\frac{\partial \phi}{\partial y} = 0 + 4x + \frac{\partial c}{\partial y}$$

But from eq<sup>n</sup> (iii) we have  $\frac{\partial \phi}{\partial y} = y + 4x$

Equating the two values of  $\frac{\partial \phi}{\partial y}$  we get.

$$4x + \frac{\partial c}{\partial y} = y + 4x \quad \text{or} \quad \frac{\partial c}{\partial y} = y$$

Integrating the above eq<sup>n</sup> we get.

$$c = \frac{y^2}{2} + C_1$$

Where  $C_1$  is a constant of integration which is independent of  $x$  and  $y$

Taking it equal to zero we get  $c = \frac{y^2}{2}$

Substituting the value of  $c$  in eq<sup>n</sup> (ii)

$$\boxed{\phi = -\frac{x^2}{2} + 4xy + \frac{y^2}{2}}$$

\* Value of Stream f<sup>n</sup>.

Let  $\psi = \text{Stream f}^n$

The velocity components in terms of stream f<sup>n</sup> are

$$\frac{\partial \psi}{\partial x} = V = -y - 4x \quad \dots (ix)$$



$$\frac{\partial \psi}{\partial y} = -x = -(x - 4y) = -x + 4y \quad \text{--- (v)}$$

Integrating eq<sup>n</sup> (iv) w.r.t  $x$  we get

$$\psi = -4x - \frac{4x^2}{2} + k \quad \text{--- (vi)}$$

Where  $k$  is a constant of integration which is independent of  $x$  but can be a fn of  $y$ .

Differentiating eq<sup>n</sup> (vi) w.r.t  $y$  we get

$$\frac{\partial \psi}{\partial y} = -x - 0 + \frac{\partial k}{\partial y}$$

But from eq<sup>n</sup> (v) we have

$$\frac{\partial \psi}{\partial y} = -x + 4y$$

eq<sup>n</sup>ing the two values of  $\frac{\partial \psi}{\partial y}$  we get

$$-x + \frac{\partial k}{\partial y} = -x + 4y \quad \text{or} \quad \frac{\partial k}{\partial y} = 4y$$

Integrating the above eq<sup>n</sup> we get

$$k = \frac{4y^2}{2} = 2y^2$$

Substituting the values of  $k$  in eq<sup>n</sup> (vi) we get

$$\psi = -4x - 2x^2 + 2y^2$$

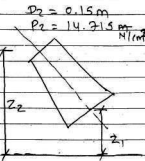


Ex-4 Water is flowing through a pipe having dia 30 cm 15 cm at the bottom and upper end respectively. The intensity of pressure at the bottom end is  $29.43 \text{ N/cm}^2$  and the pressure at the upper end is  $14.715 \text{ N/cm}^2$ . Determine the distance in datum head if the rate of flow through pipe is  $50 \text{ lit/s}$ .

→

$D_1 = 0.30 \text{ m}$   
 $P_1 = 29.43 \text{ N/cm}^2$   
 $= 29.43 \text{ N/m}^2$   
 $A_1 = 0.071 \text{ m}^2$   
 $D_2 = 0.15 \text{ m}, A_2 = 0.018 \text{ m}^2$   
 $P_2 = 14.715 \text{ N/cm}^2$   
 $= 14.715 \times 10^4 \text{ N/m}^2$   
 Rate of flow =  $50 \text{ lit/s}$

$$Q = \frac{50}{1000} = 0.05 \text{ m}^3/\text{s}$$



$$A_1 V_1 = A_2 V_2 = \text{rate of flow (Q)} = 0.05$$

$$V_1 = \frac{Q}{A_1} = \frac{0.05}{0.071} = 0.704 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.05}{0.018} = 2.78 \text{ m/s}$$

Applying Bernoulli's eq<sup>n</sup> at section  
1 & 2 w.r.g.





$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\therefore Z_2 - Z_1 = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} - \frac{P_2}{\rho g} - \frac{V_2^2}{2g}$$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{(0.076)^2}{2 \times 9.81} - \frac{14.715 \times 10^4}{1000 \times 9.81}$$

$$- \frac{(2.28)^2}{2 \times 9.81}$$

$$= 14.60 \text{ m}$$

$\therefore$  Difference in datum head = 14.60 m