

GUJARAT TECHNOLOGICAL UNIVERSITY

Semester: 1

PDDC- CIVIL ENGINEERING

Subject Name MATHEMATICS-1

Sr.No	Course content
1.	Matrices : A. Rank of a matrix (1) by row echelon form (2) by determinant method B. Method to solve system of linear equations by (1) Gaussian elimination (row echelon form) (2) Gauss-Jordan method (reduced row echelon form) C. Inverse of matrices by Gauss Jordan-method D. Eigen values and eigen vectors
2.	Partial differentiation : A. Functions of several variables B. Partial derivatives of first and higher orders C. Homogenous functions D. Euler's theorem on homogenous functions E. Jacobians F. Errors and approximations G. Maxima and minima of functions of two variables
3.	Differential equations of first order and first degree : A. Methods for solving differential equation by (1) Variable separable (2) Homogenous differential equation (3) Linear differential equation (4) Bernoulli's differential equation (5) Exact differential equation
4.	Modeling of differential equations : A. Orthogonal trajectories B. Electrical circuits
5.	Tracing of curves : A. Tracing of Cartesian curves B. Tracing of polar curves
6.	Multiple integrals : A. Double integrals, triple integrals B. Change of order of integration. C. Change of variable from Cartesian to polar coordinates. D. Evaluation of area and volume.

7.	Vector calculus : A. Scalar and vector Fields B. Gradient of a scalar field. C. Curl and Divergence of vector field. D. Vector integration E. Line integral F. Green's theorem in the plane.
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Reference Books:

1. Higher Engineering Mathematics by Dr. B.S. Grewal,
Khanna Publishers, New Delhi.
2. Elementary Engineering Mathematics by Dr. B.S. Grewal,
Khanna Publishers, New Delhi.
3. A Textbook of Engineering Mathematics by N.P. Bali, Ashok Saxena & Iyengar,
Laxmi Publications (P) Ltd., New Delhi.
4. Advanced Engineering Mathematics by H.K. Dass
S. Chand & Co. (Pvt.) Ltd., New Delhi.

GUJARAT TECHNOLOGICAL UNIVERSITY**PDDC - SEMESTER-I • EXAMINATION – SUMMER 2013****Subject Code: X10001****Date: 03-06-2013****Subject Name: Mathematics - I****Time: 02.30 pm - 05.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1

- (a) Reduce the matrix $A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$ into row echelon form and find its rank. **05**
- (b) Solve following linear system by Gauss – Jordan elimination method. **05**
- $$3x + 3y + 2z = 1$$
- $$x + 2y = 4$$
- $$10y + 3z = -2$$
- $$2x - 3y - z = 5$$
- (c) Solve $(x^2 - yx^2) \frac{dy}{dx} + (y^2 + xy^2) = 0$. **04**

Q.2

- (a) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. **07**
- (b) 1) Solve $(x^2 - y^2)dx - xydy = 0$. **04**
- 2) Solve $(x^2 - ay)dx = (ax - y^2)dy$ **03**

OR

- (b) Trace the curve $y^2(a + x) = x^2(a - x)$ **07**

Q.3

- (a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x + y + z)^2}$ **05**
- (b) If $x^2 = au + bv$ and $y^2 = au - bv$ then show that $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \frac{1}{2}$. **05**
- (c) Show that $\text{grad}\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$. **04**

OR**Q.3**

- (a) Find the maximum and minimum value of the function $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$. **07**
- (b) Evaluate the line integral $\int_C [(5xy - 6x^2)\vec{i} + (2y - 4x)\vec{j}]$ where C is the curve $y = x^3$ from the point (1,1) to (2,8). **07**

Q.4

- (a) If $u = \tan^{-1}\left(\frac{y^2}{x}\right)$ prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\sin^2 u \sin 2u$. **07**
- (b) 1) Find curl \vec{F} at the point (1,2,3), where $\vec{F} = x^2 yz \vec{i} + xy^2 z \vec{j} + xyz^2 \vec{k}$. **04**
- 2) Show that $\vec{F} = (-x^2 + yz)\vec{i} + (4y - z^2 x)\vec{j} + (2xz - 4z)\vec{k}$ is solenoidal. **03**

OR

- Q.4** (a) If $x = r \cos \theta$, $y = r \sin \theta$, prove that $JJ' = 1$. **07**
- (b) Verify Green's theorem for $\int_C [(xy + y^2)dx + x^2 dy]$, where C is bounded by $y = x$ and $y = x^2$. **07**
- Q.5** (a) Change the order of integration in $I = \int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy dx$ and hence evaluate. **05**
- (b) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dx dy$. **05**
- (c) Find the orthogonal trajectories of the family of the parabolas $y = ax^2$. **04**
- OR**
- Q.5** (a) Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dy dx$ by changing to polar coordinates. **05**
- (b) Evaluate $\int_0^a \int_0^b \int_0^c (x^2 + y^2 + z^2) dx dy dz$. **05**
- (c) Find the directional derivatives of $f(x, y, z) = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the vector $\bar{i} + 2\bar{j} + 2\bar{k}$. **04**

GUJARAT TECHNOLOGICAL UNIVERSITY
PDDC - SEMESTER – I • EXAMINATION – WINTER 2012

Subject code: X 10001**Date: 11/01/2013****Subject Name: Mathematics - I****Time: 10.30 am - 01.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) (i) Define the rank of Matrix. Determine the rank of Matrix $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 2 & 3 & 1 \end{bmatrix}$ **03**

(ii) Is the Matrix $\begin{bmatrix} 1 & -5 & 4 & 5 \\ 0 & 1 & 2 & 3 \end{bmatrix}$ in row echelon form or reduced row echelon form? **02**

(iii) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$. **02**

(b) (i) Solve the following system of equations:
 $x + y + z = 6, x + 2y + 3z = 4, x + 4y + 9z = 6$. by Gauss-elimination and back substitution method. **04**

(ii) Using Gauss-Jordan method find the inverse of the matrix **03**

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}.$$

Q.2 (a) (i) Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. **04**

(ii) State the Euler's theorem for homogeneous function and if **03**

$$u = \sin^{-1} \left(\frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{5}} + y^{\frac{1}{5}}} \right) \text{ then prove that } x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{20} \tan u.$$

(b) (i) If $y = f(x + 2t) + g(x - 2t)$, prove that $\frac{\partial^2 y}{\partial t^2} = 4 \frac{\partial^2 y}{\partial x^2}$. **04**

(ii) If $x = r \cos \theta, y = r \sin \theta$ then find $\frac{\partial(x, y)}{\partial(r, \theta)}$. **03**

OR

(b) (i) Discuss the maxima and minima of $x^2 + y^2 + 6x + 12$. **04**

(ii) Find the equation of tangent plane at the point (1,1,1) on the surface $x^2 + y^2 + z^2 = 3$. **03**

Q.3 (a) Solve the following differential equations:

(i) $(x^2 - y^2)dx = (2xy)dy$. **04**

03

(ii) $\frac{dx}{dy} + x = y$.

- (b) (i) Solve the exact differential equation 04
 $(x^2 + y^2 - a^2)xdx + (x^2 - y^2 - b^2)ydy = 0$.
(ii) Find the orthogonal trajectories of the family of the curve $x^2 - y^2 = c$. 03

OR

- Q.3** (a) Solve the following differential equations: 04
(i) $\frac{dy}{dx} - \frac{y}{x+1} = e^x(x+1)$, 03
(ii) $(e^y + 1)\cos x dx + e^y \sin x dy = 0$. 04
(b) (i) Solve $\cos(x+y)dy = dx$. 03
(ii) Find the orthogonal trajectories of the family of the curve $r^2 = c \sin(2\theta)$. 10

- Q.4** (a) Trace the curves: 04
(i) $r = a(1 + \cos \theta)$, $a > 0$. (ii) $xy^2 = 9a^2(2a - x)$.
(b) Evaluate $\int_0^2 \int_0^2 (x^2 + y^2) dx dy$. 04

OR

- Q.4** (a) Trace the curves: 10
(i) $y^2(a - x) = x^3$, (ii) $r^2 = a^2 \cos 2\theta$.
(b) Change the order of integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$ and evaluate it. 04

- Q.5** (a) (i) Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dx dy$ by changing into polar coordinates. 04
(ii) Evaluate $\int_0^a \int_0^{a-x} \int_0^{a-x-y} x dz dy dx$. 03
(b) (i) Find the constants a, b, c so that vector $(x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (-x + cy + 2z)\hat{k}$ is irrotational. 03
(ii) Obtain the area of an Ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, by using Green's theorem. 04

OR

- Q.5** (a) (i) Change the order of integration $\int_0^{\frac{1}{\sqrt{2}}} \int_x^{\sqrt{1-x^2}} y^2 dA$ and evaluate it. 04
(ii) Evaluate $\int_0^2 \int_0^x \int_0^{\sqrt{x+y}} z dx dy dz$. 03
(b) (i) Is the vector $\vec{v} = (x - 3y)\hat{i} + (y - 2z)\hat{j} + (x - 3z)\hat{k}$ solenoidal? 03
(ii) Find $\int_c \vec{F} d\vec{r}$, where $\vec{F} = \frac{y\hat{i} - x\hat{j}}{x^2 + y^2}$ and c is the circle $x^2 + y^2 = 4$ traversed counter clockwise. 04

GUJARAT TECHNOLOGICAL UNIVERSITY**PDDC - Ist Semester–Examination – May/June- 2012****Subject code: X10001****Subject Name: Mathematics-I****Date: 29/05/2012****Time: 10:30 am – 01:30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) (i) Solve $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$ **03**
- (ii) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$ **04**
- (b) (i) If $\theta = t^n e^{-r^2/4t}$, what value of n will make $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \theta}{\partial r} \right) = \frac{\partial \theta}{\partial t}$? **04**
- (ii) Trace the curve $y^2 (2a - x) = x^3$ **03**
- Q.2** (a) (i) Using Gauss Jordan method, find the inverse of the matrix **03**
- $$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$
- (ii) Find the Eigen values and Eigen vectors of the matrix **04**
- $$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$
- (b) Solve the following differential equations:
- (i) $\left(1 + e^{x/y}\right) dx + e^{x/y} \left(1 - \frac{x}{y}\right) dy = 0$. **04**
- (ii) $(e^y + 1) \cos x dx + e^y \sin x dy = 0$. **03**
- OR**
- (b) Solve the following differential equations:
- (i) $(1 + y^2) dx = (\tan^{-1} y - x) dy$ **04**
- (ii) $x \frac{dy}{dx} + y \log y = xy e^x$ **03**
- Q.3** Attempt the following:
- (a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that **05**
- $$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = \frac{-9}{(x + y + z)^2}$$
- (b) If $u = \sin^{-1} \left[\frac{x + y}{\sqrt{x} + \sqrt{y}} \right]$, prove that **05**
- $$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{-\sin u \cos 2u}{4 \cos^3 u}$$
- (c) If $x = r \cos \theta$ and $y = r \sin \theta$; evaluate $\frac{\partial(x, y)}{\partial(r, \theta)}$ & $\frac{\partial(r, \theta)}{\partial(x, y)}$ **04**

OR

Q.3 Attempt the following:

(a) If $u = f(r)$, where $r^2 = x^2 + y^2$, **05**

prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

(b) If $z = x f\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$; **05**

show that $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0$

(c) Examine the function $f(x, y) = x^3 + y^3 - 3axy$ for maxima & minima. **04**

Q.4 Attempt the following:

(a) Evaluate $\iint_R y \, dx \, dy$ where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. **05**

(b) Change the order of integration & evaluate $\int_0^a \int_y^a \frac{x \, dx \, dy}{x^2 + y^2}$ **05**

(c) Using double integration, find area lying between the parabola $y = 4x - x^2$ and the line $y = x$. **04**

OR

Q.4 Attempt the following:

(a) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} \, dx \, dy$ by changing to polar coordinates. **05**

(b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx \, dy \, dz}{\sqrt{(1-x^2-y^2-z^2)}}$ **05**

(c) Calculate the volume of the solid bounded by the planes $x = 0$, $y = 0$, $x + y + z = 1$ and $z = 0$. **04**

Q.5 Attempt the following:

(a) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 3$ where t is the time. Find the components of its velocity and acceleration at $t = 1$ in the direction $i + j + 3k$. **05**

(b) Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$ at the point $(1, 2, 3)$ **05**

(c) The rate at which body cools is proportional to the difference between the temperature of the body and that of the surrounding air. If body in air at 25°C will cool from 100° to 75° in one minute, find its temperature at the end of three minutes. **04**

OR

Q.5 Attempt the following:

(a) Find the directional derivative of the scalar function $f(x, y, z) = x^2 + xy + z^2$ at the point $A(1, -1, -1)$ in the direction of the line AB where B has coordinates $(3, 2, 1)$ **05**

(b) Use Green's theorem to evaluate $\int_C (x^2 + xy) \, dx + (x^2 + y^2) \, dy$ where C is the square formed by the lines $y = \pm 1$, $x = \pm 1$. **05**

(c) Find the orthogonal trajectories of the family of curves $x^2 - y^2 = c$. **04**

GUJARAT TECHNOLOGICAL UNIVERSITY
PDDC SEM-I Examination-Dec-2011

Subject code: X10001

Date: 19/12/2011

Subject Name: Mathematics-I

Time: 10.30 am -1.30 pm

Total marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) i. Determine the rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$. **02**

ii. If $x = r \cos \theta$, $y = r \sin \theta$ show that $\frac{\partial r}{\partial x} = \frac{\partial x}{\partial r}$. **02**

iii. What is degree and order of $\frac{dy}{dx} = 2xy$? Solve it. **03**

(b) i. Discuss the nature of the origin for $y^2 = x(x+2) - 3$. **01**

ii. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} dy dx$ by changing into polar coordinates. **03**

iii. Find a vector normal to the surface $xy^3z^2 = 4$ at the point $(-1, -1, 2)$. **03**

Q.2 (a) i. Solve: $x + 2y + 3z = 0$, $3x + 4y + 4z = 0$, $7x + 10y + 12z = 0$. **03**

ii. Using Gauss-Jordan method find the inverse of $B = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$. **04**

(b) i. Find the eigen values of $C = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. Find eigen vector corresponding to **04**

its smallest eigen value.

ii. If $u = e^{xyz}$, find u_{xyz} . **03**

OR

(b) i. If $D = \begin{bmatrix} 1 & 5 \\ 0 & -1 \end{bmatrix}$ find the eigen values of D^9 . **03**

ii. Find u_{xy} for $u = x^2 \tan^{-1} \frac{y}{x} - y^2 \tan^{-1} \frac{x}{y}$. **04**

Q.3 (a) i. If $u = \sin^{-1} \frac{x^2 y^2}{x+y}$, show that $xu_x + yu_y = 3 \tan u$. **03**

ii. If $x = r \cos \theta$, $y = r \sin \theta$, prove that $JJ' = 1$. **04**

(b) Solve: **07**

i. $xy \frac{dy}{dx} = 1 + x + y + xy$

ii. $(x^2 - y^2) dx - xy dy = 0$

OR

Q.3 (a) i. Examine $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for extreme values. **04**

- ii. In estimating the cost of a pile of bricks measured as $2m \times 15m \times 1.2m$, the tape is stretched 1% beyond the standard length. Find the percentage error in the volume of the pile. **03**
- (b) Solve : **07**
- $(x+1) \frac{dy}{dx} - y = e^{3x} (x+1)^2$
 - $(x^2 - ay) dx = (ax - y^2) dy$
- Q.4 (a)** If the stream lines of a flow around a corner are $xy = \text{constant}$, find their orthogonal trajectories. **04**
- (b) Trace the curves **10**
- $r^2 = a^2 \cos 2\theta$
 - $x^2 y = a^2 (a - y) ; a > 0$
- OR**
- Q.4 (a)** When a resistance R ohms is connected in series with an inductance L henries with an e.m.f. of E volts, the current i amperes at time t is given by $L \frac{di}{dt} + Ri = E$. If $E = 10 \sin t$ volts and $i = 0$ when $t = 0$, find i as a function of t . **04**
- (b) Trace the curves: **10**
- $y^2 (2a - x) = x^3 ; a > 0$
 - $r = a (1 + \cos \theta) ; a > 0$
- Q.5 (a)** i. Evaluate by changing the order of integration $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dy dx$. **03**
- ii. By triple integration, find the volume of the sphere $x^2 + y^2 + z^2 = 1$. **04**
- (b) i. Find the value of a if $(ax^2y + yz) i + (xy^2 - xz^2) j + (2xyz - 2x^2y^2) k$ is solenoidal. **03**
- ii. Verify Green's theorem for $\int_C [(xy + y^2) dx + x^2 dy]$, where C is bounded by the curves $y = x$, $y = x^2$. **04**
- OR**
- Q.5 (a)** i. Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) dy dx$ **03**
- ii. Find, by double integration, the area lying inside the curve $r = 2a \cos \theta$ **04**
- (b) i. Find the curl of $(-2x^2y + yz) i + (xy^2 - xz^2) j + (2xyz - 2x^2y^2) k$. **03**
- ii. Apply Green's theorem to evaluate $\int_C [(2x^2 - y^2) dx + (x^2 + y^2) dy]$, where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = 1$. **04**

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC Sem-I June-July Examination 2011

Subject code: X10001**Subject Name: Mathematics - I****Date: 18/06/11****Total Marks: 70****Time: 10:30am to 1:30pm****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 Do as directed.

(a) Find a unit vector normal to the surface $xy^3z^2 = 4$ at the point $(-1, -1, 2)$. (02)

(b) Evaluate $\int_0^{\pi} \int_0^{a(1+\cos\theta)} r dr d\theta$. (03)

(c) Trace the curve $y^2(a-x) = x^2(a+x)$. (03)

(d) Determine Rank of the following matrix by row echelon form (03)

$$A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}.$$

(e) Using Gauss-Jordan method, find the inverse of following Matrix (03)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}.$$

Q.2 Attempt the following:

(a) Solve the following system of equations: (03)

$$x + y + 2z = 8; -x - 2y + 3z = 1; 3x - 7y + 4z = 10.$$

By Gaussian elimination and back substitution.

(b) Find the Eigen values and Eigen vectors of the following Matrix

$$A = \begin{bmatrix} 8 & 0 & 3 \\ 2 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}. \quad (04)$$

Solve the following differential equations:

(c) (i) $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ (04)

(ii) $(x^2 - y^2)dy = 2xydx$ (03)

OR

Solve the following differential equations:

(c) (i) $(x^2 + y^2 - a^2)xdx + (x^2 - y^2 - b^2)ydy = 0$ (04)

(ii) $x \frac{dy}{dx} - ay = x + 1$ (03)

- Q.3 Attempt the following:
- (a) If $u = \log(x^3 + y^3 - x^2y - xy^2)$ prove that (05)
- $$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2 u = \frac{-4}{(x+y)^2}.$$

(b) If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, prove that $x \frac{du}{dx} + y \frac{du}{dy} = \tan u$. (05)

(c) If $x = u(1-v)$, $y = uv$ evaluate $\frac{\partial(x,y)}{\partial(u,v)}$. (04)

OR

- Q-3 Attempt the following :
- (a) If $f(x,y,z) = \log(x^2 + y^2 + z^2)$, prove that $xfyz = yfzx = zfxy$. (05)
- Find the maximum and minimum values of the function
- (b) $f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$. (04)
- Find the approximate value of $f(x,y,z) = \sqrt{x^2 + y^2 + z^2}$ at the point
- (c) (3.01, 4.02, 11.98). (05)

- Q.4 Attempt the following:
- (a) Evaluate $\iint_R y dy dx$, where R is the positive quadrant of the circle $x^2 + y^2 = 1$. (05)
- (b) Find the volume of ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. (05)
- (c) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz dy dx$. (04)

OR

- Q.4 Attempt the following:
- (a) Change the order of integration and evaluate (05)
- $$\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dy dx.$$
- (b) Change into polar co-ordinate and evaluate (05)
- $$\int_0^a \int_y^a \frac{x^2 dx dy}{\sqrt{x^2 + y^2}}.$$
- © By double integration, find the area common to the (04)
- Curves $y^2 = x$ and $x^2 = y$.

- Q.5** Attempt the following:
- (a) A particle moves along the curve $x = 1 + t^3$, $y = t^2$ and $z = 2t + 5$. Find the components of its velocity and acceleration at time $t=1$ in the direction of $2\hat{i} + \hat{j} + 2\hat{k}$. (05)
- A vector field is given by $\vec{F} = (x + 3y)\hat{i} + (y - 3z)\hat{j} + (x - 2z)\hat{k}$ (05)
- (b) Show that \vec{F} is solenoidal.
show that the differential equation for the current i in an electrical circuit containing an inductance L and resistance R in series and acted on by an electromotive force $E \sin \omega t$ satisfies the equation
- (c) $L \frac{di}{dt} + Ri = E \sin \omega t$. (04)
Find the value of the current at any time t , if initially there is no current in the circuit.

OR

- Q.5** Attempt the following:
- (a) Find the directional derivative of $f(x, y, z) = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of vector $\hat{i} + 2\hat{j} + 2\hat{k}$. (05)
- (b) Apply Green's theorem to evaluate $\int_c (2x^2 - y^2)dx + (x^2 + y^2)dy$, (05)
 Where c is the boundary of the area enclosed by the x axis and the upper half of the circle $x^2 + y^2 = a^2$.
- (c) Find the orthogonal trajectories of the family of parabolas $y = ax^2$. (04)

GUJARAT TECHNOLOGICAL UNIVERSITY
P.D.D.C. Sem- I Regular / Remedial Examination January. 2011

Subject code: X10001

Date: 03 / 01 /2011

Subject Name: Mathematics – I

Time: 10.30 am – 01.30 pm

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** Do as directed. **02**
- (a) Which of the following matrices are in row-echelon form, reduced row-echelon form, both or neither. **03**
- $$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -7 & 5 & 5 \\ 0 & 1 & 3 & 2 \end{bmatrix}.$$
- (b) Using determinant method find the rank of a matrix **03**
- $$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}.$$
- (c) Using Gauss-Jordan method find the inverse of a matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$. **03**
- (d) Evaluate : $\int_0^5 \int_0^{x^2} x(x^2 + y^2) dx dy$. **03**
- (e) Find $\text{curl } \vec{F}$, where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$. **03**
- Q.2** (a) 1) Solve **04**
- $$\begin{aligned} -2y + 3z &= 1 \\ 3x + 6y - 3z &= -2 \\ 6x + 6y + 3z &= 5 \end{aligned}$$
- By Gaussian elimination and back-substitution.
- 2) Solve : $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$. **03**
- (b) 1) If $u = \tan^{-1}\left(\frac{y^2}{x}\right)$, prove that **04**
- $$x \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\sin^2 u \sin 2u.$$
- 2) If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$, Find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. **03**
- OR**
- (b) 1) If $z(x+y) = x^2 + y^2$, then show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$. **04**
- 2) Solve : $x \frac{dy}{dx} = y^2 + y$. **03**

- Q.3 (a)** Find the eigen values and eigen vectors of a matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. **05**
- (b)** Find the extreme values of a function $f(x, y) = x^3 y^3 (1 - x - y)$. **05**
- (c)** Find the orthogonal trajectories of the family of parabola $y = ax^2$. **04**
- OR**
- Q.3 (a)** Solve : $x \log x \frac{dy}{dx} + y = \log x^2$. **05**
- (b)** Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing to polar coordinates. **05**
- (c)** Solve : $(x^2 + y^2 - a^2)xdx + (x^2 - y^2 - b^2)ydy = 0$. **04**
- Q.4 (a)** Trace the curve $y^2(a - x) = x^2(a + x)$. **05**
- (b)** Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy dx dy$, and hence evaluate the same. **05**
- (c)** Evaluate : $\int_0^4 \int_0^x \int_0^{x+y} z dz dy dx$. **04**
- OR**
- Q.4 (a)** Trace the curve $r^2 = a^2 \cos 2\theta$. **05**
- (b)** Find, by double integration, the area lying between the parabola $y = 4x - x^2$ and the line $y = x$. **05**
- (c)** Solve : $\frac{dy}{dx} + y \tan x = y^2 \sec x$. **04**
- Q.5 (a)** Find the directional derivative of $\phi = x^2 yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the vector $2\bar{i} - \bar{j} - 2\bar{k}$. **05**
- (b)** If $\vec{F} = (5xy - 6x^2)\bar{i} + (2y - 4x)\bar{j}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the curve C in the xy -plane, $y = x^3$ from the point $(1, 1)$ to $(2, 8)$. **05**
- (c)** If $u = x^2 + y^2 + z^2$, and $\vec{r} = x\bar{i} + y\bar{j} + z\bar{k}$, then find $\text{div}(\vec{r}u)$ in terms of u . **04**
- OR**
- Q.5 (a)** Verify Green's theorem for $\int_C [(3x - 8y^2)dx + (4y - 6xy)dy]$ where C is the boundary of the region bounded by $x = 0$, $y = 0$ and $x + y = 1$. **05**
- (b)** Find constant 'a' so that $\vec{F} = y(ax^2 + z)\bar{i} + x(y^2 - z^2)\bar{j} + 2xy(z - xy)\bar{k}$ is solenoidal. **05**
- (c)** Prove that $\nabla r^n = nr^{n-2}\vec{r}$, where $\vec{r} = x\bar{i} + y\bar{j} + z\bar{k}$. **04**

GUJARAT TECHNOLOGICAL UNIVERSITY**P.D.D.C. Sem- I Remedial Examination March / April 2010****Subject code: X 10001****Subject Name: Mathematics – I****Date: 30 / 03 / 2010****Time: 12.00 noon – 2.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 Do as directed.(a) Find the unit normal vector to the surface $x^2 + y^2 + z^2 = 7$ at $(1, -1, 2)$. **02**(b) Trace the curve $y^2(2a - x) = x^3$. **03**(c) Solve $(x^2 + y^2 - a^2)xdx + (x^2 + y^2 - b^2)ydy = 0$. **03**(d) Determine rank of the following matrix by row echelon form. **03**

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 6 \\ 4 & 6 & 8 & 10 \\ 7 & 10 & 13 & 16 \end{bmatrix}$$

(e) Find the inverse of the matrix **03**

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}, \text{ using gauss-Jordan method.}$$

Q.2 Attempt the following:(a) Solve $x + y + 2z = 9$ **03**

$$2x + 4y - 3z = 1$$

$$3x + 6y - 5z = 0 \text{ by Gaussian elimination and back substitution.}$$

(b) Find the eigen values and eigen vectors of the matrix **04**

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

(c) Solve the following differential equations:

$$(i) \frac{dy}{dx} = \cos x \cos y - \sin x \sin y. \quad \mathbf{04}$$

$$(ii) \left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0. \quad \mathbf{03}$$

OR

(c) Solve the following differential equations:

$$(i) \frac{dy}{dx} + 2y \tan x = \sin x \text{ given that } y = 0 \text{ when } x = \frac{\pi}{3} \quad \mathbf{04}$$

$$(ii) \frac{dy}{dx} + \frac{1}{x}y = x^2y^6 \quad \mathbf{03}$$

- Q.3** Attempt the following:
- (a) If $v = \log r$, where $r^2 = x^2 + y^2$, show that **05**
- $$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$
- (b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, show that **05**
- (i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$
- (ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \sin u \cos 3u$
- (c) If $r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1} \frac{y}{x}$, evaluate $\frac{\partial(r, \theta)}{\partial(x, y)}$. **04**

OR

- Q.3** Attempt the following:
- (a) If $u = \log(\tan x + \tan y + \tan z)$, prove that **05**
- $$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 0$$
- (b) Find the maximum and minimum values of **05**
- $$x^3 + 3xy^2 - 3x^2 - 3y^2 + 4.$$
- (c) The period of a simple pendulum is given by $T = 2\pi \sqrt{\frac{l}{g}}$. **04**
- If T is computed using $l = 8 \text{ ft}$, $g = 32 \text{ ft/sec}^2$, find approximate error in T if true values are $l = 8.05 \text{ ft}$ and $g = 32.01 \text{ ft/sec}^2$.

- Q.4** Attempt the following:
- (a) Evaluate $\iint_R xy \, dy \, dx$ where R is the positive quadrant of the circle $x^2 + y^2 = a^2$. **05**
- (b) change the order of integration in the integral $\int_0^{\frac{x^2}{2a}} \int_{\frac{x^2}{4a}}^{\frac{x^2}{2a}} xy \, dy \, dx$ **05**
- and hence evaluate it.
- (c) Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) \, dy \, dx \, dz$. **04**

OR

- Q.4** Attempt the following:
- (a) Evaluate $\int_0^a \int_0^{\sqrt{a^2 - y^2}} (x^2 + y^2) \, dx \, dy$ by changing into polar co-ordinates **05**
- (b) By double integration, find the area common to the curves $y^2 = 8x$ and $x^2 = 8y$. **05**
- (c) Find the volume of the solid bounded by the surfaces $x = 0, y = 0, z = 0$ and $x + y + z = 1$. **04**

- Q.5** Attempt the following:
- (a) A particle moves along the curve $x = t^3 + 1, y = t^2, z = 2t + 5$ where t is the time. Find the component of its velocity and acceleration at time $t = 1$ in the direction $\hat{i} + \hat{j} + 3\hat{k}$. **05**

- (b) A vector field is given by $\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$. Show that \vec{F} is irrotational and find its scalar potential. **05**
- (c) The current i flowing in the circuit containing resistance R , inductance L and e.m.f E Satisfies the differential equation $L \frac{di}{dt} + Ri = E$. Prove that **04**

$$i = \frac{E}{R} \left(1 - e^{-\frac{Rt}{L}} \right), \text{ if } i = 0, \text{ when } t = 0.$$

OR

Q.5 Attempt the following:

- (a) Using the line integral, compute the work done by the force $\vec{F} = y\hat{i} + xz\hat{j} - x\hat{k}$ When it moves a particle from the point $(0, 0, 0)$ to the point $(2, 1, 1)$ along the Curve $x = 2t^2, y = t, z = t^3$. **05**
- (b) Verify Green's theorem in the plane for $\oint_c (2x - y^2)dx - xydy$, where c is the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. **05**
- (c) Find the orthogonal trajectories of the circles $(x - a)^2 + y^2 = a^2$. **04**

Seat No. _____

Enrollment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

P.D.D.C Sem – I Examination, January – 2010

Subject code: X10001

Subject Name : Mathematics - I

Date: 01 / 01 / 2010

Time: 11.00 am – 2.00 pm

Total marks:70

Instructions:

- 1. Attempt all questions.**
- 2. Make suitable assumption wherever necessary.**
- 3. Figure to the right indicate full marks.**

Q.1. Do as directed.

(a) Find vector normal to the surface $x^2 + y^2 - z = 1$ at the point (1, 1, 1). (02)

(b) Solve the differential equation $\frac{dy}{dx} + \frac{1}{x}y = x^2y^6$. (03)

(c) Trace the curve $y^2(a-x) = x^2(a+x)$. (03)

(d) Determine Rank of the following matrix by row echelon form (03)

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 7 & 8 & 9 & 10 \\ 12 & 13 & 14 & 15 \end{bmatrix}$$

(e) Using Gauss-Jordan method, find inverse of following Matrix (03)

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Q.2. Attempt the following:

(a) Solve the following system of equations: (03)

$$x + y + z = 6;$$

$$x + 2y + 3z = 14; \text{ by Gaussian elimination and back substitution.}$$

$$x + 4y + 9z = 36.$$

(b) Find Eigen values and Eigen vectors of following Matrix (04)

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

(c) Solve the following differential equation

(i) $x^4 \frac{dy}{dx} + x^3y - \sec xy = 0$ (04)

(ii) $(5x^4 + 6x^2y^2 - 8xy^3)dx + (4x^3y - 12x^2y^2 - 5y^4)dy = 0$. (03)

OR

(c) Solve the following differential equation

(i) $x^2ydx - (x^3 + y^3)dy = 0$ (04)

(ii) $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$. (03)

Q.3. Attempt the following:

(a) If $V = r^m$, where $r^2 = x^2 + y^2 + z^2$ prove that (05)

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = m(m+1)r^{m-2}.$$

(b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, show that (05)

$$(i) x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$$

$$(ii) x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \sin u \cos 3u.$$

(c) If $x = r \sin^2 \theta$, $y = r \cos^2 \theta$, prove that $\frac{\partial(x, y)}{\partial(r, \theta)} = r \sin 2\theta$. (04)

OR

Q.3. Attempt the following:

(a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that (05)

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x + y + z)^2}.$$

(b) If $u = f\left(\frac{y}{x}\right) + x\phi\left(\frac{x}{y}\right) + 4xy$ prove that (05)

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 8xy.$$

(c) Find the approximate value of $f(x, y) = \sqrt{x^2 + y^2}$ at the point (3.01, 4.02). (04)

Q.4. Attempt the following:

(a) Change the order of integration and evaluate (05)

$$\int_0^\infty \int_0^x x e^{-\frac{x^2}{y}} dy dx.$$

(b) Change into polar co-ordinate and evaluate (05)

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2 + y^2) dy dx.$$

(c) Find the area bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ (04)

OR

Q.4. Attempt the following:

(a) Evaluate $\iint \frac{xy}{\sqrt{1-y^2}} dx dy$ over the +ve quadrant of the circle $x^2 + y^2 = 1$. (05)

(b) Evaluate $\int_{-1}^1 \int_0^x \int_{x-y}^{x+y} (z - 2x - y) dz dy dx$. (05)

- (c) Find the volume of the solid bounded by the surfaces $x = 0, y = 0, z = 0$ and $x + y + z = 1$. (04)

Q.5. Attempt the following:

- (a) If $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2yz + 2z)\hat{k}$ (05)

Show that \vec{F} is both solenoidal and irrotational.

- (b) If $\phi = 2xy^2z + x^2y$, evaluate $\int_c \text{grad} \phi \cdot d\vec{r}$, where c is the curve $x = t$, (05)

$y = t^2, z = t^3$ from $t = 0$ to $t = 1$.

- (c) The charge Q on the plate of a condenser of capacity C charged through a resistance R by a steady voltage V satisfies the differential equation (04)

$$R \frac{dQ}{dt} + \frac{Q}{C} = V, \text{ if } Q = 0 \text{ at } t = 0, \text{ show that } i = \frac{V}{R} e^{\frac{-t}{RC}}.$$

OR

Q.5. Attempt the following:

- (a) Find $f(r)$ such that $\nabla^2 f(r) = 0$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ (05)

- (b) Verify Green's theorem for $\oint_c [(xy + y^2)dx + x^2dy]$, where c is (05)

bounded by $y = x$ and $y = x^2$.

- (c) Find the orthogonal trajectories of confocal and coaxial parabolas (04)

$$r = \frac{2a}{1 + \cos \theta}.$$
