GUJARAT TECHNOLOGICAL UNIVERSITY

2^{nd} Semester Civil Engineering – PDDC

Subject Code & Name : X20001 - Mathematics-II

Sr. No.	Course content
1.	**Beta and Gamma Functions :** Definition, Basic properties, Relation between Beta and Gamma functions, Use in evaluation of definite integrals, Duplication formula via Beta Gamma.
2.	**Laplace Transforms :** Definition, Linearity property, Laplace transforms of elementary functions, First shifting theorem, Differentiation and integration of Laplace transforms. Inverse Laplace transform , Laplace transforms of derivatives and integrals, Convolution theorem, Application of Laplace transforms to solve ordinary differential equations
3.	**Fourier Series :** Periodic functions, Dirichlet's conditions, Fourier Series, Euler's formulae, Fourier expansion of periodic functions with period 2л, Fourier Series of even and odd functions, Fourier series of periodic functions with arbitrary periods, Half – range Fourier series.
4.	**Fourier Integrals and transforms :** Fourier integral theorem (Only statement), Fourier Sine and Cosine integrals, Fourier Transforms, Fourier Sine and Cosine transforms
5.	**Higher Order Differential Equation :** Linear differential equations of higher order with constant coefficient, Method of variation of parameter, Cauchy's homogeneous linear equation, Legendre's homogeneous linear equation, Simultaneous linear differential equations, Application of linear differential equations, Modelling : Mechanical vibration system, Electrical circuit system & Deflection of beams.
6.	**Partial differential equations :** Formation of partial differential equations, Directly integrable equations, Lagrange's equation, Solution of special types of non-linear partial differential equation of the first order, Equations reducible to the standard forms, Application of partial differential equations, Boundary value problems and method of separation of variables, Vibrations of a stretched Elastic string.
7.	**Z – transforms :** Z transforms of the standard functions like e^{kx}, λ^n, Sin hx, Coshx. Linearity property, Damping rule, Initial value and final value problem.

References Books:

1. Elementary Engineering Mathematics by Dr. B.S. Grewal,
   Khanna Publishers, New Delhi.
2. Higher Engineering Mathematics by Dr. B.S. Grewal,
   Khanna Publishers, New Delhi.
3. A Textbook of Engineering Mathematics by N.P. Bali, Ashok Saxena & Iyengar,
   Laxmi Publications (P) Ltd., New Delhi.
4. Advanced Engineering Mathematics by H.K. Dass
   S. Chand & Co. (Pvt.) Ltd., New Delhi.
5. Engineering Mathematics Vol. – I, II, III by G.V. Kumbhojkar, C.
   Jamnadas & Co., Bombay.

Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC - SEMESTER–II • EXAMINATION – WINTER 2013

Subject Code: X20001 **Date: 18-12-2013**

Subject Name: Mathematics-II

Time: 02.30 pm - 05.30 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** 1) Define Beta function. Compute $\beta(2.5,1.5)$. **03**

2) Prove that $\int_0^1 x^3\left(1-\sqrt{x}\right)^5 dx = 2\beta(8,6)$. **04**

(b) 1) Form the partial differential equation from $z = ax + by + a^2 + b^2$ **03**

2) Solve $\frac{\partial^2 z}{\partial x^2} = xy$ **04**

Q.2 **(a)** Solve $\frac{d^{3y}}{dx^3} + 2\frac{d^2 y}{dx^2} + \frac{dy}{dx} = e^{-x} + \sin 2x$ **07**

(b) Solve $y'' + y = \tan x$ by the method of variation of parameter. **07**

OR

(b) Solve $x^3\frac{d^3 y}{dx^3} + 3x^2\frac{d^2 y}{dx^2} + x\frac{dy}{dx} + 8y = 65\cos(\log x)$. **07**

Q.3 **(a)** Find the Fourier series expansion of $f(x) = 2x - x^2$ in $(0,3)$, $f(x+3) = f(x)$. **07**

(b) State Convolution theorem and using it, evaluate $L^{-1}\left\{\frac{s}{(s+2)(s^2+9)}\right\}$. **07**

OR

Q.3 **(a)** Obtain a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$, $f(x+2\pi) = f(x)$. **07**

(b) Using Laplace transform method, Solve $y'' + y = t,\ y(0) = 1, y'(0) = -2$ **07**

Q.4 **(a)** Express $f(x) = x$ as a half range cosine series in $0 < x < 2$. $f(x+4) = f(x)$. **07**

(b) 1) Find $L\left\{(t+2)^2 e^t\right\}$. **03**

2) Find $L^{-1}\left\{\tan^{-1}\left(\frac{2}{s}\right)\right\}$ **04**

OR

Q.4 **(a)** Find a Fourier series to represent x^2 in the interval $(-l,l)$, $f(x+2l) = f(x)$. **07**

(b) 1) Find $L^{-1}\left\{\frac{1}{s^2 - 5s + 6}\right\}$. **03**

2) Find $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$. **04**

Q.5 **(a)** 1) Solve $p + q = \sin x + \sin y$. **03**

2) Solve $x(y-z)p + y(z-x)q = z(x-y)$. **04**

(b) Express the function $f(x)=\begin{cases}1, & |x|<1\\ 0, & |x|>1\end{cases}$ as a Fourier integral. Hence evaluate $\int_0^{\infty}\frac{\sin\lambda\cos\lambda x}{\lambda}\,d\lambda.$ **07**

OR

Q.5 **(a)** Solve the equation $3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0,\quad u(x,0)=4e^{-x}$ by the method of separation of variables. **07**

(b)
1) Solve $(D^4-4D^2+4)y=0$ **03**
2) Solve $p(1+q)=qz.$ **04**

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Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC - SEMESTER–II • EXAMINATION – SUMMER 2013

Subject Code: X20001 **Date: 04-06-2013**

Subject Name: Mathematics-II

Time: 02.30 pm - 05.00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Define Gamma and Beta Functions. **02**

(ii) Show that $\int_0^\infty \frac{e^{-3t}}{\sqrt{t}} dt = \sqrt{\frac{\pi}{3}}$. **03**

(iii) Find $L(t \sin t)$. **02**

(b) (i) Find $L(f(t))$ if $f(t) = \begin{cases} 1, 0 < t < 2 \\ 3, t > 2. \end{cases}$. **03**

(ii) Find the inverse Laplace transform of $\frac{3s-2}{(s+2)(s^2+1)}$. **03**

(iii) State Relation between Gamma and Beta Functions. **01**

Q.2 **(a)** (i) State Convolution theorem. Using it find $L^{-1}(\frac{1}{(s+1)(s+2)})$ **04**

(ii) Solve : $y'' + y = t, y(0) = 0 \,\&\, y'(0) = 1.$ Using Laplace transform. **03**

(b) (i) Evaluate $\int_0^\infty e^{-t} \cosh t dt$ by using Laplace transform. **03**

(ii) Show that $\int_0^{\frac{\pi}{2}} \sqrt{(\cot \theta)} d\theta = \frac{1}{2} \left\lceil \frac{1}{4} \right\rceil \frac{3}{4}$. **04**

OR

(b) (i) Find the inverse Laplace transform of $\log(\frac{s+2}{s+5})$. **03**

(ii) Evaluate $\int_0^1 \frac{dx}{(1-x^3)^{1/3}}$ by using Gamma-Beta functions. **04**

Q.3 **(a)** Find the Fourier series expansion of $f(x) = x - x^2, -\pi < x < \pi.$ **05**

(b) Find the Fourier series expansion of $f(x) = x^3, -2 < x < 2.$ **05**

(c) Find the Fourier sine transform of $f(x) = e^{-2x}$. **04**

OR

Q.3 **(a)** Find the Fourier series expansion of $f(x) = e^{-x}, 0 < x < 2\pi.$ **05**

(b) Find the Fourier cosine transform of $f(x)=\begin{cases} x^2, -1<x<0 \\ 1+x, 0<x<1 \\ 0, x>1. \end{cases}$ **05**

(c) Find the Fourier sine series of $f(x)=3-x, 0<x<3.$ **04**

Q.4 **(a)** Solve $(D^2+3D+2)y=x^2+e^{-x}.$ **05**

(b) Using the method of variation of parameter, solve $y''+y=\operatorname{cosec} x.$ **05**

(c) Solve $y''+9y=3x^2.$ **04**

OR

Q.4 **(a)** Solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x.$ **05**

(b) Solve $x^2y''-xy'+4y=\cos(\log x)+x\sin(\log x).$ **05**

(c) Solve the simultaneous equations: $\frac{dx}{dt}=-wy, \frac{dy}{dt}=wx.$ **04**

Q.5 **(a)** Form the partial differential equation from **05**

(i) $z=f(x^2+y^2)$, (ii) $f(xy+z^2, x+y+z)=0.$

(b) Using the method of separation of variables, solve $u_{xx}=25u_y.$ **05**

(c) Define Z-transform. Find the Z-transform of the sequence $\{a^m\}, m\geq 0.$ **04**

OR

Q.5 **(a)** Solve : **05**

(i) $\frac{\partial^2 z}{\partial x^2}=\cos x,$, (ii) $\frac{\partial^2 z}{\partial x\partial y}=x^2+y^2$

(b) Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ by using the method of separation of variables. **05**

(c) State the linearity property of Z-transform. Find the Z-transform of $\{f(k)\}$, where $f(k)=\begin{cases} 7^k, k<0 \\ 5^k, k\geq 0. \end{cases}$ **04**

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Seat No.: ________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC - SEMESTER – II • EXAMINATION – WINTER 2012

Subject code: X 20001 **Date: 12/01/2013**

Subject Name: Mathematics - II

Time: 10.30 am - 01.30 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 Attempt the following.

a) Define Beta function. Evaluate $\int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}}dx.$. **3**

b) Evaluate $\int_0^\infty e^{-x^2}dx$ in terms of gamma function. **3**

c) Define Gamma function and evaluate $\Gamma 4.5$ **2**

d) Find the Laplace Transform of $\sin 2t \sin 3t$. **2**

e) Solve : $(D^2+6D+9)y=0$ **2**

f) Solve : $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ **2**

Q.2 (a) Find the inverse Laplace Transforms of **6**

1) $\frac{3s+2}{s^2-s-2}$

1) $\cot^{-1}\left(\frac{s}{2}\right)$

(b) Find the Laplace Transformations of **4**

1) $e^{-3t}(2\cos 5t - 3\sin 5t)$

2) $\frac{1-e^t}{t}$

(c) Using Laplace Transform solve $y'' + y = t, y(0)=1, y'(0)=0.$ **4**

Q.3 (a) Solve the following differential equations: **6**

1) $(D^2-3D+2)y=\cos 3x$

2) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x$

(b) Using method of variation of parameters, solve the differential equation : **4**

$y'' - 6y' + 9y = e^{3x}/x^2$

(c) Find half range sine series of $f(x)=x$ in $0<x<2$ **4**

Q.4

(a) Find the Fourier series of $f(x)=\left(\frac{\pi - x}{2}\right)^2$ in the range $0\, to\, 2\pi$. **5**

	(b)	Find the Fourier series for the function $f(x)=\begin{cases}\pi x & if\ \ 0\le x\le 1\\ \pi(2-x) & if\ \ 1\le x\le 2\end{cases}$	**5**
	(c)	Solve $x^2\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}+y=\log x$	**4**
Q.5	(a)	Find the Fourier series for the function $f(x)=x-x^2$ in the interval $-\pi\le x\le\pi$.	**5**
	(b)	Solve $(1+x)^2\dfrac{d^2y}{dx^2}+(1+x)\dfrac{dy}{dx}+y=2\sin[\log(1+x)]$.	**5**
	(c)	Find the Z-transforms of 1) e^{an} 2) $(n+1)^2$	**4**
Q. 6	(a)	Solve the following equations: 1) $p\tan x+q\tan y=\tan z$ 2) $(z-y)p+(x-z)q=y-x$	**6**
	(b)	Form the partial differential equation from 1) $(x-a)^2+(y-b)^2+z^2=c^2$. 2) $z=f\left(x^2-y^2\right)$	**4**
	(c)	Using the Fourier integral representation, show that $\displaystyle\int_0^\infty\frac{\omega\sin x\omega}{1+\omega^2}d\omega=\frac{\pi}{2}e^{-x}\quad(x>0).$	**4**
Q. 7	(a)	Obtain the complete solution of the equations: 1) $p(1+q)=qz$ 2) $p+q=\sin x+\sin y$	**6**
	(b)	Find the Fourier sine transform of $e^{-\lvert x\rvert}$.	**4**
	(c)	Solve the equation $\dfrac{\partial u}{\partial x}=4\dfrac{\partial u}{\partial y}$, given that $u(0,y)=8e^{-3y}$ by the method of separation of variable.	**4**

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Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC SEM-II Examination May 2012

Subject code: X20001

Subject Name: Mathematics-II

Date: 22/05/2012 **Time: 10.30 am – 01.30 pm**
Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1	**(a)**	(i) $\beta(3,4) =$ ____________	**01**
		(ii) Write relation between Beta and Gamma function.	**01**
		(iii) Show that $\beta(m,n) = \beta(n,m)$	**01**
		(iv) Define Gamma function	**01**
		(v) Express integral $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ in term of gamma function.	**03**
	(b)	(i) Find the period of cos2x	**02**
		(ii) Find $L(e^{4t} + \cos 3t + t^4)$	**02**
		(iii) Solve differential equation $D^2y - a^2y = 0$	**03**
Q.2	**(a)**	(i) Find $L^{-1}\left(\frac{2s^2 - 6s + 5}{(s-1)(s-2)(s-3)}\right)$	**04**
		(ii) Find $L(e^{-3t}(\sin 5t - \cos 5t))$	**03**
	(b)	(i) Use Convolution theorem to evaluate $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$	**03**
		(ii) Using Laplace transform solve $y'' - y = e^{2t}$, $y(0) = y'(0) = 0$	**04**
		OR	
	(b)	(i) Evaluate $L\left(e^{3t}\int_0^t \frac{\sin t}{t}dt\right)$	**03**
		(ii) Using Lapalce transform solve $y'' - 2y' + y = e^t$, $y(0) = 2$, $y'(0) = -1$.	**04**
Q.3	**(a)**	(i) Find the Fourier series for $f(x) = x^3, -\pi \le x \le \pi, f(x+2\pi) = f(x)$.	**03**
		(ii) Find the Fourier series for $f(x) = 1$ if $0 \le x \le \pi$ and $f(x) = 0$ if $\pi \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$.	**04**
	(b)	Find a Fourier series for $f(x) = x - x^2$ from $x = -\pi$ to $x = \pi$. Hence prove that $\frac{\pi^2}{12} = \frac{1}{1^1} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$	**07**
		OR	
Q.3	**(a)**	(i) Express $f(x) = x$ as a half range cosine series in $0 < x < 2$.	**03**
		(ii) Express $f(x) = 1$ for $0 \le x \le \pi$ and $f(x) = 0$ for $x > \pi$ as Fourier sine integral and hence evaluate $\int_0^\infty \frac{1-\cos\pi\lambda}{\lambda}\sin x\lambda d\lambda$	**04**
	(b)	(i) Expand $\pi x - x^2$ in half range sine series in the interval $(0,\pi)$ up to the first three term.	**03**

Q.3 **(b)** (ii) Find the Fourier sine transform of $e^{-|x|}$. Hence evaluate $\int_0^\infty \frac{x\sin mx}{1+x^2}dx$. **04**

Q.4 **(a)** In an L-C-R circuit, the charge q on a plate of a condenser is given by $L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{c}=E\sin pt$. The circuit is tuned to resonance so that p^2 = 1/LC. If initially the current i and the charge q be zero, show that, for small values of R/L, the current in the circuit at time t is given by (Et/2L) sin(pt). **07**

(b) (i) Solve differential equation $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-3y=x^2$ **03**

(ii) Solve differential equation **04**

$(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=2\sin(\log(1+x))$

OR

Q.4 **(a)** (i) Solve differential equation $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\log x$ **03**

(ii) Solve differential equation $(D^2 + 1)y = \sin x$ **04**

(b) (i) Using variation of parameter solve $\frac{d^2y}{dx^2}+4y=\tan 2x$ **04**

(ii) Solve $\frac{d^2y}{dx^2}-\frac{dy}{dx}-6y=e^{3x}$ **03**

Q.5 **(a)** (i) Form partial differential equation from **03**

(A) $z=f(x^2+y^2)$ (B) $2z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

(ii) Solve $(mz - ny)p + (nx - lz)q = ly - mx$ **04**

(b) (i) Solve $p^2 + q^2 = x + y$ **03**

(ii) Solve (A) $z = px + qy + 2\sqrt{pq}$ (B) $p^2 + q^2 = 2$ **04**

OR

Q.5 **(a)** A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y=\sin\frac{\pi x}{l}$ from which it is released at time t = 0. Show that the displacement of any point at distance x from one end at time t is given by $y(x,t)=a\sin\frac{\pi x}{l}\cos\frac{\pi ct}{l}$. **07**

(b) (i) Find the Z- transform of ka^k , $k \geq 0$ **03**

(ii) Solve deference equation $U_{k+1} + U_k = 1$ if $U_0 = 0$ **04**

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Seat No.: ________ **Enrolment No.______________**

GUJARAT TECHNOLOGICAL UNIVERSITY
PDDC SEM-II Examination-Dec-2011

Subject code: X20001 **Date: 20/12/2011**
Subject Name: Mathematics -II
Time: 10.30 am -1.30 pm **Total marks: 70**

Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Prove that $\overline{)m+1} = m\overline{)m}$ **02**

(ii) $L(e^{4t}\,t^2)$ = ________ **02**

(iii) Show that $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\, d\theta = \frac{\pi}{\sqrt{2}}$ **03**

(b) (i) Show that $\beta(m,n) = \beta(m+1, n) + \beta(m, n+1)$ **02**

(ii) $L(t\,\sin t)$ = ________ **02**

(iii) Prove that $\int_0^1 \left(\log\frac{1}{y}\right)^{n-1} dy = \overline{)n}$, $n > 0$ **03**

Q.2 **(a)** (i) Evaluate $\int_0^\infty e^{-3t}\sin 4t\,dt$ using Laplace transform. **03**

(ii) State convolution theorem. Find $L^{-1}\left(\frac{1}{(s-2)(s-1)}\right)$ **04**

(b) (i) Define unit step function. Find Laplace of unit step function. **03**

(ii) Using Laplace transform solve **04**
$y'' - y' - 6y = e^{-t}$, $y(0) = y'(0) = 0$

OR

(b) (i) Evaluate: $L(e^{5t}\,t^2\,\sin t)$ **03**

(ii) Evaluate: $L\left(e^{3t}\int_0^t \frac{\sin t}{t}\,dt\right)$ **04**

Q.3 **(a)** (i) Draw the graph of periodic function $f(x) = 1$ if $0 < x < 2$ and $f(x) = -1$ if $-2 < x < 0$. Check whether $f(x)$ is even or odd. **03**

(ii) Find the Fourier series for $f(x) = x^2$, $-\pi < x < \pi$, $f(x + 2\pi) = f(x)$. **04**

(b) (i) Write Dirichlet's Conditions for Fourier series. **03**
Fourier series of $\tan^{-1}(x)$ in interval $(0, 2\pi)$ does not exist. (true/false).

(ii) Find Fourier integral representation of the function **04**
$f(x) = 1$ if $|x| < 1$, $f(x) = 0$ if $|x| > 1$.

OR

Q.3 **(a)** (i) If f(x) = $e^{|x|}, -\pi < x < \pi$, f(x + 2π) = f(x) then check f(x) is even or odd also find Fourier coefficient b_n. **03**

(ii) Find the Fourier series for f(x) = - k if - π < x < 0 and f(x) = k if 0 < x < π , f(x + 2π) = f(x) **04**

(b) (i) Find F_c (e^{-x}). **03**

(ii) Expand π x – x^2 in a half range sine series in interval (0, π). **04**

Q.4 **(a)** (i) Solve (D^2 – D – 12)y = e^{2x} +5 **03**

(ii) Solve by method of variation of parameter $D^2y + y = \sec x$ **04**

(b) (i) Solve (D^2 – 2D +1) y = sinx **03**

(ii) Solve $x^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - 3y = x^2$ **04**

OR

Q.4 **(a)** In L-C-R circuit the charge q on a plate of condenser is given by $L\dfrac{d^2q}{dt^2} + R\dfrac{dq}{dt} + \dfrac{q}{C} = E\sin wt$. The circuit is tuned to resonance so that w^2 = 1/LC. If R^2 < 4L/C and $q = 0 = \dfrac{dq}{dt}$ when t = 0. Find q(t) **07**

(b) (i) Solve $(1+2x)^2 \dfrac{d^2y}{dx^2} - 6(1+2x)\dfrac{dy}{dx} + 16y = 8(1+2x)^2$ **03**

(ii) If in a mass spring system mass = 4kg spring constant = 64, f(t) = 8 sin4t and if there is no air resistance then find the subsequent motion of the weight. **04**

Q.5 **(a)** (i) Form partial differential equation from following equation. (A) z = ax + bx + ab (B) z = f(x^2 –y^2). **03**

(ii) (A) Solve $y^2zp + x^2zq = xy^2$ **04**

(B) Solve (y – z)p + (x – y)q = z – x

(b) (i) (A) Solve z = px + qy + p^2q^2 (B) Solve $p^2 + q^2$ = 2. **03**

(ii) Solve deference equation $U_{k+1} + U_k$ = 1 if U_0 =0 **04**

OR

Q.5 **(a)** (i) Prove that Z-transform is linear. **03**

(ii) Solve the following equation by method of separation of variables $\dfrac{\partial u}{\partial x} = 4\dfrac{\partial u}{\partial y}$ where u(0,y) = 8 e^{-3y}. **04**

(b) A tightly stretched string with fixed end points x = 0 and x = L is initially at rest in it equilibrium position. If it is set vibrating by giving to each its points a velocity λx(L – x), find the displacement of the string at any distance x from one end at any time t. **07**

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Seat No.: _____ Enrolment No._______

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC 2ND Semester Examination – July- 2011

Subject code:X20001

Subject Name: MATHEMATICS-II

Date:11/07/2011 **Time: 10:30 am – 01:30 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 Attempt the following.

a) Prove that $\frac{\beta(m+1,n)}{m} = \frac{\beta(m,n+1)}{n} = \frac{\beta(m,n)}{m+n}$. **3**

b) Express the integral $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ in terms of Gamma functions. **3**

c) Define Beta function and compute $\beta(2.5,1.5)$.

d) Find : $L\{(t+2)^2 e^t\}$. **2**

e) Solve : $\frac{d^2y}{dx^2} + a^2 y = 0$. **2**

f) Solve : $\frac{\partial^2 z}{\partial x^2} = xy$. **2**

2

Q.2 **(a)** (1) Find the Laplace Transformations of **4**

a) $\frac{1-e^t}{t}$

b) $te^{-2t}\sin 2t$

(2) Using Convolution theorem evaluate $L^{-1}\left\{\frac{1}{s(s^2+4)}\right\}$. **3**

(b) (1) Find the inverse Laplace Transforms of **4**

a) $\frac{3s}{s^2+2s-8}$

b) $\log\left(\frac{s+a}{s+b}\right)$

(2) Evaluate : $\int_0^\infty te^{-2t}\sin t\,dt$ **3**

OR

(b) (1) Using Laplace Transform, solve the differential equation **4**

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^t$ with y (0) = 2, y′(0) = -1.

(2) Find : $L^{-1}\left\{\frac{5s+3}{(s-1)(s^2+2s+5)}\right\}$. **3**

Q.3 **(a)** Solve the following differential equations : **6**

(1) $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}+\frac{dy}{dx}=e^{-x}+\sin 2x.$

(2) $\frac{d^2y}{dx^2}+\frac{dy}{dx}=x^2+2x+4.$

(b) Using method of variation of parameters, solve the differential equation: $(D^2+4)y=\tan 2x$. **4**

(c) The differential equation for a circuit in which self-inductance and capacitance neutralize each other is $L\frac{d^2i}{dt^2}+\frac{i}{C}=0$. Find the current i as a function of t given that I is the maximum current and $i = 0$ when $t=0$. **4**

OR

Q.3 **(a)** Solve the following differential equations : **6**

(1) $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=e^{-2x}\sin 2x$

(2) $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=4\cos^2 x.$

(b) Solve : $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\cos(\log x).$ **4**

(c) The deflection of a strut of length l with one end (x = 0) built-in and the other supported and subjected to end thrust P satisfies the equation $\frac{d^2y}{dx^2}+a^2y=\frac{a^2R}{P}(l-x)$. Prove that the deflection curve is $y=\frac{R}{P}\left(\frac{\sin ax}{a}-l\cos ax+l-x\right)$, where *al = tan al*. **4**

Q.4 **(a)** Find the Fourier series to represent $x-x^2$ from $x=-\pi$ to $x=\pi$. **5**

(b) Find the Fourier series in the interval (-2,2) if $f(x)=\begin{cases}0;-2<x<0\\1;0<x<2.\end{cases}$ **5**

(c) Find the Fourier cosine transform of $f(x)=e^{-ax}$; Hence evaluate $\int\frac{\cos\lambda x}{x^2+a^2}dx.$ **4**

OR

Q.4 **(a)** Find the Fourier series to represent the function *f(x)* given by $f(x)=\begin{cases}x;\ for\ 0\le x\le\pi\\2\pi-x;\ for\ \pi\le x\le 2\pi.\end{cases}$ **5**

(b) Find the Fourier series to represent πx in the interval $0\le x\le 2$. **5**

(c) Using the Fourier sine transform of $e^{-ax}(a>0)$, show that $\int\frac{x\sin kx}{a^2+x^2}dx=\frac{\pi}{2}e^{-ak}(k>0).$ **4**

Q.5 **(a)** Attempt the following: **6**

(1) Solve : $x(y-z)p+y(z-x)q=z(x-y).$

(2) Solve : $x^2p^2+y^2q^2=z^2.$

(b) Attempt the following : **4**

(1) Form the partial differential equation from $z = f\left(\frac{xy}{z}\right)$.

(2) Solve : $pq+p+q=0$.

(c) Attempt the following : **4**

(1) Find z-transform of $a^k \cos \alpha k; k \geq 0$.

(2) Find the inverse z-transform of $\frac{z}{(z-2)(z-3)}; |z| > 3$.

OR

Q.5 **(a)** Attempt the following : **6**

(1) Solve : $(y+z)p-(z+x)q = x-y$.

(2) Solve : $(x^2-yz)p+(y^2-zx)q = z^2-xy$

(b) Solve by the method of separation of variable $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t}+u$, where $u(x,0) = 6e^{-3x}$. **4**

(c) Attempt the following : **4**

(1) Find the z-transform of $ka^k; k \geq 0$.

(2) Find the inverse z-transform of $\frac{z^2}{(z-1)(z-\frac{1}{2})}; |z| > 1$.

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Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

P.D.D.C. Sem- II Remedial Examination Nov / Dec. 2010

Subject code: X20001 **Subject Name: Mathematics-2**

Date: 27 / 11 / 2010 **Time: 10.30 am – 01.30 pm**

Instructions: **Total Marks: 70**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 Do as directed. **14**

(a) DefineBetafunction.Provethat

$$\beta(m,n)=2\int_0^{\frac{\pi}{2}}\sin^{2m-1}\theta\cos^{2n-1}\theta d\theta$$

(b) Prove that $\beta(m,n+1)=\dfrac{n}{m+n}\beta(m,n)$.

(c) Define Gamma Function. Prove that $\int_0^{\infty}e^{-x^2}dx=\dfrac{\sqrt{\pi}}{2}$.

(d) Prove that $\int_0^{\frac{\pi}{2}}\sqrt{\sin\theta}d\theta\times\int_0^{\frac{\pi}{2}}\dfrac{d\theta}{\sqrt{\sin\theta}}=\pi$.

(e) Show that $\int_0^{\infty}x^2e^{-x^4}dx=\dfrac{1}{4}\overline{\left|\dfrac{3}{4}\right.}$.

(f) Prove that $\int_0^1\dfrac{dx}{\sqrt{-\log x}}=\sqrt{\pi}$.

(g) Prove that $\overline{\left|\dfrac{1}{2}\right.}=\sqrt{\pi}$.

Q.2 **(a)** Find the Laplace transform of the following: (any two) **04**

(i) $e^{3t}\sin^2 t$ (ii) $t\cos 2t$ (iii) $\dfrac{\cos at-\cos bt}{t}$

(b) State convolution theorem.Using it find inverse Laplace transform of $\dfrac{1}{(s+2)(s+3)}$. **03**

(c) (I)Find the Laplace transform of the following: (any two) **03**

(i) $\cos 2t\sin 2t$ (ii) $te^{-t}\cos 2t$ (iii) $\dfrac{1-e^t}{t}$

(II)Find the inverse Laplace transforms of **04**

(i) $\dfrac{s+2}{s^2-4s+13}$ (ii) $\log\left(\dfrac{s+1}{s-1}\right)$.

OR

(c) (I)Find the inverse Laplace transforms of **04**

(i) $\tan^{-1}\left(\dfrac{2}{s^2}\right)$ (ii) $\dfrac{2s^2-6s+5}{(s-1)(s-2)(s-3)}$.

(II)By using method of Laplace transform solve the initial value problem $y''+4y'+3y=e^{-t}, y(0)=y'(0)=1$. **03**

Q.3 **(a)** Solve: **05**

(i) $(D^2+5D+6)y=e^x$

(ii) $(D^2 - 5D + 6)y = \sin 3x$.

(b) Using the method of variation of parameter solve the differential equation $y'' + y = \sec x$. **05**

(c) Solve $x^2 \dfrac{d^2 y}{dx^2} - x\dfrac{dy}{dx} + y = \log x$. **04**

OR

Q.3 (a) Solve: **05**

(i) $(D^2 + D)y = x^2 + 2x + 4$

(ii) $(D^2 - 2D + 4)y = e^x \cos x$.

(b) Solve $(1+x)^2 \dfrac{d^2 y}{dx^2} + (1+x)\dfrac{dy}{dx} + y = 2\sin(\log(1+x))$. **05**

(c) Show that the frequency of free vibration in a closed electrical Circuit with inductance L and capacity C in series is $\dfrac{30}{\pi\sqrt{LC}}$ per minute. **04**

Q.4 (a) Find the Fourier series of the function $f(x) = x^2, -\pi < x < \pi$. **05**

(b) Find the Fourier series expansion for $f(x)$, if **05**

f(x)= $\{-\pi, -\pi < x < 0$

$\{x, 0 < x < \pi$

(c) Find the Fourier transform of f(x)= $\{1, |x| < 1$ **04**

$\{0, |x| > 1$

Hence evaluate $\int_0^\infty \dfrac{\sin x}{x} dx$

OR

Q.4 (a) Obtain the Fourier series for f(x)=e^{-x} in the interval $0 < x < 2\pi$. **05**

(b) Express $f(x) = x$ as a half range sine series in $0 < x < 2$. **05**

(c) Find the Fourier sine transform of $\dfrac{e^{-ax}}{x}$. **04**

Q.5 (a) Form the partial differential equation from: **05**

(i) $2z = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$

(ii) $z = f(x^2 - y^2)$.

(b) Solve: **05**

(i) $(mz - ny)\dfrac{\partial z}{\partial x} + (nx - lz)\dfrac{\partial z}{\partial y} = ly - mx$

(ii) $((x^2 - y^2 - z^2)p + 2xyq = 2xz$.

(c) (i)State and prove linearity property of Z transform. **04**

(ii)state and prove Damping rule of Z transform.

OR

Q.5 (a) Solve by the method of separation of variable **05**

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

(b) Solve (i) $\sqrt{p} + \sqrt{q} = 1$ **05**

(ii) $p^2 + q^2 = x + y$.

(c) Define Z transform . **04**

Solve (i) $z(a^n)$

(ii) $z(n^p)$.

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