4

Sediment Transport and Design of Irrigation Channels

Whenever water flows in a channel (natural or artificial), it tries to scour its surface. Silt or gravel or even larger boulders are detached from the bed or sides of the channel. These detached particles are swept downstream by the moving water. This phenomenon is known as *Sediment Transport.*

4.1. Importance of Sediment Transport

(*i*) The phenomenon of sediment transport causes large scale scouring and siltation of irrigation canals, thereby increasing their maintenance. Many poorly designed artificial channels get silted up so badly, that they soon become inoperable, causing huge economic loss to the public exchequer. The artificial channels should, therefore, be properly designed, and should not fail to carry the sediment load admitted at the canal headworks.

(*ii*) The design and execution of a flood control scheme is chiefly governed by the peak flood levels, which, in turn, depend upon the scour and deposition of sediment. Firstly, the bed levels may change by direct scouring or deposition of sediment, and thereby changing the flood levels. Secondly, the scouring and silting on the river banks may create sharp and irregular curves, which increase the flow resistance of the channel, and thereby, raising the flood levels for the same discharge.

(*iii*) Silting of reservoirs* and rivers is another important aspect of sediment transport. The storage capacity of the reservoir is reduced by its silting, thereby, reducing its use and life. Natural rivers used for navigation are frequently silted up, reducing the clear depth (draft) required for navigation. Sediment deposited in these rivers and harbours may often require costly dredgings.

Sediment transport, thus, poses numerous problems, and is a subject of great importance, and possesses enough potential for further research and development.

4.2. Sediment Load

The sediment in a canal is a burden to be borne by the flowing water, and is, therefore, designated as sediment load.

Bed Load and Suspended Load. The sediment may move in water either as bed load or as suspended load. Bed load is that in which the sediment moves along the bed with occasional jumps into the channel. While, the suspended load is the one in which the material is maintained in suspension due to the turbulence of the flowing water.

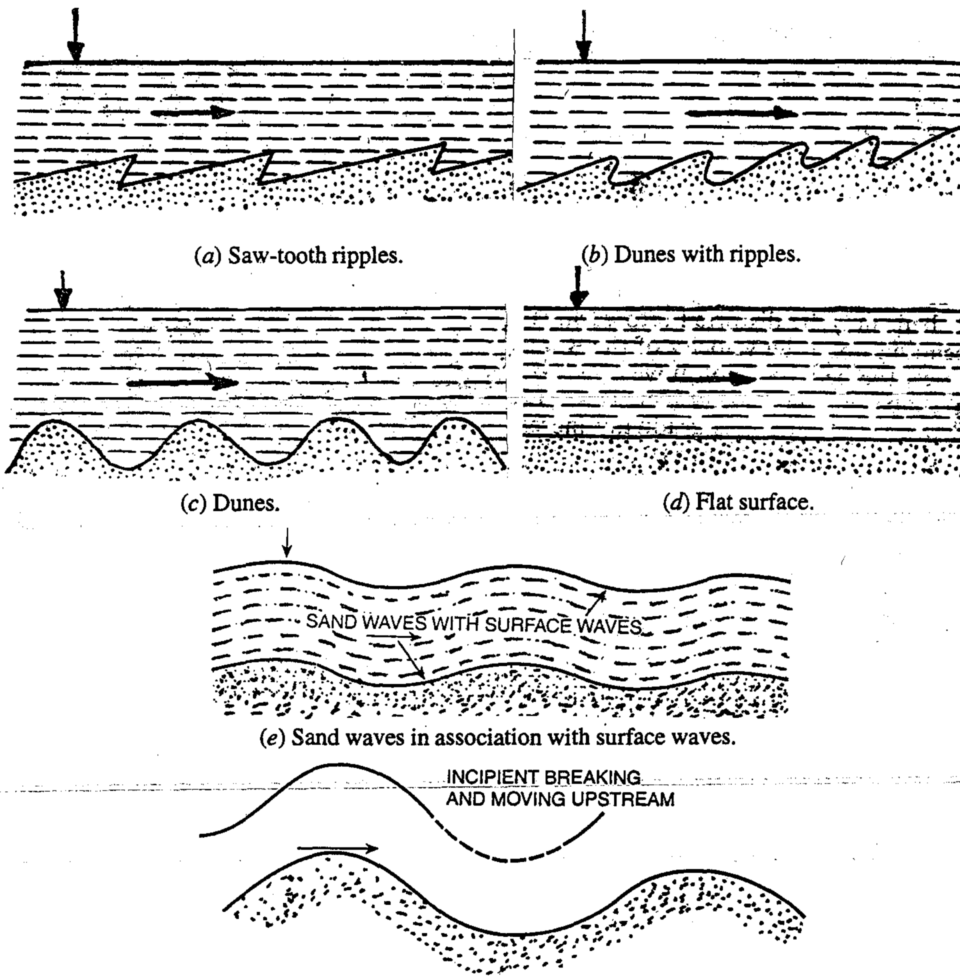
* For details of reservoir silting, please see Chapter 18.

4.3. Bed Formation (Practical Aspect)

The channel bed may be distorted into various shapes by the moving water, depending upon the discharge or the 'velocity of the water.

At low velocities, the bed does not move at all, but it goes on assuming different shapes as the velocity increases. *Let us see what happens to a channal bed made of fine sand (lesser than 2mm dia) when the velocity is gradually increased in steps.*

When the velocity is gradually increased, then first of all, a stage is reached, when the sediment load comes just at the point of motion. This stage is known as **threshold stage of motion.** On further increase of velocity, the bed develops *ripples* of the saw-tooth type, as shown in Fig. 4.1 (*a*). Such ripples can also be seen in sand on any beach. As the velocity is increased further, larger periodic irregularities appear, and are called *Dunes.* When they first appear, ripples are superimposed on them [Fig. 4.1 (*b*)]. But at still higher velocities, the ripples disappear and only the dunes are left [Fig. 4.1 (*c*)]. Dunes may form in any grain size of sediment, but ripples do not occur if the size



(*a*) Saw-tooth ripples.

(*b*) Dunes with ripples.

(*c*) Dunes.

(*d*) Flat surface.

(*e*) Sand waves in association with surface waves.

(*f*) Antidunes.

Fig. 4.1. Different shapes of bed developed along river flow with gradual increase in discharge or flow velocity.

of the bed particles is coarser than 0.6 mm. Dunes are much larger (in length and height) and more rounded than ripples. So much so that ripples seldom exceed 40 cm in length (between two adjacent troughs) and 4 cm in height (trough to crest), while the dunes in laboratory flumes may have 3m length and height up to about 40 cm, while in large rivers, they may be as high as 15 metres, with several hundred metres length. Crests of both, do not extend across the entire width of the stream, *i.e.*, both formations tend to occur in the form of "short crested waves". The flow conditions remain sub-critical in both these regimes. While most of the sediment particles move along the bed, some finer particles of sediment may go in suspension.

When the velocity is increased beyond formation of dunes, the dunes are erased by the flow, leaving very small undulations or virtually a flat surface with sediment particles in motion [Fig. 4.1 (*d*)]. Further increase in velocity, results in the formation of sand waves in association with surface waves [Fig. 4.1 (*e*)]. As the velocity is further increased, so as to make the Froude number $\left(i.e.\ \frac{V}{\sqrt{gy}}\right)$ exceeding unity, the flow becomes super-critical, and the surface waves become so steep that they break intermittently and move upstream, although the sediment particles keep on moving downstream only [Fig. 4.1 (*f*)]. Sand Waves are then called **anti-dunes**, since the direction of movement of bed forms in this regime is opposite to that of the dunes. The sediment transport rate in this regime is obviously very high. The resistance to flow is, however, small compared to that of the ripple and dune regime. In case of canals and natural streams, anti-dunes rarely occur.

4.4. Mechanics of Sediment Transport

In the study of mechanics of sediment transport, we will throughout assume that the soil is *incoherent.* By incoherent soil, we mean that there are no cohesive forces between the particles, or in other words $c = 0$, such as in sands or gravels.

Most of our river beds are made up of sands and gravels, and hence, we confine ourselves to the mechanism of movement of such a soil only. Though cohesive clays, etc. are also sometimes met with, but no systematic study upon such soils has been undertaken and only very little work has been done in that direction.

By assuming the soil to be incoherent, each soil grain can be studied individually. The basic mechanism behind the phenomenon of sediment transport is the drag force exerted by water (or fluid) in the direction of flow, on the channel bed. This force, which is nothing but a pull of water on the wetted area, is known as *Tractive Force* or *Shear Force* or *Drag Force.*

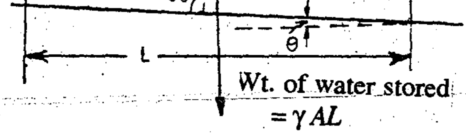


Fig. 4.2.

Let us consider a channel of length L and cross-sectional area A.

The volume of water stored in this channel reach $= AL$

Wt. of water stored $= \gamma_w AL$

where γ_w = unit wt. of water $= \rho_w g$, where ρ_w is the density of water.

Horizontal component of this wt. $= \gamma_w AL \sin\theta = \gamma_w ALS$

where S = channel bed slope.

This horizontal force exerted by water is nothing but Tractive force.

Average Tractive force per unit of wetted area

$$= \text{Unit Tractive Force } (\tau_0) = \frac{\gamma_w ALS}{\text{Wetted area}}$$

$$= \frac{\gamma_w ALS}{\text{Wetted perimeter} \times \text{Length}} = \frac{\gamma_w ALS}{P \cdot L} = \gamma_w \left(\frac{A}{P}\right) S = \gamma_w R \cdot S \left(\because \frac{A}{P} = R\right)$$

where R = is the hydraulic mean depth of channel.

S = channel bed slope

γ_w = unit wt. of water

P = wetted perimeter

Hence, **Average Unit Tractive force,** also called Shear stress

$$= \tau_0 = \gamma_w RS \qquad \text{...(4.1)}$$

It may be noted that the unit tractive force in channels, except for wide open channels, is not uniformly distributed along the wetted perimeter. A typical distribution of shear stress (unit-tractive force) on a trapezoidal channel section is shown in Fig. 4.3.

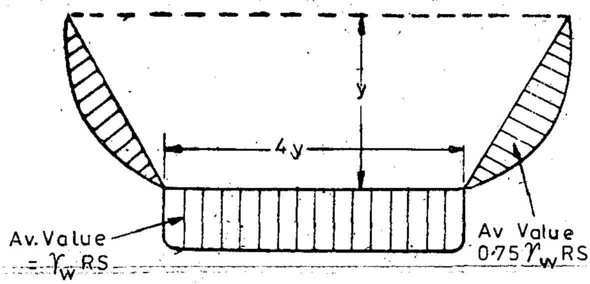


Fig. 4.3. Distribution of tractive force generated in a trapezoidal channel section.

Before entering into the mathematical aspect of sediment transport, we will again visualise the "threshold movement of the sediment", and its application for design of non-scouring channels.

4.4.1. Threshold Motion of the Sediment. When the velocity of flow through a channel is very small, the channel bed does not move at all, and the channel behaves as a **rigid boundary channel.** As the flow velocity increases steadily, a stage is reached when the shear force exerted by the flowing water on the bed particles will just exceed the force opposing their movement. At this stage, a few particles on the bed will just start moving intermittently. This condition is called the **incipient motion condition** or simply the **critical condition** or the **threshold point.**

A knowledge of the velocity at which such a critical condition occurs is quite helpful in designing stable non-scouring channels admitting clear waters, since this velocity will help us in fixing the hydraulic mean depth (R)* and bed slope (S_0) of the channel. The knowledge of this incipient motion condition is also required in some of the methods adopted for computation of sediment load.

* This will help in fixing depth (y) and bed width (b).

The experimental data on incipient motion condition was analysed using *critical tractive force approach,* for the first time by Shield, so as to help in designing stable channels in alluviums. This design basically assumes the entry of clean and clear water in the channel, which is designed to develop non-scouring highest possible flow velocity at the peak flow.

4.5. Shield's Entrainment Method for Design of Non-Scouring Stable Channels having Protected Side Slopes in Alluviums

Shield was the first investigator, who provided a semi-theoretical analysis of the problem of incipient condition of bed motion, and used it for designing non-scouring channels. He defined the **critical tractive stress** (τ_c) as that average shear stress (τ_0) acting on the bed of the channel, at which the sediment particle just begins to move. According to him, the bed particle begins to move when the drag force (F_1) exerted by the fluid on the particle, just equals or exceeds the resistance (F_2) offered by the particle to its movement.

(*i*) The drag force (F_1) exerted by the flow is given by :

$$F_1 = K_1\left[C_D \cdot d^2 \cdot \frac{1}{2} \cdot \rho_w \cdot V_0^2\right] \quad ...(4.2)$$

where K_1 = *a* factor depending on the shape of the particle.

C_D = coefficient of drag.

d = The dia of the particle.

ρ_w = The density of the flowing fluid *i.e.* water.

V_0 = The velocity of flow at the top of the particle *i.e.* at the bottom of the channel.

Using *Karman-Prandtl equation* for the velocity distribution along a channel *x*-section, the velocity of flow at the bottom of the channel (V_0) can be expressed as :

$$\frac{V_0}{V^*} = f_1\left(\frac{Vd}{\nu}\right) \quad ...(4.3)$$

$$= f_1 \cdot R_e^*$$

or

$$V_0 = V^* f_1 \cdot R_e^* \quad ...(4.4)$$

where V^* = **Shear friction velocity** $= \sqrt{\frac{\tau_0}{\rho_w}}$

where τ_0 is the shear stress acting on the boundary of the channel.

ν = Kinematic viscosity of the flowing fluid *i.e.* water.

R_e^* = **Particle Reynold Number** = V^*d/ν

Also, the coefficient of drag C_D is given by :

$$C_D = f' \cdot \left(\frac{V_0 \cdot d}{\nu}\right)$$

or $$C_D = f_2 \cdot \left(\frac{V^* d}{\nu}\right)$$
$$= f_2 \cdot R_e^* \quad \text{...(4.5)}$$

Substituting values of V_0 and C_D from Eqns. (4.4) and (4.5) in Eq. (4.2), we get

$$F_1 = K_1 \left[(f_2 \cdot R_e^*) \cdot d^2 \cdot \tfrac{1}{2} \rho_w \cdot (V^* f_1 \cdot R_e^*)^2\right]$$

or $$F_1 = \left[K_1 \cdot f_2 \cdot f_1^2 \cdot \tfrac{1}{2} d^2 \cdot \rho_w \cdot V^{*2} R_e^{*2}\right] \quad \text{...(4.6)}$$

(*ii*) The particle resistance (F_2) is further given by :

$$F_2 = K_2 \, [d^3 \cdot (\rho_s - \rho_w) \, g] \quad \text{...(4.7)}$$

where ρ_s = density of particle

ρ_w = density of fluid or water.

S_s = Specific gravity of particle

γ_w = unit wt. of fluid or water.

K_2 = a factor dependent on the shape of the particle and internal friction of soil.

$$\therefore \quad F_2 = K_2 \left[d^3 \cdot \left(\frac{\rho_s}{\rho_w} - 1\right) \rho_w \cdot g\right] = K_2 \, [d^3 \cdot (S_s - 1) \cdot \gamma_w]$$
$$= K_2 \cdot \gamma_w d^3 \, (S_s - 1) \quad \text{...(4.8)}$$

At critical condition, equating Eqs. (4.6) and (4.8), and introducing subscript (c) to denote critical conditions, we get

$$K_1 f_2 \cdot f_1^2 \frac{1}{2} \cdot d^2 \cdot \rho_w \cdot V_{(c)}^{*2} \, R_{e\,(c)}^{*3} = K_2 \cdot \gamma_w d^3 \cdot (S_s - 1) \quad \text{...(4.9)}$$

or $$\frac{\rho_w \cdot V_{(c)}^{*2}}{\gamma_w \cdot d \, (S_s - 1)} = \left(\frac{2K_2}{K_1 f_2 f_1^2}\right) \cdot R_{e(c)}^{*-3} \, ; \quad \text{But} \quad \rho_w \cdot V_{(c)}^{*2} = \tau_c$$

$$\therefore \quad \frac{\tau_c}{\gamma_w \cdot d \cdot (S_s - 1)} = F \cdot R_{e\,(c)}^* \quad \text{...(4.14)}$$

(some function of $R_{e(c)}$)

The left hand side term is a dimensionless number and is called the **Shield's Entrainment function,** and is usually denoted by F_s

$$\therefore \quad F_s = F \cdot (R*_e) \quad \text{...(4.10 a)}$$

at critical stage of bed movement in a channel in alluviums.

The above mathematical work shows that F_s *i.e.* $\dfrac{\tau_c}{\gamma_w \cdot d \, (S_s - 1)}$ is a function of R_e^* at critical stage of bed movement ; and based on the experimental work done by Shield, graphs have been plotted between F_s and R_e^*, as shown in Fig. 4.4. The obtained curve, forms a suitable basis for the design of channels, where it is required to prevent bed movement or to keep it to the minimum.

The application of this curve becomes more simple when Particle Reynold number is more than 400, and as such F_s becomes constant and equal to 0.056. Particle Reynold number, representing roughness, has been found to be more than 400, when the particle size exceeds 6 mm, such as for 'coarse alluvium soils'.

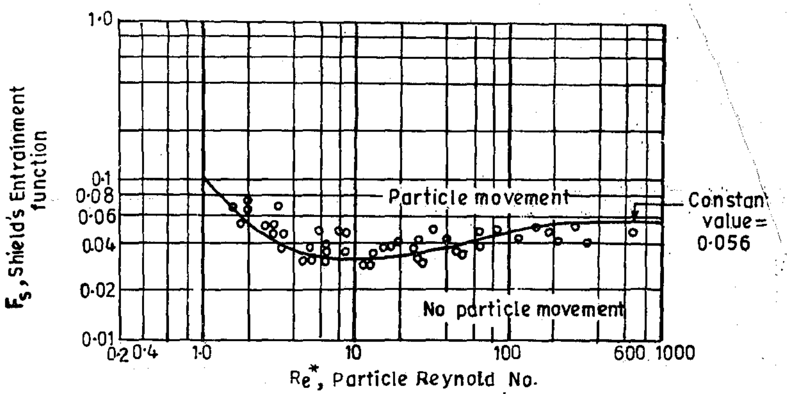


Fig. 4.4. Shield's curve for incipient motion condition.

Hence, for **designing non-scouring channels in coarse alluviums**

$$\frac{\tau_c}{\gamma_w d\,(S_s - 1)} = 0.056 \text{ (for } d > 6 \text{ mm)*} \qquad ...(4.11)$$

where γ_w = unit wt. of water = 9.81 kN/m^3
or 1 t/m^3 or 1000 kgf/m^3.

The average shear stress caused on the bed of a channel by the flowing water is given by Eq. (4.1) as :

$$\tau_0 = \gamma_w RS$$

where, R = Hydraulic mean radius of the channel, *i.e.* A/P.
S = Bed slope.

Moreover, $\tau_0 \le \tau_c$;

$\therefore \quad \tau_0 \le \gamma_w d\,(S_s - 1)\,(0.056)$

or $\quad \gamma_w RS \le \gamma_w d(S_s - 1)\,(0.056)$

or $\quad RS \le d\,(S_s - 1)\,(0.056)$

or $\quad RS \le d\,(2.65 - 1)\,(0.056)$

or $\quad RS \le \dfrac{d}{11}$

or $\quad d \ge 11\,RS \qquad ...(4.13)$

Equation (4.13) gives the minimum size of the bed material or lining stone that will remain at rest in a channel of given R and S.

Since with the passage of time, the channel bed becomes Armored (*i.e.* the smaller stones are flushed out of the surface lining of the coarser stones), actual size of bed or lining to be used should be somewhat more than what is calculated from the equation $d = 11\,RS$.

*Mittal and Swamee has worked out a general relation between τ_c and d which gives results within +5% of the values given by Shield's curve, for all values of d. The relation for water and soil of $S_s = 2.65$, is given by equation

$$\tau_c\,(\text{N/m}^2) = 0.155 + \frac{0.409 d^2_{mm}}{\sqrt{1 + 0.177 d^2_{mm}}} \qquad ...(4.12)$$

Example 4.1. *An irrigation channel is to be constructed in coarse alluvium gravel with D-75 size of 5 cm. The channel has to carry 3 cumecs of discharge and the longitudinal slope is 0.01. The banks of the channel will be protected by grass against scouring. Find the minimum width of the channel.*

Solution. d = grain dia = 5 cm = 0.05 m (> 6 mm)

By **Strickler's formula***, we know that Manning's rugosity coefficient (n) is given as:

$$n = \frac{1}{24} d^{1/6} \qquad \text{where } d \text{ is in metres}$$

$$= \frac{1}{24} \times \left(\frac{5}{100}\right)^{1/6} = \frac{1}{24}(0.05)^{0.16} = \frac{1}{24} \times 0.619 = 0.0258 \text{ m.}$$

Now, $d \geq 11\ RS$, or $R \leq \frac{d}{11\ S} = \left(\frac{5}{100} \times \frac{1}{11 \times 0.01}\right) = 0.455$ m,

or $R_{max} = 0.455$ m

Now using $V = \frac{1}{n} R^{2/3} S^{1/2}$ (Manning's formula), we have

$$V_{max} = \frac{1}{0.0258}(0.455)^{2/3}(0.01)^{1/2}$$

$$= \frac{1}{0.0258}(0.592)(0.1) = 2.29 \text{ m/sec.}$$

Assuming $R = y$ (1st app.), $Q = AV = by \times V = b{\cdot}RV$

$Q = b{\cdot}RV$, if R and V are taken maximum b will be minimum.

$\therefore$ $3 = b_{min} \times 0.455 \times 2.29$

or $b_{min} = \frac{3}{0.455 \times 2.29} = 2.88$ m.

Use a conservative value of base width as 3 m. **Ans.**

Example 4.2. *Water flows at a depth of 0.6 m in a wide stream having a bed slope of 1 in 2500. The median diameter of the sand bed is 1.0 mm. Determine whether the soil grains are stationary or moving, and comment as to whether the stream bed is scouring or non-scouring.*

Solution. Since the given size of bed particles is 1.0 mm, which is less than 6 mm, we can not use Shield's Eq. (4.11), since R_e^* in this case will be less than 400.

We will, therefore, use the general Eq. (4.12), which is valid for all sizes of d.

$$\tau_c \,(\text{N/m}^2) = 0.155 + \frac{0.409\, d^2_{mm}}{\sqrt{1 + 0.177\, d^2_{mm}}}$$

$$= 0.155 + \frac{0.409 \times 1}{\sqrt{1 + 0.177 \times 1}} = \mathbf{0.53\ N/m^2}.$$

*Strictly speaking, the **Strickler's formula** is applicable to the *rigid boundary channels* only and not to the *moveable boundary channels*, since it gives the roughness coefficient (n') due to grain roughness alone, and does not account for form roughness, which is caused by the undulations in the bed of a moveable boundary channel. The true value of n in Manning's equation will, on the other hand, represent roughness of bed consisting of grain roughness as well as the form roughness, together. This formula is therefore valid for rivers with beds of coarse materials, practically free from ripples.

Also using Eq. (4.1), we have

$$\tau_0 = \gamma_w \cdot R \cdot S = 9.81 \times 0.6 \times \frac{1}{2500} \text{ kN/m}^2$$

$$= 2.35 \times 10^{-3} \text{ kN/m}^2 \qquad \begin{pmatrix} \gamma_w = 9.81 \text{ kN/m}^3 \text{ for water} \\ \because R \approx y \text{ for wide streams} \end{pmatrix}$$

$$= 2.35 \text{ N/m}^2$$

$$= 2.35 \text{ N/m}^2\text{; which is more than } 0.53 \text{ N/m}^2.$$

Since $\tau_0 > \tau_c$, the soil grains will not be stationary, and the scouring and sediment transport will occur. **Ans.**

4.6. Stability of Channel Slopes (Design of Non-Scouring Channels with Unprotected Side Slopes)

Upto now, we have considered the stability of horizontal beds, where the shear stress τ_0 (given by $\tau_0 = \gamma_w RS$) was the only disturbing force. But on side slopes of channels, one more disturbing force, *i.e.*, the component of the weight of the particle, also comes into picture.

We will now consider a grain on the side slope of a channel. Various forces acting on this grain are shown in Fig. 4.5 (*a*).

Now, let τ_c' be the shear stress required to move the grain of weight W on the side slopes. The free-body diagram of various forces acting on the grain is shown in Fig. 4.5 (*c*).

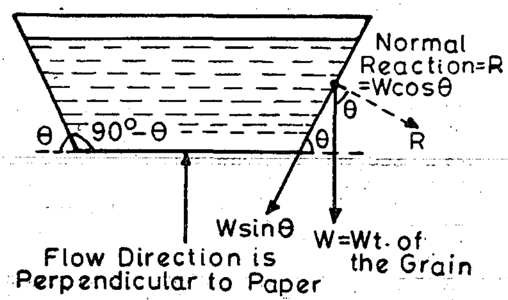
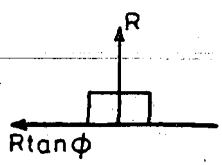
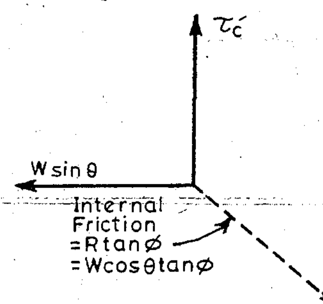


Fig. 4.5. (*a*) Forces acting on a grain on the side slope of a channel.



(*b*) where ϕ = Angle of repose of soil.



(*c*) Free body diagram of forces.

Fig. 4.5.

Let τ_e represents the critical shear stress or the shear stress required to move a similar grain on a horizontal bed, as shown in Fig. 4.6.

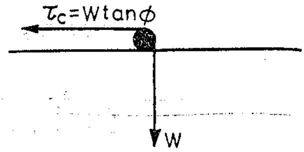


Fig. 4.6.

Now, $\tau_c = W \tan \phi$

From Fig. 4.5 (*b*),

$$(\tau_c')^2 + (W \sin \theta)^2 = [(W \cos \theta) \tan \phi]^2$$

or $$\tau_c'^2 + \left[\frac{\tau_c}{\tan \phi} \sin \theta\right]^2 = \left[\frac{\tau_c}{\tan \phi} \cos \theta \tan \phi\right]^2$$

or $$r_c'^2 + \frac{\tau_c^2}{\tan^2 \phi} \sin^2 \theta = \tau_c^2 \cos^2 \theta$$

or $$\tau_c'^2 = \tau_c^2 \left[\cos^2 \theta - \frac{\sin^2 \theta}{\tan^2 \phi}\right] \qquad ...(4.14)$$

or $$\frac{\tau_c'^2}{\tau_c^2} = \cos^2 \theta - \frac{\sin^2 \theta}{\tan^2 \phi} = \cos^2 \theta \left[1 - \frac{\tan^2 \theta}{\tan^2 \phi}\right]$$

or $$\frac{\tau_c'}{\tau_c} = \cos \theta \sqrt{1 - \frac{\tan^2 \theta}{\tan^2 \phi}} \qquad ...(4.15)$$

Equation (4.14) can also be written as

$$\left(\frac{\tau_c'}{\tau_c}\right)^2 = \left[\cos^2 \theta + (\sin^2 \theta - \sin^2 \theta) - \frac{\sin^2 \theta}{\tan^2 \phi}\right] = \left[1 - \sin^2 \theta - \frac{\sin^2 \theta}{\tan^2 \phi}\right]$$

$$= \left[1 - \sin^2 \theta \left(1 + \frac{1}{\tan^2 \phi}\right)\right] = \left[1 - \sin^2 \theta \times \frac{1 + \tan^2 \phi}{\tan^2 \phi}\right]$$

$$= \left[1 - \sin^2 \theta \cdot \frac{\sec^2 \phi}{\tan^2 \phi}\right] = \left[1 - \frac{\sin^2 \theta}{\sin^2 \phi}\right]$$

or $$\boxed{\frac{\tau_c'}{\tau_c} = \sqrt{1 - \frac{\sin^2 \theta}{\sin^2 \phi}}} \qquad ...(4.16)$$

The above equation shows that $\tau_c' < \tau_c$: which means *that the shear stress required to move a grain on the side slopes is less than the shear-stress required to move the grain on canal bed.*

Moreover, on the channel bed, the average value of actual shear stress generated by the flowing water in a channel of given R and S, is given by equation (4.1), as

$$\tau_0 = \gamma_w RS$$

while on slopes, this value is given by $(\tau_0') = 0.75 \gamma_w RS$ ...(4.17)

The above distribution of shear-stresses was shown earlier in Fig. 4.3.

Example 4.3. *A canal is to be designed to carry a discharge of 56 cumec. The slope of the canal is 1 in 1000. The soil is coarse alluvium having a grain size of 5 cm. Assuming the canal to be unlimited and of a trapezoidal section, determine a suitable section for the canal, ϕ may be taken as 37°.*

Solution. First of all, let us choose suitable side slopes, such that $\theta < \phi$. Let θ be 30°.

Now $$\frac{\tau_c'}{\tau_c} = \sqrt{1 - \frac{\sin^2 \theta}{\sin^2 \phi}} = \sqrt{1 - \frac{\sin^2 30°}{\sin^2 37°}} = \sqrt{1 - \left(\frac{0.50}{0.62}\right)^2} = 0.557$$

$$\therefore \quad \frac{\tau_c'}{\tau_c} = 0.557$$

Therefore, minimum shear stress required to dislodge the grain on side slope is given by

$$\tau_c' = 0.557\, \tau_c.$$

Hence, for stability, the shear stress actually going to be generated on the slopes of a channel of given R and S must be less than or equal to $0.557\, \tau_c$

i.e., $$\tau_0' \le 0.557\, \tau_c \text{ (for stability)}$$

But the shear stress actually going to be generated on the side slopes of a channel of given R and S, from Eq. (4.17), is

$$= \tau_0' = 0.75\, \gamma_w RS$$

$$0.75\, \gamma_w RS \le 0.557\, \tau_c$$

But $$\tau_c = \gamma_w RS = \frac{\gamma_w \cdot d}{11}$$

$$\therefore\ 0.75\, \gamma_w RS \le 0.557 \cdot \frac{\gamma_w d}{11}$$

or $$RS \le \frac{0.557}{0.75 \times 11} d$$

or $$RS \le 0.0676\, d$$

or $$RS \le (0.0676 \times 0.05) \text{ m.}$$

or $$R \times \frac{1}{1000} \le (0.0676 \times 0.05) \quad \text{when } R \text{ is in metres}$$

or $$R \le 3.38 \text{ m}$$

$$\therefore \quad y \le 3.38 \text{ m.} \quad (\because\ y \approx R)$$

With 20% factor of safety, use $y = 2.8$ m.

Hence, choose the depth as 2.8 m. Let us now choose the base width b in such a way as to have a discharge of 56 cumec.

Let us use hit and trial method to determine b.

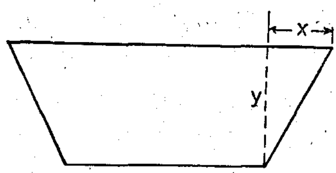
$$A = y\,(b + x) = 2.8\,(b + 4.85)$$

and $$P = b + 2\sqrt{x^2 + y^2} \quad \text{(w.r. to Fig. 4.7)}$$
$$= b + 2\sqrt{23.6 + 7.84} = b + 11.22$$

$$n = \frac{1}{24} d^{1/6} = \frac{1}{24} \times (0.05)^{1/6} = 0.0258$$

$$V = \frac{1}{n} R^{2/3} \cdot S^{1/2}$$

$$= \frac{1}{0.0258} R^{2/3} \sqrt{\frac{1}{1000}} = 1.223\, R^{2/3}.$$



$$\frac{y}{x} = \tan 30° = \frac{1}{\sqrt{3}}$$

or $$x = 2.8\sqrt{3} = 4.85 \text{ m}$$

Fig. 4.7.

Choose a number of trial values of b and assuming a depth of 2.8 m, proceed by the given table 4.1, till a discharge of 56 cumec is reached at $b = 6.3$ m.

Table 4.1

b in metres	$A = 2.8 \times (b + 4.85)$ m^2	P *in m* = $(b + 11.22)$	$R = \frac{A}{P}$ *m*	$R^{2/3}$	$1.223\, R^{2/3} = V$ *m/sec*	$Q = A \times V$ *cumecs*
3	22	14.22	1.544	1.337	1.635	36
4	24.8	15.22	1.625	1.382	1.695	42
5	27.6	16.22	1.695	1.422	1.745	48.3
6	30.4	17.22	1.764	1.470	1.800	54.8
6.3	31.22	17.52	1.782	1.470	1.798	**56.1 ; say 56**

Hence, use 6.3 m base width and 2.8 m depth **Ans.**

Example 4.4. *A trapezoidal channel with side slopes 1.5 horizontal : 1 vertical is required to carry 15 m^3/s of flow with a bed slope of 1 in 4000. If the channel is lined, the Manning's coefficient n will be 0.014, and it will be 0.028 if the channel is unlined. Calculate the average boundary shear stress if a hydraulically efficient lined channel is adopted. What percentage of earthwork is saved in a lined section, relative to an unlined section, when hydraulically efficient section is used in both the cases ? The free board can be assumed to be 0.75 m in both the cases and the lining can be assumed to be up to the top of the section.* *(Civil services, 1991)*

Solution. $Q = 15\ m^3/s$ (given)

$N = 0.014$ for lined section, and

0.028 for unlined section (given)

$$S = \frac{1}{4000} \text{ (given)}$$

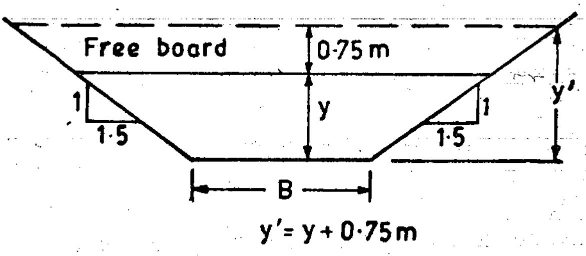


Fig. 4.8.

A hydraulically and economically efficient channel is the one *which possesses minimum perimeter for a given area ;*

For a trapezoidal channel having 1.5 H : 1V side slopes, bed width B, and depth y, we have

$$P = B + 2\,[\text{ sloping side }] = B + 2\,[1^2 + 1.5^2]\,y = B + 2 \times 1.8y$$

or $$P = B + 3.6y \qquad ...(i)$$

Also $$A = [B + y \times 1.5]\,y = (B + 1.5y)\,y$$

or $$\frac{A}{y} = B + 1.5y$$

or $$B = \frac{A}{y} - 1.5y \qquad ...(ii)$$

Substituting in (*i*), we get

$$P = \left[\frac{A}{y} - 1.5y\right] + 3.6y$$

or

$$P = \frac{A}{y} + 2.1y \qquad ...(iii)$$

For minimum perimeter (with constant A), $\frac{dP}{dy} = 0$

$$\therefore \quad \frac{dP}{dy} = A\,(-1y^{-2}) + 2.1 \times 1 = -\frac{A}{y^2} + 2.1 = 0$$

or $\frac{A}{y^2} = 2.1$ or $\mathbf{A = 2.1\,y^2}$

Substituting in (*iii*), we get

$$P = \frac{2.1y^2}{y} + 2.1y = 4.2y$$

$$\therefore \quad R = \frac{A}{P} = \frac{2.1y^2}{4.2y} = \frac{\mathbf{y}}{\mathbf{2}}$$

For a channel having 1.5 H : 1 V slopes, therefore, the most efficient section will have $A = 2.1y^2$, and $R = \frac{y}{2}$.

Now, using $Q = \frac{1}{n} A \cdot R^{2/3} S^{1/2}$, we have

$$15 = \frac{1}{0.014} \times (2.1\,y_1^2)\left(\frac{y_1}{2}\right)^{2/3} \frac{1}{\sqrt{4000}}$$

or $y_1 = (10.039)^{3/8}$ or $y_1 = \mathbf{2.37\ m}$

$\therefore \quad B_1 = 0.6 \times 2.37 = \mathbf{1.42\ m} \qquad \therefore \quad R_1 = \frac{y_1}{2} = 1.19$ m

Now, **average boundary shear stress** τ_0 *is given by eqn. (4.1) as :*

$$\tau_0 = \gamma_w \cdot RS$$

where γ_w = unit wt. of water = 9.81 kN/m^3

$R = 1.19$ m

$S = \frac{1}{4000}$

$$\therefore \quad \tau_0 = 9.81 \times 1.19 \times \frac{1}{4000}$$

$$= 2.918 \times 10^{-3}\ \text{kN/m}^2 = \mathbf{2.918\ N/m^2}. \quad \textbf{Ans.}$$

Now, we compute X-sectional areas of excavation in both cases :

Using free-board of 0.75 m

(for lined section) $= A_1 = (B_1 + 1.5\,y'_1)\,y'_1$

where $y_1' = y_1 + 0.75 = 2.37 + 0.75 = 3.12$ m

or $A_1 = (1.42 + 1.5 \times 3.12)\ 3.12\ \text{m}^2 = \mathbf{19.04\ m^2} \qquad ...(i)$

For unlined section of bed width B_2 and depth y_2, we have

$$n = 0.028$$

$$\therefore \quad 15 = \frac{1}{0.028}(2.1\, y_2^2)\left(\frac{y_2}{2}\right)^{2/3} \cdot \frac{1}{\sqrt{4000}}$$

or $$y_2 = (2 \times 10.039)^{3/8} = 3.08 \text{ m}$$

$$\therefore \quad B_2 = 0.6 y_2 = 0.6 \times 3.08 = 1.85 \text{ m}$$

$$A_2 = (B_2 + 1.5\, y_2')\, y_2' \qquad \text{where } y_2' = 3.08 + 0.75 = 3.83 \text{ m}$$

$$= (1.85 + 1.5 \times 3.83)\, 3.83 \text{ m}^2 = \mathbf{29.09\ m^2} \qquad ...(ii)$$

Percentage saving in earth work due to lining

$$= \frac{A_2 - A_1}{A_2} \times 100 = \frac{29.09 - 19.04}{29.09} \times 100 = \mathbf{34.55\%\ Ans.}$$

Example 4.5 *A most efficient trapezoidal section is required to give a maximum discharge of 21.5 m^3/s of water. The slope of the channel bottom is 1 in 2500, Taking $C = 70$ m$^{1/2}$/s in Chezy's equation, determine the dimensions of the channel. Also determine the value of Manning's 'n', taking the value of velocity of flow as obtained for the channel by Chezy's equation.* *(Engg Services 1997)*

Solution. We shall first derive the equations to be used in the question as follows.

For the most efficient channel, the wetted perimeter must be minimum for a given area. Thus, for a trapezoidal channel of bed width B, depth y, and side slopes $m : 1$ $(H : V)$, we have

$$A = (B + m \cdot y)\, y \qquad ...(i)$$

$$P = 2 \cdot \sqrt{(1 + m^2)}\; y + B \qquad ...(ii)$$

or $$P = 2\sqrt{(1+m^2)} \cdot y + \left(\frac{A}{y} - m \cdot y\right)$$

or $$P = \frac{A}{y} - m \cdot y + 2\,(1 + m^2)^{1/2} \cdot y$$

or $$\frac{dP}{dy} = -A \cdot y^{-2} - m + 2\sqrt{1 + m^2} = 0$$

or $$\frac{A}{y^2} = 2\sqrt{1+m^2} - m$$

or $$\boxed{A = [2\sqrt{1+m^2} - m]\, y^2} \qquad ...(iii)$$

But from (*i*) $B = \dfrac{A}{y} - my = [2\sqrt{1+m^2} - m]\, y - my$

or $$B = 2\,[\sqrt{1+m^2} - m]\, y \qquad ...(iv)$$

Note : The second derivative of P with respect to y is worked out to be $2A/y^3$, which is + ve, and hence the condition obtained above is for minimum P.

Using the above worked out relation, we can write

$$P = 2\sqrt{1+m^2} \cdot y + B = 2\sqrt{1+m^2} \cdot y + 2\,[\sqrt{1+m^2} - m]\, y$$

$$= 4\sqrt{1+m^2} \cdot y - 2my$$

or $$P = 2 \cdot y\,[2\sqrt{1+m^2} - m]$$

$$\therefore \qquad R = \frac{A}{P} = \frac{[2\sqrt{1+m^2} - m]\,y^2}{[2\sqrt{1+m^2} - m]\,2y} = \frac{y}{2}$$

or $$\boxed{R = \frac{y}{2}} \qquad ...(1)$$

In the expression for P, there are two variables y and n. A second condition for min. P can be obtained by equating $\frac{dP}{dm} = 0$, holding y as constant.

$\therefore$ using $P = 2y\left[2\sqrt{1+m^2} - m\right]$

or $$P = 4 \cdot y\sqrt{1+m^2} - 2y \cdot m$$

or $$\frac{dP}{dm} = 4 \cdot y\frac{1}{2}(1+m^2)^{-1/2} \cdot 2m - 2y \times 1 \qquad \text{(with constant } y\text{)}$$

$$= \frac{4y \cdot m}{\sqrt{1+m^2}} - 2y = 0$$

or $\frac{4ym}{\sqrt{1+m^2}} = 2y$ or $\frac{2m}{\sqrt{1+m^2}} = 1$

or $\sqrt{1+m^2} = 2m$ or $(1+m^2) = 4m^2$ or $3m^2 = 1$

or $$m = \frac{1}{\sqrt{3}} \qquad ...(2)$$

Hence, for the most efficient trapezoidal channel, side slopes should be $\frac{1}{\sqrt{3}} : 1$ (H : V)

Using the above two conditions, **for the most efficient channel** ; *i.e.* $m = \frac{1}{\sqrt{3}}$ and $R = \frac{y}{2}$, we can solve the given question as follows :

Using $Q = C \cdot \sqrt{R \cdot S} \times A$, we have

$$Q = 21.5 \text{ m}^3/\text{s},$$
$$C = 70\sqrt{\text{m}}/\text{sec}$$
$$S = \frac{1}{2500}$$

$$\therefore \qquad 21.5 = 70 \times \sqrt{R} \cdot \frac{1}{\sqrt{2500}} \times A$$

or $$21.5 = \frac{70}{50}\sqrt{R} \cdot A$$

For the most efficient trepezoidal channel, we have side slopes $m : 1$ $(H : V)$, where $m = \frac{1}{\sqrt{3}}$; and $R = \frac{y}{2}$.

Also, $A = [2\sqrt{1+m^2} - m]\,y^2$ ...... from Eq. (*iii*)

$$= \left[2\sqrt{1+\frac{1}{3}} - \frac{1}{\sqrt{3}}\right]y^2 = (2.3094 - 0.5773)y^2 = 1.732y^2$$

$$\therefore \qquad 21.5 = \frac{70}{50} \cdot \sqrt{\frac{y}{2}} \times 1.732 y^2$$

or $\dfrac{21.5 \times 50 \times \sqrt{2}}{70 \times 1.732} = y^{2.5}$ or $y = \mathbf{2.75\ m}$ **Ans.**

Also $\quad A = 1.732 \cdot (2.75)^2 = 13.10$

But $\quad A = (B + m \cdot y)\, y = \left(B + \frac{1}{\sqrt{3}} \times 2.75\right) 2.75$

$$\therefore \qquad 13.10 = (B + 0.577 \times 2.75)\ 2.75 \quad \text{or} \quad 13.10 = (B + 1.588)\ 2.75$$

or $\quad \mathbf{B = 3.18\ m}$

The channel dimensions are thus worked out as :

$$B = 3.18 \text{ m} \qquad \text{and} \qquad y = 2.75 \text{ m}$$

Side slopes $= \frac{1}{\sqrt{3}} : 1$ (*i.e.* sides inclined at 60° to horizontal)

Velocity as per Chezy's Equation determined above is

$$V = C \cdot \sqrt{RS} = 70 \cdot \sqrt{R} \cdot \frac{1}{50} = 70 \cdot \sqrt{\frac{y}{2} \cdot \frac{1}{50}}$$

$$= 70 \times \sqrt{\frac{2.75}{2} \cdot \frac{1}{50}} = 1.64 \text{ m/s}$$

With Manning's equation

$$V = \frac{1}{n} \cdot R^{2/3} \cdot \sqrt{S} = \frac{1}{n} \cdot \left(\frac{y}{2}\right)^{2/3} \cdot \sqrt{S} = \frac{1}{n} \times \left(\frac{2.75}{2}\right)^{2/3} \cdot \sqrt{\frac{1}{50}}$$

or $\quad 1.64 = \frac{1}{n} \cdot (1.375)^{2/3} \cdot \sqrt{\frac{1}{50}}$

or $\quad n = \dfrac{(1.375)^{2/3}}{\sqrt{50} \times 1.64}$ or $n = 0.015$ **Ans.**

4.7. Design of Stable Channels in India

So long as the average shear stress (τ_0) acting on the boundary of an alluvial channel is less than the critical shear stress (τ_c), the channel shape remains unchanged, and hence the channel can be considered of rigid boundary. The resistance equations, such as those given by Chezy's formula and Manning's formula, remain well applicable to such channels. However, as soon as the sediment movement starts, undulations develop on the bed, which increases the boundary resistance of the channel. Besides this, some energy is required to move the grains. The suspended load, carried due to turbulence in the flow, further affects the resistance of the alluvial streams. All these factors render the evaluation of resistance of alluvial streams to be a very complex problem, and the complexity further increases if one includes the effects of the channel shape, non-uniformity of sediment size, discharge variation, and other such factors. None of the resistance equations developed so far, takes all these factors into account. The direct accurate mathematical solution to the design of channels in alluvial soils is, therefore, not an easy job ; and hence in India, alluvial channels are designed on the basis of hypothetical theories given by Kennedy and Lacey. These theories are based on experiments and experience gained on the existing channels over the past many years. The semi-theoretical approach discussed in the previous article, is however, used to design channels in countries like America and Europe.

4.7.1. Problem in India. In prehistoric periods, the area bounded by Indo-Gangetic plain, *i.e.* the area starting from Himalayas to Vindhya mountains, used to be in the form of depressions with water flowing over it. But with the passage of time, it was filled up with loosely filled fine silt particles, thereby forming nothing, but what is known as **alluvial soil.** Almost all the north Indian rivers flow through such soils and, therefore, do carry a certain amount of sediment. Artificial channels have to carry their water supply from such rivers, and thus carrying sediment.

We also know that water moving with a given velocity and a certain depth can carry in suspension, only a certain amount of silt of a certain nature. If water of a given velocity and depth is not fully charged with silt (that it can carry in suspension) it will scour the bed and sides of the channel, till it is fully charged with silt. *Hence, if the velocity of flow in the channel is more, the bed and the banks are likely to be eroded, and similarly, if the velocity is less, the silt which was formerly carried in suspension is likely to be dropped.*

Silting and scouring in channels is not very uncommon and is understandable, but must be avoided by proper designs. Scouring lowers the full supply level and causes loss of command. It may also cause breaching of canal banks and failure of foundations of irrigation structures. Silting interferes with the proper working of a channel, as the channel section gets reduced by siltation, thereby reducing the discharging capacity of the channel.

Therefore, while thinking to design a properly functioning channel, one must think to design such a channel in which neither silting nor scouring takes place. Such channels are known as stable channels or regime channels.

4.7.2. Regime Channels. A channel is said to be in a state of 'Regime', if the flow is such that 'silting and scouring' need no special attention. Such a state is not easily possible in rivers, but in artificial channels, such a state can be obtained by properly designing the channel.

The basis for designing such an ideal, non-silting, non-scouring channel is that, whatever silt has entered the channel at its head is kept in suspension, so that it does not settle down and deposit at any point of the channel. Moreover, the velocity of the water should be such that it does not produce local silt by erosion of channel bed and slopes.

4.7.3. Kennedy's Theory (1895). R.G. Kennedy, an Executive Engineer of Punjab P.W.D, carried out extensive investigations on some of the canal reaches in the upper Bari Doab Canal System. He selected some straight reaches of the canal section, which had not posed any silting and scouring problems during the previous 30 years or so.

From the observations, he concluded that the silt supporting power in a channel cross-section was mainly dependent upon the generation of the eddies, rising to the surface. These eddies are generated due to the friction of the flowing water with the channel surface. The vertical component of these eddies try to move the sediment up, while the weight of the sediment tries to bring it down, thus keeping the sediment in suspension. So if the velocity is sufficient to generate these eddies, so as to keep the sediment just in suspension, silting will be avoided. Based upon this concept, he defined the **critical velocity** (V_0) in a channel as the *mean velocity* (across the section) which will just keep the channel free from silting or scouring, and related it to the depth of flow by the equation

$$V_0 = c_1 \cdot y^{c_2}$$

where c_1 and c_2 are constants depending upon silt charge.

c_1 and c_2 were found to be 0.55 and 0.64 (in M.K.S. or S.I. units), respectively.

Therefore, $V_0 = 0.55\, y^{0.64}$ ...(4.18)

Since this formula was worked out especially for the upper Bari Doab canal system, it could not have been applicable in toto to other canals or canal systems due to variation in the type of soil (or silt) at various canal sites. Realising this lacuna, Kennedy later introduced a factor (m) in this equation, to account for the type of soil through which the canal was to pass. This factor, which was dependent upon the silt grade, was named as **critical velocity ratio (C.V.R.)** and denoted by *m*.

The equation for critical velocity was, thus, modified as :

$$\boxed{V_0 = 0.55\, m y^{0.64}} \qquad \text{...(4.19)}$$

where V_0 = Critical velocity in the channel in m/s.

y = water depth in channel in m

$m = C.V.R$

For sands coarser than the standard, the values of *m* were given from 1.0 to 1.2 ; and for sands finer than the standard, *m* was valued between 1.0 to 0.7, as shown in Table 4.2.

Table 4.2. Recommended Values of C.V.R. (m)

S.No.	*Type of silt*	*Value of m*
1.	Silt of River Indus (Pakistan)	0.7
2.	Light sandy silt in North Indian Rivers	1.0
3.	Light sandy silt, a little coarser	1.1
4.	Sandy, loamy silt	1.2
5.	Debris of hard soil	1.3

Design procedure. Determine the critical velocity V_0 by the above Eq. (4.19) by assuming a trial depth, and then determine area by dividing discharge by velocity. Then determine channel dimensions. Finally, compute the actual mean velocity (V) that will prevail in the channel of this cross-section, by using Kutter's formula, Manning's formula, etc. If the two velocities V_0 and V work out to be the same, then the assumed depth is all right, otherwise change it and repeat the procedure, till V and V_0 become equal.

Kutter's Formula

$$V = \left[\frac{\dfrac{1}{n} + \left(23 + \dfrac{0.00155}{S}\right)}{1 + \left(23 + \dfrac{0.00155}{S}\right)\dfrac{n}{\sqrt{R}}}\right]\sqrt{RS} \qquad \text{...(4.20)}$$

Manning's Formula

$$V = \frac{1}{n} R^{2/3} \cdot S^{1/2} \qquad \text{...(4.21)}$$

where V = Velocity of flow in metres/sec.

R = Hydraulic mean depth in metres.

S = Bed slope of the channel.

n = Rugosity coefficient.

The values of n in both these equations depend upon channel condition and also upon discharge. The values of n may be taken as given in Table 4.3.

Table 4.3. Recommended Values of Manning's Coefficient n for Unlined Channels

Condition of channel	*Value of n*
Very good	0.0225
Good	0.025
Indifferent	0.0275
Poor	0.030

The Central Board of Irrigation and Power (India) has recommended the following values of n for different discharges.

Table 4.4. Values of Manning's n for Different Discharges

Discharge in cumec	*Value of n for Unlined Channels*
14 to 140	0.025
140 to 280	0.0225
280 and above	0.020

Chezy's Formula

$$V = C\sqrt{RS} \quad \text{...(4.22)}$$

where C = a constant depending upon the shape and surface of the channel.

R and S have the same meaning as in eq. (4.21).

The actual mean velocity(V) generated in the channel can be computed by any of these three resistance equations, but generally Kutter's equation is used with Kennedy's theory.

Example 4.6. *Design an irrigation channel to carry 50 cumecs of discharge. The channel is to be laid at a slope of 1 in 4000. The critical velocity ratio for the soil is 1.1. Use Kutter's rugosity coefficient as 0.023.*

Solution. $Q = 50$ cumecs, $S = \dfrac{1}{4000}$

$m = 1.1,\quad n = 0.023$

Use equation (4.19), as, $V_0 = 0.55m \cdot y^{0.64}$

Assume a depth equal to 2 m

$$V_0 = 0.55 \times 1.1 \times (2)^{0.64} = 0.605 \times 1.558 = \mathbf{0.942} \text{ m/sec}$$

$$A = \frac{Q}{V_0} = \frac{50}{0.942} = 53.1 \text{ m}^2.$$

Assume side slopes as $\frac{1}{2} : 1 \left(\frac{1}{2} H : 1V\right)$

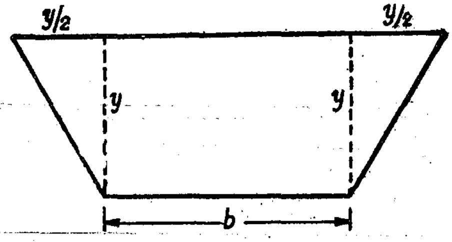


Fig. 4.9.

Now, $A = y\left(b + y \cdot \frac{1}{2}\right)$

$\therefore$ $53.1 = 2 (b + 1)$

or $26.55 = b + 1$

or $b = 25.55$ m

and $P = b + 2\sqrt{\left(1 + \frac{1}{4}\right)} \times y$

or $P = b + 2\frac{\sqrt{5}}{2} y = 25.55 + \sqrt{5} \times 2 = 30.03$

$$R = \frac{A}{P} = \frac{53.1}{30.03} = 1.77 \text{ m.}$$

But, from eqn. (4.20),

$$V = \left[\frac{\frac{1}{n} + \left(23 + \frac{0.00155}{S}\right)}{1 + \left(23 + \frac{0.00155}{S}\right)\frac{n}{\sqrt{R}}}\right]\sqrt{RS}$$

$$\therefore \quad V = \left[\frac{\frac{1}{0.023} + 23 + \frac{0.00155}{1/4000}}{1 + \left(23 + \frac{0.00155}{1/4000}\right)\frac{0.023}{\sqrt{1.77}}}\right]\sqrt{1.77 \times \frac{1}{4000}}$$

$$= \left[\frac{43.5 + (23 + 6.2)}{1 + \frac{29.2 \times 0.023}{1.33}}\right]\left[1.33 \times \frac{1}{63.3}\right]$$

$$= \frac{72.7}{1 + 0.505} \times 1.33 \times \frac{1}{63.3} = \frac{72.7}{1.505} \times 1.33 \times \frac{1}{63.3}$$

$$= 1.016 \text{ m/sec} > 0.942 \text{ ; or } V > V_0.$$

In order to *increase* the critical velocity (V_o),, we have to *increase* the depth. So increase the depth.

Use 3 m depth :

$$V_0 = 0.605 \times (3)^{0.64} = 0.605 \times 2.02 = 1.22 \text{ m/sec.}$$

$$A = \frac{50}{1.22} = 40.8 \text{ m}^2.$$

$$40.8 = 3\left(b + \tfrac{1}{2} \cdot 3\right)$$

or $13.6 - 1.5 = b = 12.1$ m.

$$P = 12.1 + 2 \times \frac{\sqrt{5}}{2} 3 = 12.1 + 6.72 = 18.82$$

$$R = \frac{A}{P} = \frac{40.8}{18.82} = 2.17 \text{ ; therefore } \sqrt{R} = 1.47.$$

$$V = \frac{43.5 + 29.2}{1 + \frac{29.2 \times 0.023}{1.47}} + \left[1.47 \times \frac{1}{63.3}\right] = \frac{72.7}{1.45} \times 1.47 \times \frac{1}{63.3}$$

$$= 1.16 \text{ m/sec.} < 1.22 \text{ ; or } V < V_0$$

So reduce the depth.

Use 2.5 m depth

$$V_0 = 0.605 \times (2.5)^{0.64} = 0.605 \times 1.797 = 1.087 \text{ m/sec.}$$

$$A = \frac{50}{1.087} = 46$$

$$46 = 2.5\,(b + \tfrac{1}{2} 2.5)$$

$$18.4 - 1.25 = b = 17.15 \text{ m}$$

$$P = 17.15 + \sqrt{5} \times 2.5 = 17.15 + 5.58 = 22.73$$

$$R = \frac{A}{P} = \frac{4}{22.73} = 2.02 \text{ ; therefore } \sqrt{R} = 1.42$$

$$V = \frac{72.7}{1 + \frac{29.2 \times 0.023}{1.42}} (1.42) \left(\frac{1}{63.3}\right) = \frac{72.7}{1.472} \times \frac{1.42}{63.3}$$

$$= 1.1 \text{ m/sec} > 1.087 \text{ ; } V > V_0$$

So increase the depth.

Use 2.7 m depth

$$V_0 = 0.605 \times 1.189 = 1.147$$

$$A = \frac{50}{1.147} = 43.5$$

$$43.5 = 2.8\,(b + \tfrac{1}{2} 2.8)$$

$$15.54 - 1.4 = b = 14.14 \text{ m}$$

$$P = 14.14 + \sqrt{5} \times 2.8 = 14.14 + 6.26 = 20.40$$

$$R = \frac{43.5}{20.4} = 2.13, \text{ therefore, } \sqrt{R} = 1.46$$

$$\therefore \quad V = \left[\frac{72.7}{1 + \frac{29.2 \times 0.023}{1.46}}\right]\left[\frac{1.46}{63.3}\right] = \left[\frac{72.6}{1.46}\right]\left[\frac{1.46}{68.3}\right]$$

$$= 1.148 \text{ m/sec} \approx 1.147 \text{ or } V \approx V_0.$$

Actual velocity V tallies with V_0.

Hence, use the depth equal to 2.7 m and base width 14.14 m. (say 14.2 m) with slopes $\frac{1}{2}$: 1 of trapezoidal section. **Ans.**

Example 4.7. *Design an irrigation channel to carry 40 cumecs of discharge, with B/D, i.e. base width to depth ratio as 2.5. The critical velocity ratio is 1.0. Assume a suitable value of Kutter's rugosity coefficient and use Kennedy's method.*

Solution. $V_0 = 0.55\,(y)^{0.64}$ ($\because$ $m = 1$)

Here $\quad y = D$

$\therefore \quad V_0 = 0.55 \cdot D^{0.64}$

$$Q = AV$$

Using $\frac{1}{2}$: 1 slopes, area (A) of trapezoidal section is given as :

$$A = BD + 2 \cdot \frac{1}{2} \cdot D \frac{D}{2} = D\left[B + \frac{D}{2}\right]$$

$$\therefore \quad 40 = D\left[B + \frac{D}{2}\right] V_0$$

But $\quad B/D = 2.5$; or $B = 2.5D$

$$\therefore \quad 40 = D\,[2.5\,D + 0.5D]\,V_0 = D\,[3D]\,V_0 = 3D^2 \cdot V_0$$

But $\quad V_0 = 0.55 \cdot D^{0.64} \qquad \therefore \quad 40 = 3D^2\,(0.55 \cdot D^{0.64})$

or $$D^{2.64} = \frac{40}{3 \times 0.55} = 24.2$$

or $$D = (24.2)^{\frac{1}{2.64}} = (24.2)^{0.379} = \mathbf{3.34\ m}$$

Now $\quad B = 2.5\,D = 2.5 \times 3.34 = \mathbf{8.35\ m}$

Now determine the slope S

$$A = 3D^2 = 3 \times (3.34)^2 = 33.5\ \text{m}^2.$$

$$P = \left[B + 2 \cdot \frac{\sqrt{5}}{2} D\right] = (8.35 + \sqrt{5} \times 3.34) = (8.35 + 7.46) = 15.81\ \text{m}$$

$$R = \frac{33.5}{15.81} = 2.12, \text{ or } \sqrt{R} = 1.456$$

$$V_0 = 0.55\,(3.34)^{0.64} = 0.55 \times 2.163 = \mathbf{1.19}$$

Assume $\quad n = 0.023$.

Using Eq. (4.20), we get

$$V = \left[\frac{\dfrac{1}{0.023} + \left(23 + \dfrac{0.00155}{S}\right)}{1 + \left(23 + \dfrac{0.00155}{S}\right)\dfrac{0.023}{1.456}}\right] 1.456\sqrt{S} \qquad \ldots(i)$$

Assume $\quad S = \dfrac{1}{4000}$.

Putting this value of S and computing the value of V, we get

$$V = \frac{43.5 + (23 + 6.2)}{1 + 29.2 \times \dfrac{0.023}{1.456}} \times \frac{1.456}{63.3} = 1.114$$

$$1.114 < 1.19 \qquad \text{or} \qquad V < V_0$$

Therefore, to increase the value of V, we must increase/steepen the slope ; hence, use a slope = 1 in 3700 (say)

Putting $\quad S = \dfrac{1}{3700}$ in (i) above, we get

$$V = 1.189 \approx 1.19$$

or $\quad V = V_0$ for value of $S = \dfrac{1}{3700} \qquad$ So use $S = \dfrac{1}{3700}$

Hence, use a trapezoidal channel section as follows :

$$\left.\begin{aligned} \text{Depth} &= 3.34 \text{ m} \\ \text{Base width} &= 8.35 \text{ m} \\ \text{Side slopes} &= \tfrac{1}{2} H : 1V \\ \text{Bed slope} &= 1 \text{ in } 3700 \end{aligned}\right] \quad \textbf{Ans.}$$

4.7.3.1. Use of Garret's Diagrams for Applying Kennedy's Theory. A lot of mathematical calculations are required in designing irrigation channels by the use of Kennedy's method. To save mathematical calculations, graphical solution of Kennedy's and Kutter's equations, was evolved by Garret. The original diagrams given by him were in F.P.S. system, but here they have been changed into M.K.S./S.I. system. The diagrams are shown in Plates 4.1 (*a*), (*b*) and (*c*). The procedure adopted for design of irrigation channels using Garret's diagrams is explained below :

(*i*) The discharge, bed slope, rugosity coefficient, value of C.V.R. are given for the channel to be designed.

(*ii*) Find out the point of intersection of the given slope line and discharge curve. At this point of intersection, draw a vertical line intersecting the various bed width curves.

(*iii*) For different bed widths (B), the corresponding values of water depth (y) and critical velocity (V_0) can be read on the right hand ordinate. Each such pair of bed width (B) and depth (y) will satisfy Kutter's equation, and is capable of carrying the required discharge at the given slope and rugosity coefficient. Choose one such pair and determine the actual velocity of flow (V).

(*iv*) Determine the critical velocity ratio (V/V_0) taking V as calculated and V_0 as read.

(*v*) If the value of C.V.R. is not the same as given in question, repeat the procedure with other pairs of B and y.

The diagrams have been drawn for a trapezoidal channel with side slopes as $\frac{1}{2}H : 1V$ ($\frac{1}{2}$: 1) on the assumption that irrigation channels adopt approximately this shape, even though they were constructed on different side slopes.

Another important point which should be noted in these diagrams is that from the *Nomogram* provided at the top, the same curves can be used for different values of rugosity coefficient n. In the nomogram, a vertical arrow has been shown. It represents the value of n for which the curves have been drawn. When the same curves are used for some other value of n (marked on right and left sides of central value), the point of intersection of discharge and slope curves, has to be shifted to the extent given in the nomogram and also in the same direction, for drawing the vertical line.

Example 4.8. *Design an irrigation channel to carry 30 cumec of discharge. The channel is to be laid at a slope of 1 in 5000. The critical velocity ratio for the soil is 1.1. Use Kutter's rugosity coefficient as 0.0225.*

Solution. $Q = 30$ cumec, $\quad S = \dfrac{1}{5000} = 0.2 \times 10^{-3} = 0.2$ m/km

$m = 1.1, \quad n = 0.0225$

$$\frac{V}{V_0} = m = 1.1,$$

$$\therefore \quad V = 1.1\, V_0$$

Using Plate 4.1 (*c*), find out the point of intersection of the given slope line and given discharge curve. Draw a vertical line through this point. Choose a point of say bed width = 12 m, as first approximation. Calculate m as shown in the last column of Table 4.5.

Table 4.5

S. No.	B metres	y metres	V_0 m/sec	$A=(2B+y)\frac{y}{2}\,m^2$	$V=\frac{Q}{A}$ m/sec.	$\frac{V}{V_0}=m$
1	12.0	2.3	0.95	$(24+2.3)\frac{2.3}{2}$ $=30.25$	0.99	1.04
2	12.5	2.25	0.92	30.66	0.98	1.07
3	13.0	2.15	0.90	30.26	0.99	1.1

The first approximation of using $B = 12.0$ m gives a value of m equal to 1.04 ; while as per given data its value should be 1.1. So we have to decrease V_0 in order to increase m. Therefore, in order to decrease the critical velocity, we have to reduce depth and thus to increase B. Hence, in the 2nd approximation, we increase B from 12.0 to 12.5 m, and find that m comes out to be 1.07. Again, we increase B to 13.0 m as third approximation, when we finally get the value of m equal to 1.1 (as given). Hence, choose the final values of $B = 13.0$ m, $y = 2.15$ m for the channel of bed slope 1 in 5000 and side slopes $\frac{1}{2}H : 1V$ **Ans.**

4.7.4. Lacey's Theory (1939). Lacey, an eminent civil engineer of U.P. Irrigation Department, carried out extensive investigations on the design of stable channels in alluviums. On the basis of his research work, he found many drawbacks in Kennedy's Theory (1895) and he put forward his new theory. The essential points which he argued, and the design procedure which he suggested, is briefly described here.

4.7.4.1. Lacey's regime channels. It was stated by Kennedy that a channel is said to be in a state of 'regime' if there is neither silting nor scouring in the channel. But Lacey came out with the statement that even a channel showing no silting no scouring may actually not be in regime. He, therefore, differentiated between three regime conditions : (*i*) True regime ; (*ii*) *Initial regime* ; and (*iii*) *Final regime.*

According to him, a channel which is under 'initial' regime, is not a channel in regime (though outwardly it appears to be in regime, as there is no silting or scouring) and hence, regime theory is not applicable to such channels. His theory is therefore applicable only to those channels, which are either in *true regime* or *in final regime.*

4.7.4.2. True regime. A channel shall be in regime, if there is neither silting nor scouring. For this condition to be satisfied, the silt load entering the channel must be carried through, by the channel section. Moreover, there can be only one channel section and one bed slope at which a channel carrying a given discharge and a particular quantum and type of silt, would be in regime. Hence, an artificially constructed channel having a certain fixed section and a certain fixed slope can behave in regime only if the following conditions are satisfied :

(*i*) *Discharge is constant ;*

(*ii*) *Flow is uniform ;*

(*iii*) *Silt charge is constant* ; *i.e.* the amount of silt is constant ;

(iv) *Silt grade is constant* ; *i.e.*, the type and size of silt is always the same, and

(v) *Channel is flowing through a material which can be scoured as easily as it can be deposited (such soil is known as* **incoherent alluvium***), *and is of the same grade as is transported.*

Hence, a designed channel shall be in 'true regime' if the above conditions are satisfied. But in practice, all these conditions can never be satisfied. *And, therefore, artificial channels can never be in 'true regime'; they can either be in initial regime or final regime,* as explained below :

4.7.4.3. Initial regime and Final regime. When only the bed slope of a channel varies due to dropping of silt, and its cross-section or wetted perimeter remains unaffected, even then the channel can exhibit 'no silting no scouring' properties, called *Initial regime*. Thus, when water flows through an excavated channel with somewhat narrower dimensions and defective slopes, the silt carried by the water may get dropped in the upper reaches, thereby increasing the channel bed slope. Consequently, the velocity is increased, and a non-silting equilibrium is established, called 'Initial regime'. Sides of such channels are subjected to a lateral restraint and could have scoured if the bank soil would have been a true alluvium. But in practice, they may either be grassed or be of clayey soil, and therefore, they may not get eroded at all. Hence, such channels will exhibit 'non-silting, non-scouring' properties, and they will appear to be in regime ; but in fact, they are not. *They have achieved only a working stability due to the rigidity of their banks.* Their slopes and velocities are higher and cross-sections narrower than what would have been if the sides were not rigid. *Such channels are termed as channels in initial regime, and regime theory is not applicable to them, as they are infact, not the channels in alluvium.*

But, if there is no resistance from the sides, and all the variables such as perimeter, depth, slope, etc. are equally free to vary and finally get adjusted according to discharge and silt grade, then the channel is said to have achieved permanent stability, called *Final Regime.* Regime theory is applicable to such channels only, and not to all regime channels, as was envisaged by Kennedy.

Such a channel in which all variables are equally free to vary, has a tendency to assume a semi-elliptical section. The coarser the silt, the flatter is the semi-ellipse, *i.e.* greater is the width of the water-surface. The finer the silt, the more nearly the section attains a semi-circle (Fig. 4.10).

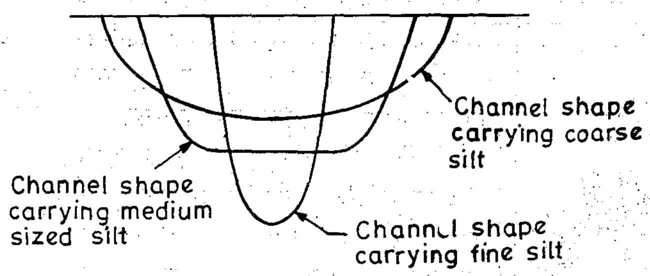


Fig. 4.10

The second point which Lacey argued was that the sediment is kept in suspension not only by the vertical component of the eddies which are generated on the channel

* **incoherent** soil, as defined in article 4.4, is the soil, which does not possess any cohesion between its soil grains. The alluvial soil is defined in article 3.2, as the soil formed by continuous deposition of silt by the moving water. Such sandy soil deposits which donot possess any cohesion (c = 0) and are formed by the agency of water are called incoherent alluvium.

bed, but also by the eddies generated on the sides of the channel. Kennedy had neglected the eddies that are generated on the sides of the channel, by presuming that such eddies has horizontal movement for greater part, and, therefore, do not have sediment supporting power. Lacey thus, argued that the silt supporting power of a channel is proportional to the wetted perimeter of the channel and not to its width, as was presumed by Kennedy.

Thirdly, Lacey argued that the grain size of the material forming the channel is an important factor, and should need much more rational attention than what was given to it by Kennedy (different values of critical velocity ratio(m) for different types of soils). He, therefore, introduced a term called **silt factor** (f) in his equations, and connected it to the average particle size (as per equation (4.24)).

The various equations put forward by Lacey for the design of stable channels are given below :

4.7.4.4. Design procedure for Lacey's theory

(1) Calculate the velocity from equation

$$V = \left[\frac{Qf^2}{140}\right]^{1/6} \text{ m/sec.} \quad \text{...(4.23)}$$

where Q is in cumec :
V is in m/s ; and
f is the **silt factor**, given by

$$f = 1.76 \cdot \sqrt{d_{mm}} \quad \text{...(4.24)}$$

where d_{mm} = Average particle size in mm, as given in Table 4.6.

Table 4.6. Values of Particle size (d_{mm}) for Various types of Alluvial materials for use in eq. (4.24)

S. No. *(1)*	*Type of material (soil)* *(2)*	*Av. grain size in mm (d_{mm})* *(3)*
1	*Silt,*	
	Very fine	0.05 to 0.08
	Fine	0.12
	Medium	0.16
	Standard	0.32 (f = 1.0)
2	*Sand,*	
	Medium	0.51
	Coarse	0.73
3	*Bajri and Sand,*	
	Fine	0.89
	Medium	1.29
	Coarse	2.42
4	*Gravel,*	
	medium	7.28
	heavy	26.10
5	*Boulders,*	
	small	50.10
	medium	72.50
	large	188.80

(2) Work out the hydraulic mean depth (R) from the equation

$$R = \frac{5}{2}\left(\frac{V^2}{f}\right) \quad \text{...(4.25)}$$

where V is in m/sec ;
R is in m.

(3) Compute area of channel section $A = \frac{Q}{V}$ ...(4.26)

(4) Compute wetted perimeter, $P = 4.75\sqrt{Q}$ ...(4.27)

where P is in m ; Q is in m^3/sec.

(5) Knowing these values, the channel section is known ; and finally the bed slope S is determined by the equation

$$S = \left[\frac{f^{5/3}}{3340\,Q^{1/6}}\right] \quad \text{...(4.28)}$$

where f is the silt factor, given by Eq. (4.24)
Q is the discharge in cumec.

Lacey's Regime Width and Scour Depth for Alluvial Rivers :

For wide streams or rivers, as we know, wetted perimeter P, approximately equals the river width. Therefore, according to Lacey, for alluvial rivers :

The regime width $= W = 4.75\sqrt{Q}$ ...(4.29)

For such streams, Lacey has also defined the regime scour depth, as

*Lacey's Normal Regime Scour Depth** $= \mathbf{R'}_r = 0.473\left(\frac{Q}{f}\right)^{1/3}$ ...(4.30)

The above scour depth equation will be applicable only when the river width equals the regime width of $4.75\sqrt{Q}$. For any other value of active river width, the normal scour depth is given by the equation :

*Lacey's Normal Scour Depth** ($\mathbf{R'}$)

$$= \mathbf{R'} = 1.35\left(\frac{q^2}{f}\right)^{1/3} \quad \text{...(4.31)}$$

where q is the discharge intensity per unit width of stream $= Q/L$, where L is the actual river width at the given site.

Example 4.9. *Design a regime channel for a discharge of 50 cumecs and silt factor 1.1, using Lacey's Theory.*

Solution. $Q = 50$ cumecs, $f = 1.1$

$$V = \left[\frac{Qf^2}{140}\right]^{1/6} = \left[\frac{50 \times (1.1)^2}{140}\right]^{1/6}$$

$$A = \frac{Q}{V} = \frac{50}{0.869} = 56.3\ \text{m}^2$$

* Symbol R is frequently used for representing Lacey's scour depth, but to avoid confusion with the symbol R used for the hydraulic Radius (i.e., hydraulic mean depth = A/P), we are using the symbol $\mathbf{R'}$ here for scour depth. When river width equals the regime width, then the regime scour depth is being represented by $\mathbf{R'}_r$

$$R = \frac{5}{2} \cdot \frac{V^2}{f} = \frac{5}{2} \cdot \frac{(0.869)^2}{1.1} = 1.675 \text{ m.}$$

$$P = 4.75\sqrt{Q} = 4.75 \cdot \sqrt{50} = 33.56 \text{ m}$$

For a trapezoidal channel with $\frac{1}{2}H : 1V$ slopes

$$P = b + \sqrt{5} \cdot y$$

and $$A = \left(b + \frac{y}{2}\right)y$$

$\therefore$ $$33.56 = b + \sqrt{5} \cdot y \quad \text{...(}i\text{)}$$

and $$56.3 = by + \frac{y^2}{2} \quad \text{...(}ii\text{)}$$

From Eq. (i), we get, $b = 33.56 - 2.24y$

Putting this value of b in Eq. (ii)

$$56.3 = [33.56 - 2.24y]\, y + \frac{y^2}{2}$$

$$= 33.56y - 2.24y^2 + 0.5y^2 = 33.56y - 1.74y^2$$

or $$1.74y^2 - 33.56y + 56.3 = 0$$

or $$y^2 - 19.3y + 32.4 = 0$$

$\therefore$ $$y = \frac{19.3 \pm \sqrt{372 - 129.6}}{2}$$

$$= \frac{19.3 \pm \sqrt{242.4}}{2} = \frac{19.3 \pm 15.6}{2}$$

Neglecting unfeasible + ve sign, we get

$$y = \frac{19.3 - 15.6}{2} = 1.65 \text{ m}$$

$\therefore$ $y = 1.65$ m. **Ans.**

$b = 33.56 - 2.24 \times 1.65 = $ **29.77 m**

or $b = 29.77$ m. **Ans.**

$$S = \frac{f^{5/3}}{3340\, Q^{1/6}} = \frac{(1.1)^{5/3}}{3340 \cdot (50)^{1/6}} = \frac{1}{5420}$$

Use a bed slope of 1 in 5420. **Ans.**

4.7.4.5. Use of Lacey's diagrams. Lacey's equations for design of irrigation channels (explained earlier) have been converted into graphical solutions. Pl see enclosed Plate Figs 4.2 (a), (b) and (c) pasted on pages 118-119. From these diagrams, the slopes and dimensions of any channel can be easily determined, if discharge and silt factor are known.

4.7.5. Comparison of Kennedy's and Lacey's Theories and Improvements over Lacey's Theory. (1) The concept of silt transportation is the same in both the cases. Both the theories agree that the silt is carried by the vertical component of the eddies generated by the friction of the flowing water against the channel surface. The difference is that Kennedy considered a trapezoidal channel section and, therefore, he neglected the eddies generated from the sides, on the presumption that these eddies has horizontal movement for greater part and, therefore, did not have silt supporting power. For this reason, kennedy's critical velocity formula was derived only in terms of depth of flow (y). On the other hand, Lacey considered that an irrigation channel achieves a cup-shaped section (semi-ellipse) and that the entire wetted perimeter (P) of the channel contributes to the generation of silt supporting eddies. He, therefore, used hydraulic mean radius $\left(R = \frac{A}{P}\right)$ as a variable in his regime velocity formula instead of depth (y).

(2) Kennedy stated all the channels to be in a state of regime provided they did not silt or scour. But, Lacey differentiated between the two regime conditions, *i.e.* Initial regime and final regime.

(3) According to Lacey, the grain size of the material forming the channel is an important factor and should need much more rational attention than what was given to it by Kennedy. Kennedy has simply stated that Critical velocity ratio ($V/V_0 = m$) varies according to the silt conditions (*i.e.* silt grade and silt charge). Lacey, however, connected the grain size (d) with his silt factor (f) by the equation $f = 1.76 \sqrt{d_{mm}}$. The silt factor (f) occurs in all those Lacey's equations, which are used to determine channel dimensions.

(4) Kennedy has used Kutter's formula for determining the actual generated channel velocity. The value of Kutter's rugosity coefficient (n) is again a guess work. Lacey, on the other hand, after analysing huge data on regime channels, has produced a general regime flow equation, stating that

$$V = 10.8\, R^{2/3}\, S^{1/3} \qquad \text{...(4.32)}$$

(5) Kennedy has not given any importance to bed width and depth ratio. Lacey has connected wetted perimeter (P) as well as area (A) of the channel with discharge, thus, establishing a fixed relationship between bed width and depth.

(6) Kennedy did not fix regime slopes for his channels, although his diagrams indicate that *steeper slopes* are required for *smaller channels* and flatter slopes are required for larger channels. Lacey, on the other hand, has fixed the regime slope, connecting it with discharge by the formula given by eqn. (4.28) as

$$S = \frac{f^{5/3}}{3340 \cdot Q^{1/6}}, \qquad \text{(Eq. 4.28)}$$

This regime slope formula, given by Lacey, gives excessive slope values. Not even a single channel has been constructed according to this regime slope equation, either on the lower Chenab Canal System or on the Lower Bari Doab Canal System or on the Jhelum Canal System. The rigidity of this regime slope equation was, therefore, later changed by Lacey (in C.B.I. publication No. 20) to the form

$$S \propto \frac{f^{5/3}}{q^{1/3}} \qquad \text{...(4.28 a)}$$

which was his final formula, showing no rigidity of the constants.

ESTIMATION OF TRANSPORTED SEDIMENT IN A CANAL

The quantity of sediment entering a channel from the head-works is an important factor. It fully controls the cross-section and shape of the true regime channel. Regime theories do not consider this vital important factor in channel design. *However, it has now been realised that the channel design shall not be successful unless full provision is made for the effects produced by the actual quantity of sediment moving in the channel. Hence, a detailed study of the subject becomes important.*

The sediment moving in water has already been classified into

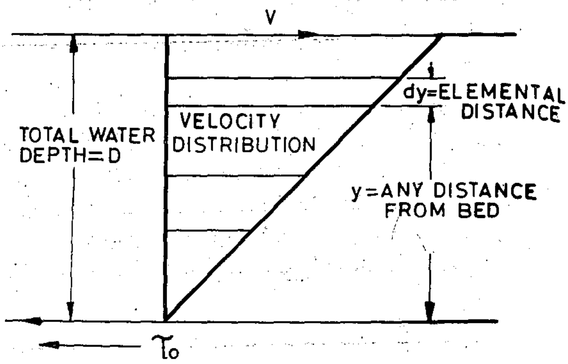
(*i*) *Bed Load*, and (*ii*) *Suspended Load*

We shall now derive equations for evaluating the amounts of these loads in a canal.

4.8. Suspended Load and its Measurement

After the threshold of motion has been passed due to increase in flow velocity, and the sediment movement is well established, some of the sediment will be carried in suspension. *The material is kept in suspension by the turbulence, or in other words, by the generation of the eddies that rise from the regions of higher sediment concentration to the regions of lower sediment concentrations.*

In a laminar flow, the shear stress caused at the base is due to one factor, and that is due to the difference of velocities at the top and at the base, as shown in Fig. 4.11. Therefore,



τ_0 = Shear stress at bed = Rate of change of velocity with distance = $\mu \dfrac{\partial V}{\partial y}$

Fig. 4.11

$$\tau_0 = \mu \frac{\partial V}{\partial y} \text{ for a laminar flow,}$$

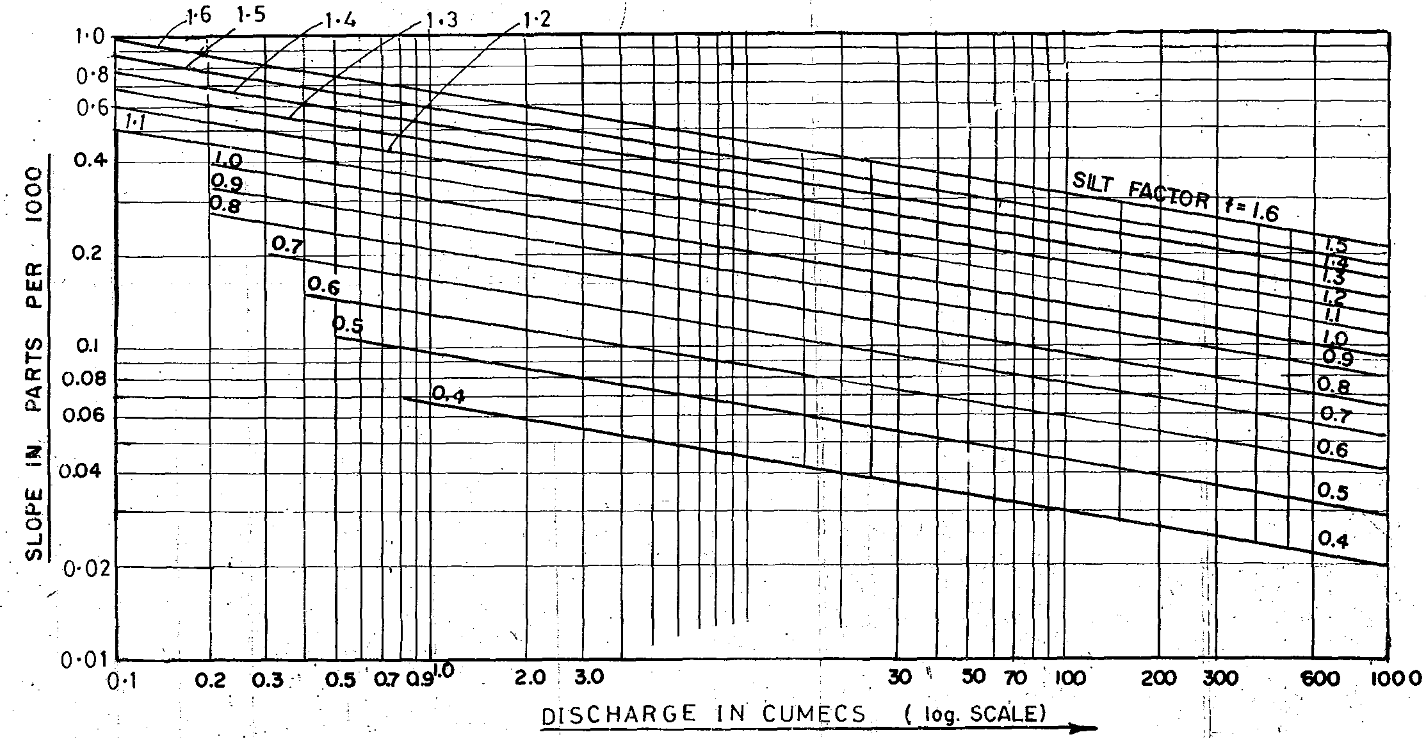
where μ is the **dynamic viscosity** of the water (fluid.) But in a turbulent flow, another factor that comes into play, is the jumping of particles from higher velocity region to lower velocity region. This is known as momentum transfer or mass exchange. Due to this transference of mass or momentum between the two adjacent fluid layers, an effective shear stress is caused at the interface between the layers, as shown in Fig. 4.12.

Fig. 4.12

The shear stress produced by this action is given by $\eta \cdot \dfrac{dV}{dy}$, where η is the **eddy viscosity,** and is defined as the rate of *mass* exchange per unit area between the adjacent layers.

Just as we define *kinematic viscosity of water* $\nu = \left(\dfrac{\mu}{\rho_w}\right)$ we here define $\varepsilon = \dfrac{\eta}{\rho_w}$ where ε is defined as the *rate of volume exchange per unit area*, between the *adjacent layers*, and is called **eddy kinematic viscosity.**

This momentum transfer caused by the generation of eddies brings about the transfer of sediment from the regions of higher concentration to the regions of lower concentration.



LACEY'S REGIME SLOPE DIAGRAM

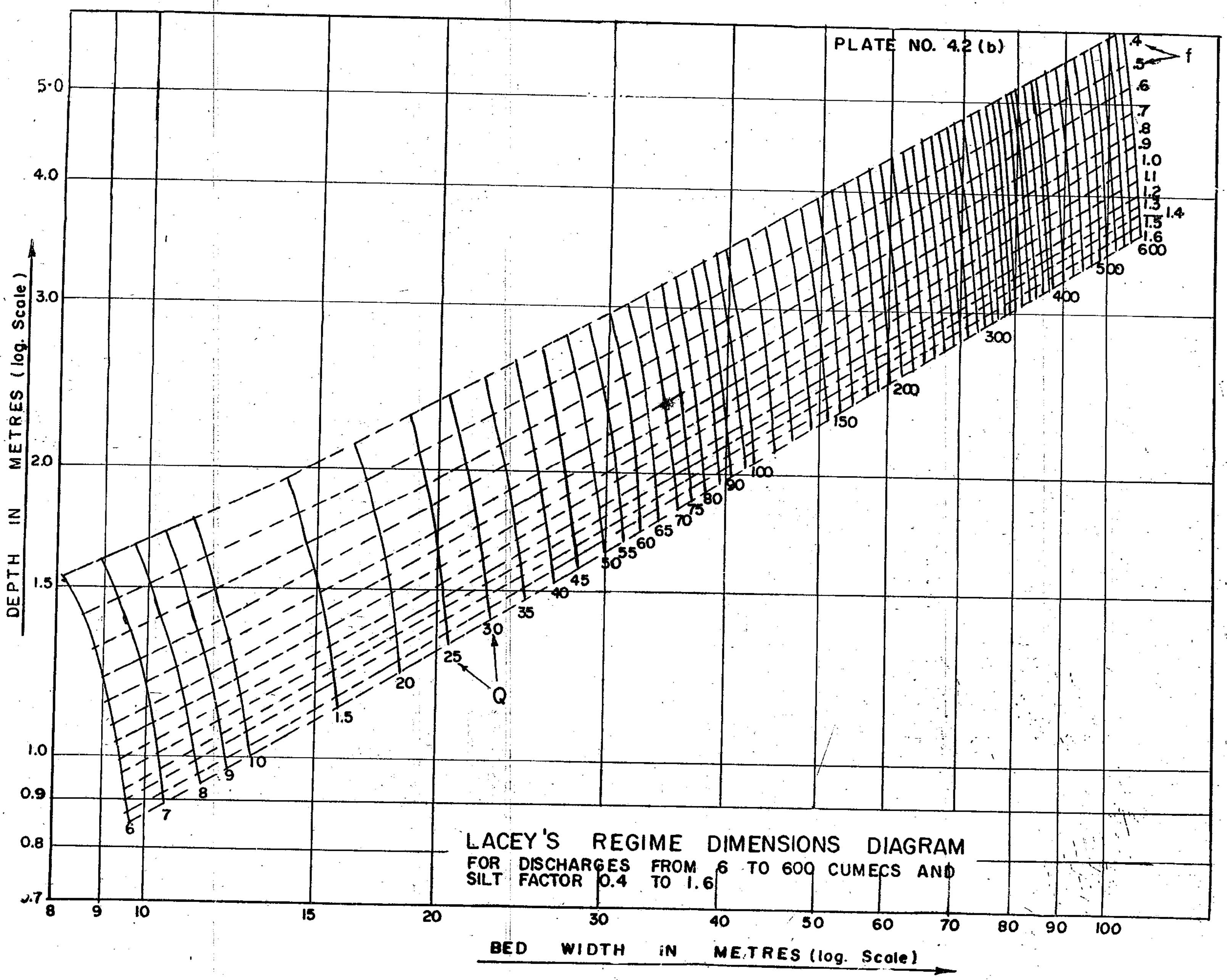


PLATE NO. 4.2 (b)
LACEY'S REGIME DIMENSIONS DIAGRAM
FOR DISCHARGES FROM 6 TO 600 CUMECS AND
SILT FACTOR 0.4 TO 1.6
BED WIDTH IN METRES (log. Scale)
DEPTH IN METRES (log. Scale)
Q
f

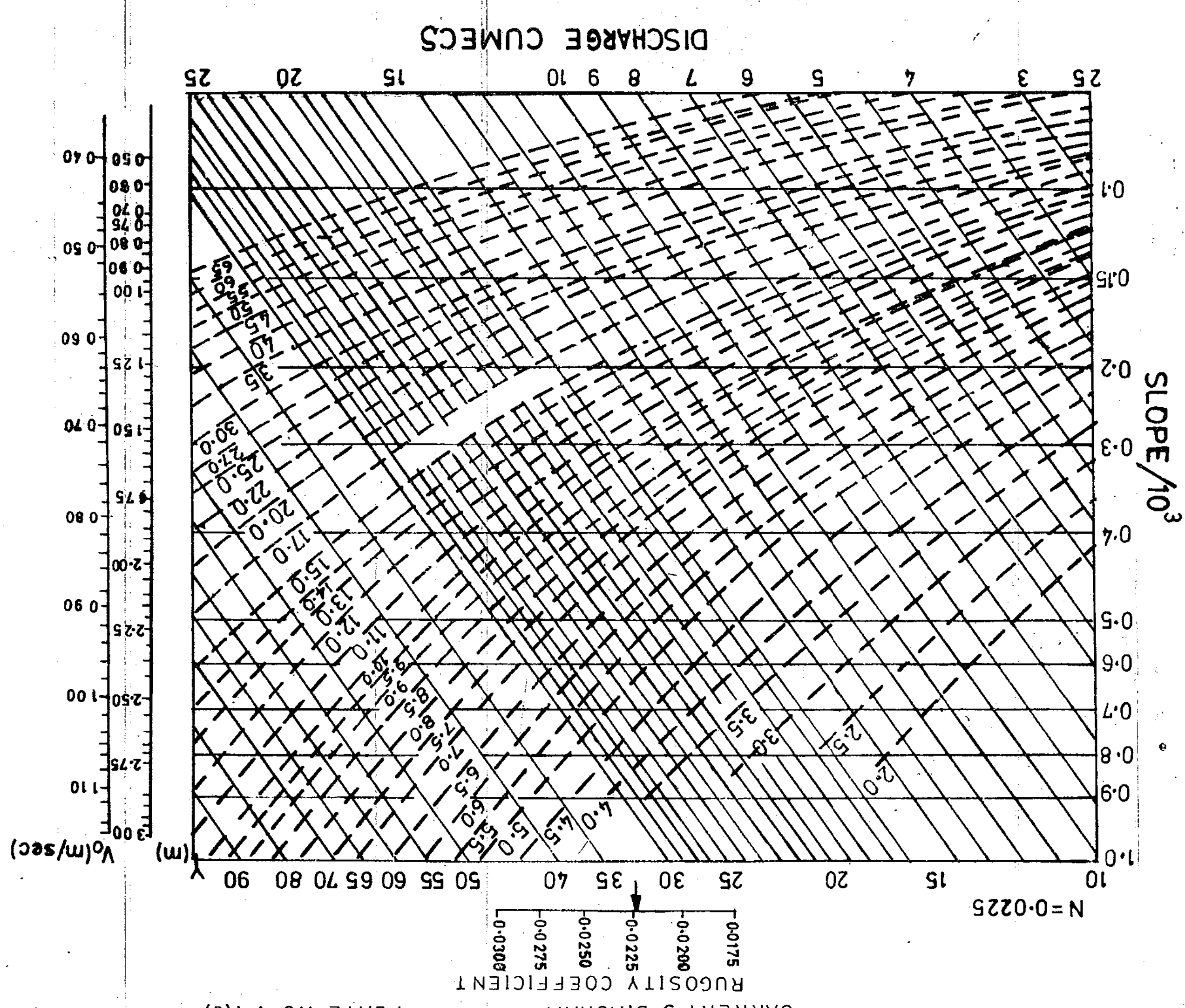


PLATE NO:4.1(c)
GARRENT'S DIAGRAM
RUGOSITY COEFFICIENT
N=0·0225
SLOPE/10^3
DISCHARGE CUMECS
V_o(m/sec)

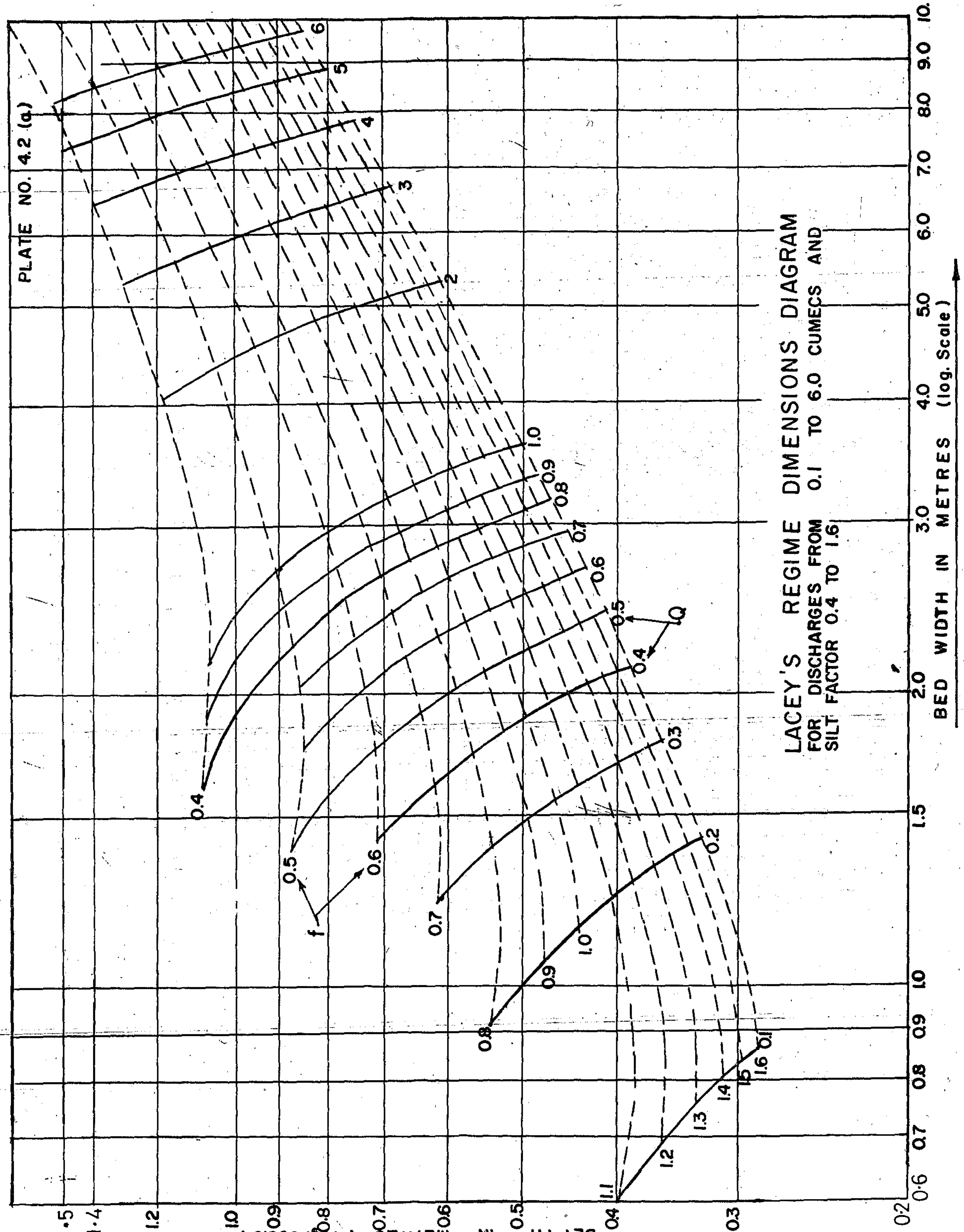
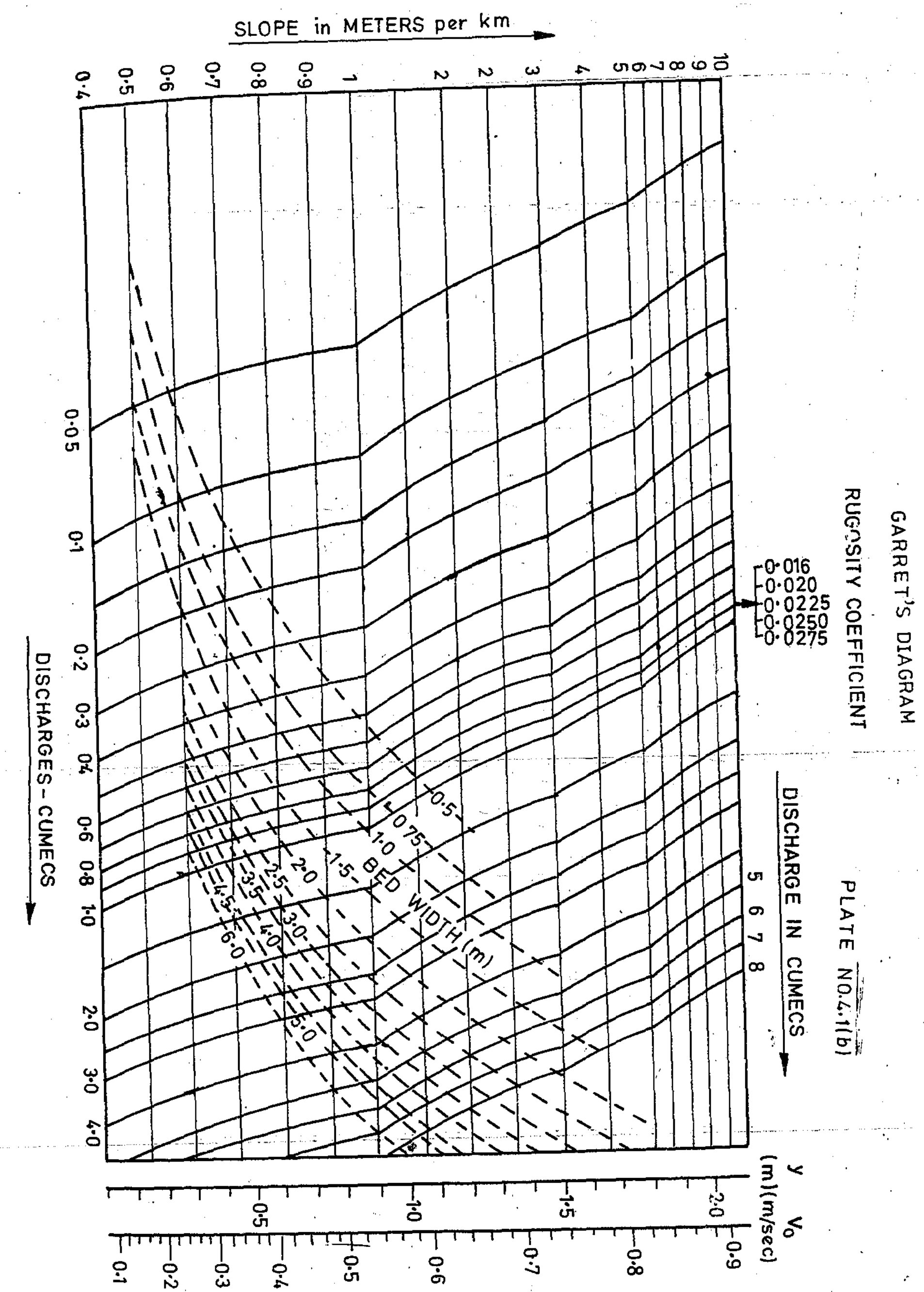
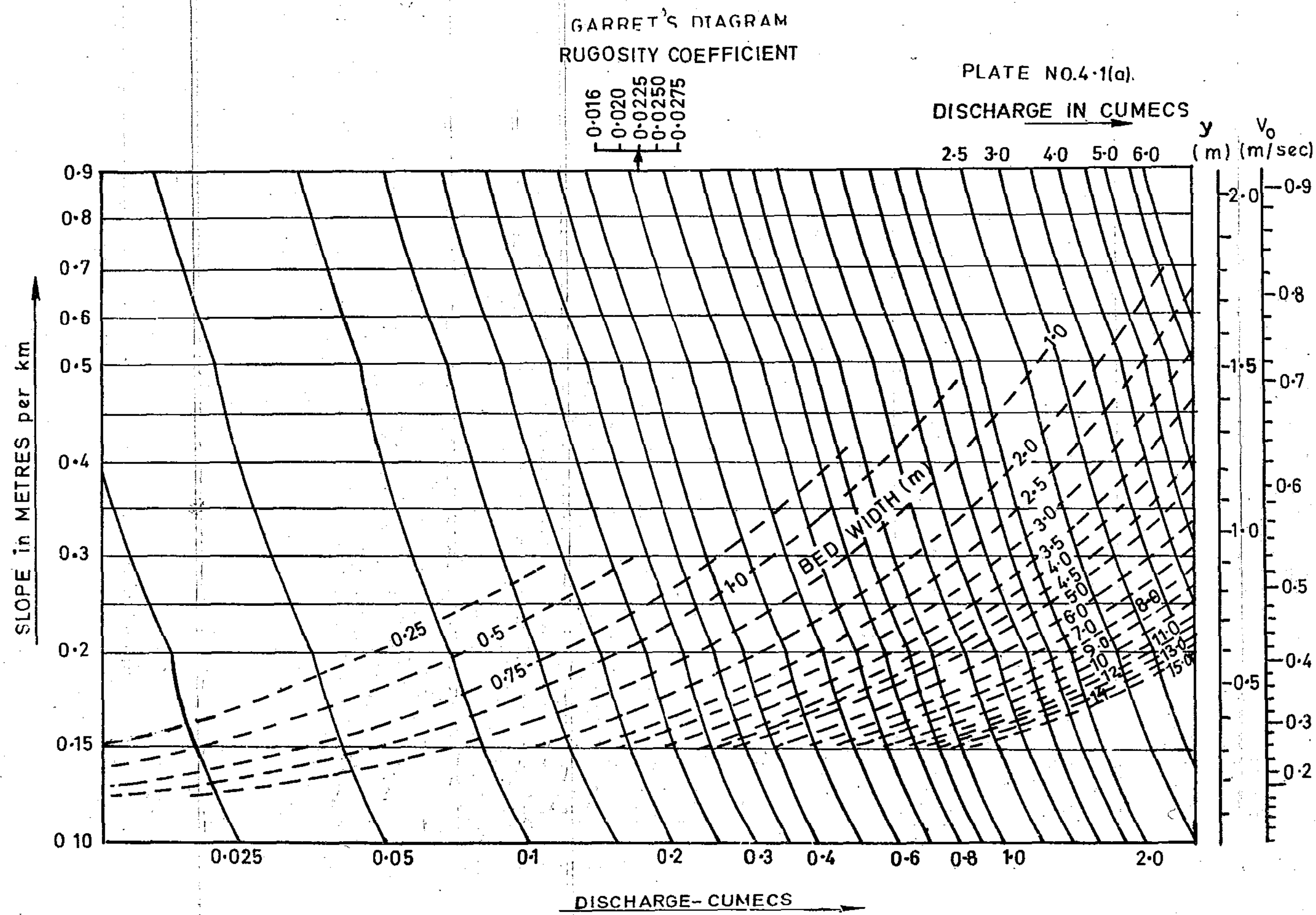


PLATE NO. 4.2 (a)
LACEY'S REGIME DIMENSIONS DIAGRAM
FOR DISCHARGES FROM 0.1 TO 6.0 CUMECS AND
SILT FACTOR 0.4 TO 1.6
BED WIDTH IN METRES (log. Scale)
DEPTH IN METRES (log. Scale)



GARRET'S DIAGRAM
PLATE NO.4.1(b)
SLOPE in METERS per km
RUGOSITY COEFFICIENT
0·016
0·020
0·0225
0·0250
0·0275
DISCHARGE IN CUMECS
DISCHARGES-CUMECS
BED WIDTH (m)
y (m)
Vo (m/sec)


GARRET'S DIAGRAM
RUGOSITY COEFFICIENT
0·016
0·020
0·0225
0·0250
0·0275
PLATE NO.4·1(a).
DISCHARGE IN CUMECS
2·5 3·0 4·0 5·0 6·0
y (m)
Vo (m/sec)
SLOPE in METRES per km
0·9 0·8 0·7 0·6 0·5 0·4 0·3 0·2 0·15 0 10
0·025 0·05 0·1 0·2 0·3 0·4 0·6 0·8 1·0 2·0
DISCHARGE- CUMECS
BED WIDTH (m)
0·25 0·5 0·75 1·0 1·0 2·0 2·5 3·0 3·5 4·0 4·5 5·0 6·0 7·0 8·0 9·0 10 11·0 12 13·0 14 15·0
2·0 1·5 1·0 0·5
0·9 0·8 0·7 0·6 0·5 0·4 0·3 0·2

If $\frac{dc}{dy}$ represents the gradient of sediment concentration between the adjacent layers (Fig. 4.13), then the rate of sediment transfer (in terms of mass between the adjacent layers)

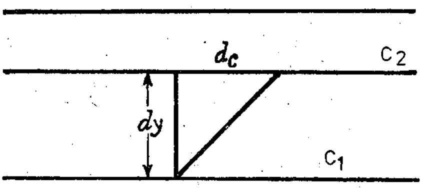


Fig. 4.13.

$$= -\eta \frac{dc}{dy}.$$

(– ve sign indicates that concentration is decreasing in the positive direction of y.)

∴ Rate of sediment transfer (by volume between the adjacent layers)

$$= -\frac{\eta}{\rho_w} \cdot \frac{dc}{dy} = -\varepsilon \cdot \frac{dc}{dy}$$

The sediment moved up by these eddies is brought down by its self-weight.

The rate of sediment falling down due to gravity

$$= w_0 \, c$$

where w_0 is the velocity of free fall of the grain, and c is sediment concentration.

When the material is just under suspension, the two forces must be equal.

Therefore, $-\varepsilon \frac{dc}{dy} = w_0 \, c$

or $$w_0 \, c + \varepsilon \frac{dc}{dy} = 0 \qquad \text{...(4.33)}$$

is the governing differential equation.

Integrating both sides after separating the variables, we get

$$\int \frac{dc}{c} = -\int \frac{w_0}{\varepsilon} dy$$

Two cases now arise.

Case I. *When ε (eddy kinematic viscosity) is constant*

$$\int \frac{dc}{c} = -\frac{w_0}{\varepsilon} \int dy$$

or $$\log_e c = -\frac{w_0}{\varepsilon} \cdot y + k \qquad \text{...}(i)$$

If c_a is the sediment concentration at a distance a from the bed, then

$$\log_e c_a = -\frac{w_0}{\varepsilon} \cdot (a) + k$$

or $$k = \log_e c_a + \frac{w_0}{\varepsilon} (a)$$

using this value of k ; the equation (i) reduces to

$$\log_e c = -\frac{w_0}{\varepsilon} \cdot y + \left(\log_e c_a + \frac{w_0}{\varepsilon} \cdot a \right)$$

or $$\log_e c - \log_e c_a = -\frac{w_0}{\varepsilon} \cdot y + \frac{w_0}{\varepsilon} \cdot a$$

or $$\log_e \frac{c}{c_a} = -\frac{w_0}{\varepsilon} \cdot (y - a)$$

or $$\boxed{\frac{c}{c_a} = e^{-\frac{w_0}{\varepsilon}(y-a)}} \qquad ...(4.34)$$

Hence, the sediment concentration c at any distance y can be determined, if the concentration c_a at a given distance a is known.

Case II. *When ε (eddy kinematic viscosity) is not constant.*

Generally, ε is not constant, and it has been shown by Prandtl that $\varepsilon = l^2 \cdot \frac{dV}{dy}$ where, l is **'mixing length'** and is analogous to 'mean free path' of kinetic theory of gases. The mixing length is a measure of turbulence and is an indicator of the size of the generated eddies.

Now, we know that $c = (\mu + \eta) \cdot \frac{dV}{dy}$.

But μ is very small compared to η, and can be neglected.

Therefore, $$c = \eta \frac{dV}{dy}$$

In the close neighbourhood of a solid boundary, it has been found from dimensional considerations that the 'Mixing length' (l) is directly proportional to the distance (y) from the boundary

or $l \propto y$ or $l = Ky$

where K is known as **Von-Karman Universal constant** and is equal to 0.4 for all homogeneous fluids.

Now, $$\tau = \eta \cdot \frac{dV}{dy} = \rho_w \cdot \varepsilon \cdot \frac{dV}{dy} = \rho_w \cdot \left(l^2 \cdot \frac{dV}{dy}\right)\frac{dV}{dy}$$

$$= \rho_w \cdot (Ky)^2 \cdot \left(\frac{dV}{dy}\right)^2 = \rho_w K^2 \cdot y^2 \cdot \left(\frac{dV}{dy}\right)^2.$$

Again in the same immediate neighbourhood, we can assume,

$$\tau = \tau_0.$$

$$\therefore \quad \tau_0 = \rho_w K^2 \cdot y^2 \cdot \left(\frac{dV}{dy}\right)^2 \quad \text{or} \quad \sqrt{\frac{\tau_0}{\rho_w}} = K \cdot y \cdot \frac{dV}{dy}$$

But $\sqrt{\frac{\tau_0}{\rho_w}} = V^*$ $\quad\therefore\quad V^* = K \cdot y \cdot \frac{dV}{dy}$

or $$\boxed{\frac{dV}{dy} = \frac{V^*}{K \cdot y}} \qquad ...(4.35)$$

The equation (4.35) gives the well known law of logarithmic velocity distribution ; given by

$$V = \frac{V^*}{K} \log_e y + \text{constant} \qquad ...(4.36)$$

But, in fact, the shear stress at any distance y from the boundary is given by

$$\tau = \tau_0 \left(1 - \frac{y}{D}\right) \qquad ...(4.37)$$

where D is the total depth.

Also assume that at sufficient distance (y) from the boundary, the mixing length is given by

$$l = K \cdot y \cdot \sqrt{1 - \frac{y}{D}}$$

Now, $$\varepsilon = l^2 \cdot \frac{dV}{dy} = K^2 \cdot y^2 \cdot \left(1 - \frac{y}{D}\right) \cdot \frac{dV}{dy}$$

But $$\frac{dV}{dy} = \frac{V^*}{K \cdot y} \text{ from equation (4.35)}$$

$\therefore$ $$\varepsilon = K^2 \cdot y^2 \cdot \left[1 - \frac{y}{D}\right] \cdot \frac{V^*}{Ky}$$

or $$\varepsilon = V^* Ky \left[1 - \frac{y}{D}\right] \qquad ...(4.38)$$

The differential equation (4.33) was

$$w_0 c + \varepsilon \cdot \frac{dc}{dy} = 0$$

Putting the value of ε from Eqn. (4.38) in the above equation, we get

$$w_0 c + V^* K \cdot y \left[1 - \frac{y}{D}\right] \cdot \frac{dc}{dy} = 0 \quad \text{or} \quad w_0 c + V^* K \cdot y \left[\frac{D-y}{D}\right] \cdot \frac{dc}{dy} = 0$$

or $$D \cdot w_0 c + V^* Ky (D - y) \cdot \frac{dc}{dy} = 0 \quad \text{or} \quad D \cdot w_0 c = - V^* Ky (D - y) \cdot \frac{dc}{dy}$$

or $$\frac{D \cdot dy}{y (D - y)} = - \frac{V^* K}{w_0} \cdot \frac{dc}{c}$$

Integrating, we get

$$D \cdot \int \frac{dy}{y (D - y)} = - \frac{V^* \cdot K}{w_0} \int \frac{dc}{c} \qquad ...(4.39)$$

Let us evaluate the L.H.S. integral first, *i.e.*

$$D \cdot \int \frac{dy}{y (D - y)}$$

Let $$\frac{1}{y (D - y)} = \frac{A}{y} + \frac{B}{D - y}$$

or $$A (D - y) + B \cdot y = 1$$

Equating coefficients on both sides, we get

$$AD = 1, \qquad \text{or} \qquad A = \frac{1}{D}$$

and $$-A + B = 0 \qquad \text{or} \qquad B = A = \frac{1}{D}$$

$$\therefore \quad \frac{1}{y(D-y)} = \frac{1}{D \cdot y} + \frac{1}{D(D-y)} = \frac{1}{D}\left[\frac{1}{y} + \frac{1}{D-y}\right]$$

or $$D\int \frac{dy}{y(D-y)} = D \cdot \frac{1}{D} \cdot \int \left[\frac{1}{y} + \frac{1}{D-y}\right] dy = \int \frac{1}{y}\, dy + \int \frac{1}{D-y}\, dy$$

$$= \log_e y - \log_e \overline{D-y} = \log_e \left(\frac{y}{D-y}\right)$$

Putting this value in Eqn. (4.39), we get

$$\log_e \left(\frac{y}{D-y}\right) = -\frac{V^* K}{w_0} \int \frac{dc}{c}$$

or $$\log_e \left(\frac{y}{D-y}\right) = -\frac{V^* K}{w_o} \log_e c + \text{constant of integration}$$

or $$\log_e \left(\frac{y}{D-y}\right) = -\frac{V^* K}{w_0} \log_e c + K'$$

or $$-\log_e c = \frac{w_0}{V^* K} \log_e \left(\frac{y}{D-y}\right) + K'' \qquad \ldots(i)$$

If $y = a$, $c = c_a$

$$\therefore \quad -\log_e c_a = \frac{w_0}{KV^*} \log_e \left(\frac{a}{D-a}\right) + K''$$

or $$K'' = -\log_e c_a - \frac{w_0}{KV^*} \log_e \left(\frac{a}{D-a}\right) \qquad \ldots(ii)$$

Putting this value of K'' in (i), we get

$$-\log_e c = \frac{w_0}{KV^*} \log_e \left(\frac{y}{D-y}\right) - \log_e c_a - \frac{w_0}{KV^*} \log_e \left(\frac{a}{D-a}\right)$$

or $$\log_e c - \log_e c_a = \frac{w_0}{KV^*} \log_e \left(\frac{a}{D-a}\right) - \frac{w_0}{KV^*} \log_e \frac{y}{D-y}$$

or $$\log_e \frac{c}{c_a} = \frac{w_0}{KV^*}\left[\log_e \frac{a}{D-a} \times \frac{D-y}{y}\right] = \log_e \left[\frac{a(D-y)}{y(D-a)}\right]^{\frac{w_0}{KV^*}}$$

or $$\boxed{\frac{c}{c_a} = \left[\frac{a(D-y)}{y(D-a)}\right]^{\frac{w_0}{KV^*}}} \qquad \ldots(4.40)$$

where, c is the sediment concentration at a distance y, and this can be determined by the above **Rouse equation,** by knowing the sediment concentration (c_a) at a known distance 'a' apart ; D being the total water depth.

w_0 is the fall velocity of a grain in still water, and V^* is the **shear friction velocity,** given by

$$V^* = \sqrt{\frac{\tau_0}{\rho_w}} \qquad \text{...(4.41)}$$

where τ_0 is the shear stress at the bottom, and ρ_w is the density of water.

K is the Von-Karman Universal Constant = 0.4.

D is the total water depth.

The equation (4.40) is known as the *suspended load concentration equation.* The limitation of this equation is that it cannot be used directly to predict the sediment concentration at any point, unless the sediment concentration at some known distance 'a' above the channel bed is preknown. Moreover, this equation has been derived on the assumptions of : (*i*) *two dimensional steady flow;* (*ii*) Constant fall velocity (w_0), and (*iii*) Constant Value of Karman's Constant (K) ; although however, it is a known fact today, that both w_0 as well as K vary with sediment concentration and turbulence.

Inspite of its above limitations, this equation (4.40) remains an important equation for determining sediment concentration in a channel at any depth y above the bed of the channel. The computation of sediment concentrations at different depths above the bed of the channel, can further be used to evaluate the total suspended load transported by the channel, as discussed below :

Rate of suspended load transport. As stated above, equation (4.40) will help us to compute the sediment concentrations at different depths, and thus plot a curve between sediment concentration (c) and depth (y) as shown in Fig. 4.14 (*a*). This curve is known as **sediment concentration curve.** When once this variation of sediment

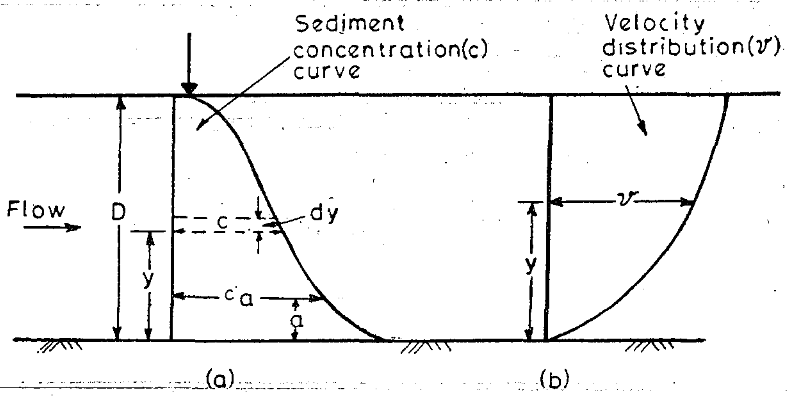


Fig. 4.14. Sediment concentration and Velocity distribution curves for a channel.

concentration with depth is known, it can be integrated over the depth to obtain total suspended load. This integration also requires plotting of **velocity distribution curve,** which is shown in Fig. 4.14 (*b*).

With reference to Fig. 4.14, let us consider a flow strip of unit width and of thickness dy at an elevation y.

The volume of suspended sediment transported past this strip in a unit time

$$=\left(\frac{c}{100}\right)v\cdot dy \qquad ...(4.41)$$

where c = The sediment concentration (volume expressed as percentage at any elevation y, where the flow velocity is v.

or $$q_s=\int\frac{c}{100}\cdot v\cdot dy \qquad ...[4.42\,(a)]$$

Since the suspended sediment moves only on top of the bed layers, the lower limit of integration can be considered as equal to the thickness of the bed layer, which is approximated to be equal to $2d$* (d being grain size), and the upper limit will certainly be D (total water depth). Hence,

$$q_s=\int_{y=2d}^{y=D}\frac{c}{100}\cdot v\cdot dy \qquad ...(4.42)$$

where q_s = Rate of suspended load transported in m^3/sec.

The reference concentration c_a required to compute various values of c by Eq. (4.40), as pointed out earlier, needs to be predetermined for the above computations. When this depth 'a' is considered equal to '$2d$', the sediment concentration c_a or 'c_{2d}' can be considered to be equal to the concentration of bed load, since bed load is assumed to be existing up to '$2d$' above the bed*. On the basis of computations carried out for bed load rate (q_b) in m^3/sec, discussed in the next article, Einstein has further computed this sediment concentration c_a at depth $a = 2d$ above bed, as equal to:

$$c_a=c_{2d}=\frac{q_b}{23.2V^{*\prime}d}\times 100 \qquad ...(4.43)$$

where $V^{*\prime}$ = shear friction velocity

$$=\sqrt{\frac{\tau_0}{\gamma_w}}=\sqrt{\frac{\gamma_w\cdot R'\cdot S}{\rho_w}}$$

$$=\sqrt{\frac{\rho_w\cdot g\cdot R'S}{\rho_w}}=\sqrt{gR'S} \qquad ...(4.44)$$

where R' is the modified value of R due to ripples, as explained in the next article, and equals $R'=R\cdot\left(\frac{n'}{n}\right)^{3/2}$ as per equation (4.47).

Example 4.10. *In a wide stream, a suspended load sample taken at a height of 0.30 m from the bed indicated a concentration of 1000 ppm of sediment by weight. The stream is 5.0 m deep and has a bed slope of 1/4000. The bed material can be assumed to be of*

* Sediment confined along and above the bed, up to a depth $2.d$ (d being grain size), is treated as bed load (q_b).

uniform size with a fall velocity of 2.0 cm/s. Estimate the concentration of the suspended load at mid-depth. *(Civil Services, 1991)*

Solution. Using Eq. (4.40), we have

$$\frac{c}{c_a} = \left[\frac{a\,(D-y)}{y\,(D-a)}\right]^{\frac{w_0}{KV^*}} \qquad \text{(Eq. 4.40)}$$

where $D = 5$ m

$S = \frac{1}{4000}$

y = up distance from bed of channel, where sediment concentration c is desired *i.e.* at mid depth $= \frac{5}{2} = 2.5$ m

c_a = known sediment concentration = 1000 ppm (of sediment by wt.)

a = known distance where sediment concentration is c_a = 0.3 m (given)

w_0 = fall velocity of sediment grain in still water = 2.0 cm/s (given) = 0.02 m/s

K = Von-karman constant = 0.4 (assumed)

V^* = shear friction velocity

$$= \sqrt{\frac{\tau_0}{\rho_w}} = \sqrt{\frac{\gamma_w RS}{\rho_w}} = \sqrt{\frac{\rho_w\, gRS}{\rho_w}} = \sqrt{g \cdot RS}$$

where $R = D$ for wide channels = 5 m.

or $$V^* = \sqrt{9.81 \times 5 \times \frac{1}{4000}} = 0.111 \text{ m/s}$$

$c = ?$

Using the above equation (4.40), we have

$$\frac{c}{1000\text{ ppm}} = \left[\frac{0.3\,(5-2.5)}{2.5\,(5-0.3)}\right]^{\frac{0.02}{0.4 \times 0.111}} = \left(\frac{0.75}{11.75}\right)^{0.45} = 0.29$$

or c = **290 ppm.** **Ans.**

4.9. Bed Load and Its Measurement

Bed load, as explained earlier, is the sediment material that remains in the bottom layers of the flow, and its movement takes place by *rolling, sliding,* and *hopping* (*i.e.* **saltation**) depending upon the velocity of flow. The bed load can be measured, though quite unsatisfactorily, by various **samplers,** such as *box type sampler, slot type sampler,* etc. Bed load is also sometimes estimated by assuming it to be between 3 to 25% of the total suspended load, depending upon the nature of response of the load material to the forces from physical, chemical and biological sectors. A figure of 10% is more commonly adopted.

Mathematical equations for bed load. The suspended load is the sediment which remains suspended in the water flowing in the channel, and this is caused by the forces of turbulence generated by the flow. The bed load, on the other hand, is the sediment which moves along the bed of the channel, and this movement is caused by the shear stress (τ_0) developed by the flowing water along the channel bed. This shear force, called the *drag force* or the *tractive force* is thus fully responsible for the bed load movement. Evidently, the amount of bed load transported in a channel of given R and S will depend on the shear stress (τ_0) developed on the bed of that channel. This bed shear or unit tractive force (τ_0) is given by the already indicated equation (4.1) as :

$$\tau_0 = \gamma_w RS$$

where R = Hydraulic mean depth of the channel $\approx y$ (or D) for wider channels.

S = Bed slope

γ_w = Unit wt. of water = 9.81 kN/m^3 = 9.81×10^3 N/m^3

We further know that a certain minimum value of shear stress is required to move the grain, depending upon the internal friction of soil. It is called critical shear stress and is represented by τ_c. For usual turbulent flow and for quartz grains, the value of τ_c has been approximated as

$$\tau_c = 0.687\, d_a \qquad \text{...(4.45)}$$

where d_a = arithmatic average diameter of sediment in mm (generally varying between d_{50} and d_{60}).

τ_c = critical shear stress in N/m^2.

when the unit tractive force caused by the flowing water (τ_0) exceeds the critical unit tractive force (τ_c) ; naturally, sediment starts moving. The rate of bed-load transported, must, therefore, be a function of ($\tau_0 - \tau_c$). But the problem becomes complicated, when we take into account the fact that as soon as the grain starts moving, the channel bed develops ripples, and a large part of shearing force is absorbed by the form resistance caused by these ripples. A part of the tractive force is, therefore, lost in overcoming ripples, and it does not play any role in transporting bed material. The quantity of shear stress lost in this process is unknown, and no perfect mathematical solution has been but forward to work out this quantity.

It has been widely suggested that the tractive force is reduced by ripples, in the following ratio :

$$\tau_0' = \tau_0 \left[\frac{n'}{n}\right]^{3/2} \qquad \text{...(4.46)}$$

where τ_0' = Resultant unit tractive force, left after the ripple resistance is overcome (*i.e.*, τ_0 − tractive force lost in overcoming ripples).

τ_c = Original tractive force exerted by the water flowing in the channel of given R and S, and is equal to $\gamma_w RS$.

n' = Rugosity coefficient that should come into play, theoretically, in an unrippled channel of given R and S. Its value may be obtained by Strickler's formula i.e. $n' = \frac{1}{24} d^{1/6}$, where d is the effective grain diameter *i.e.* **median size** (d_{50}) of the bed sediment in metres.

n = Rugosity coefficient, actually observed by experiments on the rippled bed of the channel and its value is generally taken as 0.020 for discharges over 11 cumecs, and 0.0225 for smaller canals.

From Eqn. (4.46), and using $\tau_0 = \gamma_w RS$ from Eq. (4.1), we get

$$\tau_0' = \gamma_w RS \cdot \left[\frac{n'}{n}\right]^{3/2}$$

or
$$\tau_0' = \gamma_w \left[R \left(\frac{n'}{n}\right)^{3/2} \right] S$$

or
$$\tau_0' = \gamma_w R'S.$$

$$\text{where } R' = R \cdot \left(\frac{n'}{n}\right)^{3/2} \qquad ...(4.47)$$

where R' is the corresponding hydraulic mean depth that would exist in the channel, if the bed was unrippled. In other words, if we use the value of R' in all our calculations instead of R, we can forget about bed ripples, *i.e.* the effect of ripples is only to reduce the hydraulic mean depth to a value R' from R.

Certain empirical formulas, used these days for determining bed load transport, are given below. The Einstein's semi-theoretical formula based on various assumptions is the most important and widely used these days.

4.9.1. Empirical Formula by DU-Bois. The first equation on the rate of bed load transport was proposed by DU-Bois, who assumed that the rate of bed load transportation was proportional to the excess of prevailing tractive force over the critical value required to initiate movement. Thus,

$$q_b = K_b \cdot \tau_0 (\tau_0 - \tau_c) \qquad ...(4.48)$$

where q_b = Bed load (volume) transported in m^3 per second per unit width of channel.

τ_0 = Average shear stress on the channel boundary (N/m^2).

τ_c = Minimum shear stress required to move the grain, called critical shear stress (N/m^2).

K_b = a constant depending upon the grain size, and given as :

$K_b = \dfrac{1.798 \times 10^{-3}}{(d)^{3/4}}$, where d is the effective grain diameter *i.e.* median size (d_{50}) of the bed sediment, in mm ...(4.48*a*)

4.9.2. Shield's Formula (1936). A dimensionally homogeneous equation for sediment of uniform size, taking into account the effect of specific gravity of sediment (S_s), was proposed by shield as :

$$\frac{qS_s}{q_b} = 10\left[\frac{\tau_0 - \tau_c}{\gamma_w \cdot d\,(S_s - 1)}\right] \quad ...(4.49)$$

where q_b = Bed load transported in m^3/sec per m width of channel

S_s = Specific gravity of the bed grain

q = Discharge per unit width in m^2/sec

γ_w = Unit weight of fluid in kN/m^3

d = Dia. of bed grain in m

τ_o and τ_c are stresses in kN/m^2.

4.9.3. Meyer-Peter's Formula (1948). On the lines already discussed, Meyer and Peter has suggested that the unit tractive force causing bed load to move, is reduced by ripples, in the ratio of

$$\tau_0' = \tau_0 \left(\frac{n'}{n}\right)^{3/2} \quad ...(4.50)$$

The effective unit tractive force going to cause bed load transportation, is then given by

$$\tau_{eff} = \left[\tau_0 \left(\frac{n'}{n}\right)^{3/2} - \tau_c\right]$$

On these concepts, Meyer and Peter (1948) has suggested the following formula for calculating the quantity of bed load transport

$$\boxed{g_b = 0.417\left[\tau_0 \left(\frac{n'}{n}\right)^{3/2} - \tau_c\right]^{3/2}} \quad ...(4.51)$$

where g_b is the rate of bed load transport (by wt.) in N per m width of channel per second.

($g_b = q_b \cdot \gamma_w \cdot S_s$)

where q_b = Vol. of sediment transport per metre width of channel per second.

γ_w = Unit wt. of water.

S_s = Sp. gravity of grain.

n' = Manning's coefficient pertaining to grain size on an unrippled bed and given by **Strickler's formula**

i.e. $n' = \frac{1}{24} \cdot d^{1/6}$ where d is the median size (d_{50}) of the bed sediment in metres.

n = the actual observed value of rugosity coefficient on rippled channels. Its value is generally taken as 0.020 for discharges of more than 11 cumecs, and 0.0225 for lower discharges.

τ_c = Critical shear stress required to move the grain in N/m^2, and given by equation (4.45) as

$\tau_c = 0.687\, d_a$;

where τ_c is in N/m^2 and d_a is the mean or average size of the sediment in mm. This arithmetic average size is usually found to vary between d_{50} and d_{60}.

τ_0 = Unit tractive force produced by the flowing water *i.e.* $\gamma_w RS$. Truly speaking, its value should be taken as the unit tractive force produced by the flowing water on bed = $0.97\,\gamma_w RS$.

Example 4.11. *Design a channel which has to carry 25 cumecs with a bed load concentration of 40 p.p.m. by wt. The median grain diameter of the bed material may be taken as 0.3 mm. Use, Lacey's Regime perimeter and Meyer-peter's formulas.*

Solution. Quantity of bed load transported by wt. is 40 parts per million of flowing water

$$= \frac{40}{10^6}\,[\text{wt. of flowing water}]$$

Quantity (wt.) of bed load transported per second

$$= \frac{40}{10^6}\left[25\,\frac{\text{m}^3}{\text{sec}} \times 9.81 \times 10^3\,\frac{\text{N}}{\text{m}^3}\right] = 9.81\ \text{N/sec.}$$

Lacey's Regime perimeter of the channel (P)

$$= 4.75 \cdot \sqrt{Q}$$
$$= 4.75 \cdot \sqrt{25} = 23.75\ \text{m.}$$

Let the channel bed width (B) be kept as 21 m.

Hence, the rate of bed load transport per unit width (by wt.)

$$g_b = \frac{9.81}{21}\ \text{N/m/sec.} = \mathbf{0.467\ N/m/sec.}$$

Meyer Peter's equation (4.51) is

$$g_b = 0.417 \cdot \left[\tau_0 \cdot \left(\frac{n'}{n}\right)^{3/2} - \tau_c\right]^{3/2}$$

where $n' = \frac{1}{24} \cdot d^{1/6}$

where d is median size of sediment in metres $= \frac{0.3}{1000}$ m (given)

$$\therefore \quad n' = \frac{1}{24} \cdot \left(\frac{0.3}{1000}\right)^{1/6} = 0.0108$$

The value of n may be taken as 0.020, because the discharge is more than 11 cumecs or so, and the channel may be taken in good shape and smooth soil.

$$\therefore \quad \frac{n'}{n} = \frac{0.0108}{0.0200} = \mathbf{0.54}$$

From equation (4.45),

$$\tau_c = 0.687 \cdot d_a$$

where d_a is the average particle size in mm ; and we can assume it to be either 0.3 mm or slightly more than that, since mean size is usually found to vary between d_{50} and d_{60}. Let us assume it to be equal to d_{50} *i.e.* 0.3 mm.

$$\therefore \quad \tau_c = 0.687 \times 0.3 = \mathbf{0.206\ N/m^2}$$

Putting these values in Meyer-Peter's formula, we get

$$g_b = 0.417\left[\tau_0 \cdot \left(\frac{n'}{n}\right)^{3/2} - \tau_c\right]^{3/2} \text{ N/m/sec.} \qquad \text{where } \tau_0 = \gamma_w RS$$

$$\therefore \quad 0.467 = 0.417\left[\gamma_w RS \cdot \left(\frac{n'}{n}\right)^{3/2} - \tau_c\right]^{3/2}$$

or $$0.467 = 0.417\left[(9.81 \times 10^3) \times RS\,(0.54)^{3/2} - 0.206\right]^{3/2}$$

$$0.467 = 0.417 \times (9.81)^{3/2}\left[1000RS\,(0.54)^{3/2} - 0.021\right]^{3/2}$$

or $$\left(\frac{0.467}{0.417 \times (9.81)^{3/2}}\right)^{2/3} = \left[1000\,RS\,(0.54)^{3/2} - 0.021\right]$$

or $(0.0365)^{0.67} = 1000 \times 0.397\,RS - 0.021$

or $0.11 = 397\,RS - 0.021$

or $0.131 = 397\,RS$

or $\mathbf{RS = 0.00033}$ ...(*i*)

Using Manning's equation

$$Q = \frac{1}{n} \cdot AR^{2/3} \cdot S^{1/2} \qquad \text{where } \frac{A}{P} = R\text{; or } A = P \cdot R = 23.75 \cdot R$$

$$Q = 25 = \frac{1}{0.020} \times (23.75\,R)\,R^{2/3} \cdot S^{1/2}.$$

or $$\frac{25 \times 0.02}{23.75} = R^{5/3} \cdot S^{1/2}$$

or $$0.021 = R^{5/3} \cdot S^{1/2} \qquad \text{...}(ii)$$

Solving (i) and (ii), we get

$$RS = 0.00033\,; \quad S = \frac{0.00033}{R}$$

$$\therefore \quad 0.021 = R^{5/3} \times \left(\frac{0.00033}{R}\right)^{1/2}$$

or $$\frac{0.021}{\sqrt{0.00033}} = R^{7/6}$$

$$R = \mathbf{1.13\ m}$$

$$\therefore \quad S = \frac{0.00033}{1.13} \quad \text{or} \quad S = \frac{1}{3424}\,; \ \text{say} \ \frac{\mathbf{1}}{\mathbf{3400}}.$$

Let y be the depth of water in the trapezoidal channel of $\frac{1}{2}$: 1 slopes

$$\therefore \quad P = 21 + \sqrt{5} \cdot y$$

$$A = 21y + \frac{y^2}{2}$$

$$R = \frac{A}{P} = \frac{21y + \dfrac{y^2}{2}}{21 + 2.24 \cdot y} = 1.13 \text{ (worked out earlier)}$$

or $$42y + y^2 = 2.26\,(21 + 2.24\,y)$$

or $$y^2 + 42y = 47.46 + 5.06\,y$$

or $$y^2 + 36.94y - 47.46 = 0$$

or $$y = \frac{-36.94 \pm \sqrt{(36.94)^2 + 4 \times 47.46}}{2} = \frac{-36.94 \pm 39.43}{2}$$

$$= \frac{-36.94 + 39.43}{2} \quad \text{(using only the feasible + ve sign)}$$

$$= \mathbf{1.25\ m}.$$

Use Depth = 1.25 m.

Hence, use the following channel dimensions.

$$\left.\begin{aligned} B &= 21 \text{ m} \\ y &= 1.25 \text{ m} \\ S &= \frac{1}{3400} \\ \text{Side slopes} &= \tfrac{1}{2} H : 1V. \end{aligned}\right\} \ \textbf{Ans.}$$

4.9.4. Einstein's Formula (1950). Einstein has put forward a semi-theoretical approach to the problem of bed load transport. In his formula, it is assumed, that every particle after it is dislodged from the bed, travels a certain minimum distance L (which is proportional to the grain diameter) before coming to rest.

Now, let us take a bed area of length L and of unit width. This is clear from the above assumption, that each and every grain that is dislodged from this area will cross the outer boundary (AB) of this area (because each particle has to travel a minimum distance (L). See Fig. 4.15.

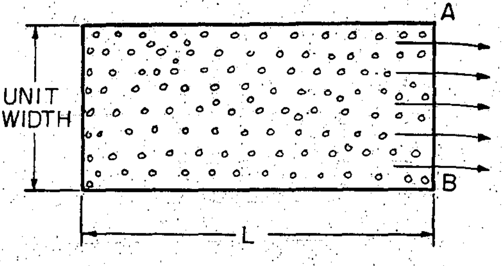


Fig. 4.15.

Now, if K_1d^2 is the bed area covered by each grain (where K_1 is a constant depending upon the shape of the grain), then the number of grains dislodged from this area is given by

$$\frac{L \times 1}{K_1 d^2}$$

If p is the probability of the grain being dislodged in the given second ($\because$ in one particular second, the particles will not be dislodged from the entire area), then the number of particles dislodged per second is given by

$$\frac{L \times 1}{K_1 d^2} \cdot p$$

If K_2d^3 represents the volume of each grain, then the volume of sediment transported per second per unit width is given by

$$q_b = \frac{L \times 1}{K_1 d^2} \cdot p \cdot K_2 \cdot d^3 = \frac{K_2}{K_1} L \cdot p \cdot d.$$

But $L \propto d$ (by assumption made in the beginning)

or $\quad L = K_3 d$

$$\therefore \quad q_b = \frac{K_2}{K_1} K_3\, p(d)^2 \qquad ...(4.52)$$

Einstein further postulated that :

$$\text{the probability, } p \propto \frac{\text{Lift force which the flow can exert on the grain}}{\text{Submerged weight of the grain}}$$

But, Lift force $\propto \rho_w V^2 d^2 = \tau_0 d^2$

and Submerged weight of the grain $\propto$ [Submerged density$\times$ volume) $\cdot g$

$$\left[\because \begin{array}{r} \text{Mass} = \text{Volume} \times \text{Density} \\ \text{Weight} = \text{Mass} \times g. \end{array}\right]$$

$\therefore$ Submerged weight of the grain $\propto [\rho_w (S_s - 1)\, g\, (K_2 d^3)]$

Using these two values, we get

$$p \propto \frac{\tau_0\, d^2}{\rho_w\, (S_s - 1)\, g\, K_2\, d^3}$$

or

$$p \propto \frac{\tau_0\, d^2}{\rho_w\, g \cdot (S_s - 1)\, K_2\, d^3}$$

But $\rho_w \cdot g = \gamma_w$.

$$\therefore \quad p \propto \frac{\tau_0 d^2}{\gamma_w (S_s - 1) K_2 \cdot d^3}$$

or
$$p \propto \frac{\tau_0}{K_2 \gamma_w (S_s - 1) d}$$

or
$$p \propto \frac{\tau_0}{\gamma_w \cdot d (S_s - 1)} \quad ...(4.53)$$

But $\dfrac{\tau_0}{\gamma_w d (S_s - 1)} = F_s$ = Shield's entrainment function

therefore, $p \propto F_s$.

But F_s is dimensionless and p has the units of time^{-1}, therefore, introducing time, we get

$$p = \frac{1}{t} f(F_s),$$ $f(F_s)$ means, a function of F_s.

Further, it was argued by Einstein that

$$t = \frac{d}{w_0} \quad i.e., \text{ time } (t) = \frac{\text{Distance travelled equal to diameter}}{\text{Fall velocity}}$$

Therefore,
$$p = \frac{w_0}{d} f(F_s)$$

or
$$p = f(F_s) \left[\frac{w_0}{d}\right] \quad ...(4.54)$$

Now Eq. (4.52) reduces to

$$q_b = \frac{K_2}{K_1} K_3 \cdot f(F_s) \left[\frac{w_0}{d}\right] \cdot d^2$$

or
$$\frac{q_b}{w_0 d} = \frac{K_2}{K_1} K_3 \cdot f(F_s)$$

or
$$\frac{q_b}{w_0 d} = \text{F}(F_s) \text{ (some other function of } (F_s)) \quad ...(4.55)$$

Now, $\dfrac{q_b}{w_0 d}$ is generally represented by ϕ, and F_s is represented by $\dfrac{1}{\psi}$; where ϕ is known as **Einstein's bed load function.**

Therefore, we get

$$\phi = F \frac{1}{\psi} \quad ...(4.56)$$

It was also given by Einstein, that

$$w_0 = G \sqrt{gd (S_s - 1)} \quad ...(4.57)$$

where
$$G = \sqrt{\frac{2}{3} + \frac{36\nu^2}{gd^3 (S_s - 1)}} - \sqrt{\frac{36\nu^2}{gd^3 (S_s - 1)}} \quad ...(4.58)$$

where ν is the kinematic viscosity of water in m^2/sec.

when $d \geq 1.6$ mm, $G \approx \sqrt{(2/3)} \approx 0.81$

The above ϕ–ψ relationship is represented by the curve, shown in Fig. 4.16, and can be used to compute bed load transport rate of a given channel; or conversely to design stable channels for a given bed load transport.

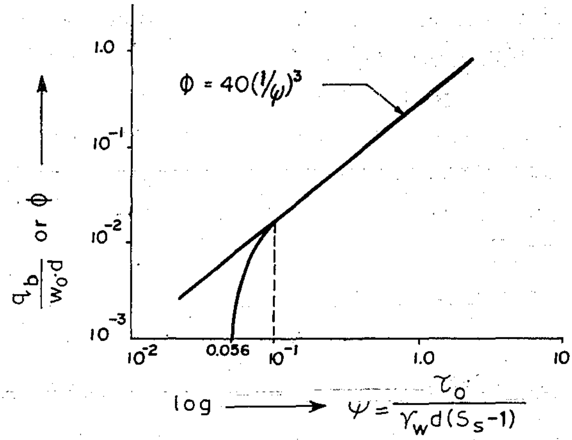


Fig. 4.16.

The straight line portion of this curve, follows the relation

$$\phi = 40 \cdot \left(\frac{1}{\psi}\right)^3 \quad \ldots(4.59)$$

or $$\boxed{\frac{q_b}{w_0 d} = 40 \cdot \left[\frac{\tau_0}{\gamma_w d\,(S_s - 1)}\right]^3} \quad \ldots(4.60)$$

Now, $w_0 = G \cdot \sqrt{gd\,(S_s - 1)}$

$$\therefore \quad \frac{q_b}{G \cdot d\sqrt{gd\,(S_s - 1)}} = 40\left[\frac{\tau_0}{\gamma_w d\,(S_s - 1)}\right]^3 \quad \ldots(4.61)$$

But $\tau_0 = \gamma_w RS$.

$$\therefore \quad \frac{q_b}{G \cdot d\sqrt{gd\,(S_s - 1)}} = 40\left[\frac{\gamma_w RS}{\gamma_w \cdot d\,(S_s - 1)}\right]^3$$

or $$\frac{q_b}{G \cdot d\sqrt{gd\,(S_s - 1)}} = 40\left[\frac{R^3 \cdot S^3}{d^3\,(S_s - 1)^3}\right]$$

or $$q_b \propto \frac{R^3 \cdot S^3 \cdot d\sqrt{d}}{d^3}$$

or $$q_b \propto \frac{R^3 S^3}{d^{3/2}} \quad \ldots(4.62)$$

Now, if it is assumed that the channel is wide and Chezy's C is constant, then

$$Q = C \cdot A\sqrt{RS}$$

$\therefore$ $$q = C \cdot y \cdot \sqrt{RS}$$ But $R \approx y$ for wide channels or rivers.

$\therefore$ $$q = C \cdot R \cdot \sqrt{RS} = C \cdot R^{3/2}\, S^{1/2}$$

or $$q^2 \propto R^3 \cdot S \quad \ldots(4.63)$$

From Eq. (4.62), $q_b \propto \dfrac{R^3 \cdot S \cdot S^2}{d^{3/2}}$

or $$q_b \propto \frac{q^2 \cdot S^2}{d^{3/2}} \qquad ...(4.64)$$

or $$\boxed{\frac{q_b}{q} \propto \frac{q \cdot S^2}{d^{3/2}}} \qquad ...(4.65)$$

This is an important form of Einstein's equation, and shows that the sediment carrying capacity of the channel depends upon the discharge per unit width. This relationship helps us to draw the following very interesting and important conclusions:

(1) If the width of a river increases, the discharge per unit width (q) will decrease (Q remaining constant) and, therefore, q_b/q *i.e.*, sediment carrying capacity will reduce. Hence, the deposition of the sediment will start, which will increase the bed slope. The deposition will continue till the slope S is increased to a value sufficient to make qS^2 a constant.

(2) Since the sediment carrying capacity depends upon the discharge, the floods will carry more sediment. In fact, the floods are mainly responsible for most of the annual sediment load.

(3) If a branch channel starts from a main channel, then an interesting phenomenon may happen, as described below :

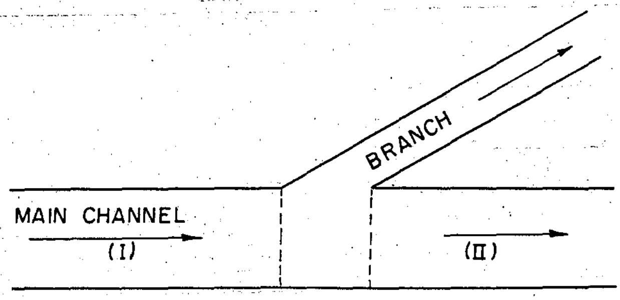


Fig. 4.17.

Let a branch channel off-takes from a main channel, as shown in Fig. 4.17.

Now, after the branch channel starts extracting some discharge, the discharge in the main channel reduces, thereby reducing its sediment carrying capacity. Hence, deposition of sediment will start, which further diverts the flow towards the off-taking channel, causing more reduction in discharge in the main channel and more silt deposition ; causing more inflow towards the branch channel. The process may continue till the entire water gets diverted towards the branch channel.

The prime cause for this phenomenon is the existence of a favourable gradient in the branch channel, but the mechanism which keeps the process going, is necessarily, what is explained above by the Einstein's equation.

A typical example exhibiting the importance of this phenomenon is quoted below:

Mississippi river is connected to another old river Atchafalya by a branch channel, which off-takes from the Mississippi river at about 480 kilometres from its mouth, as shown in Fig. 4.18.

The branch channel has been capturing flow from the main channel for the last many decades. In fact, engineers have tried to promote this flow as a means of diverting flood waters. About 26% of discharge was flowing towards this channel at one stage, with

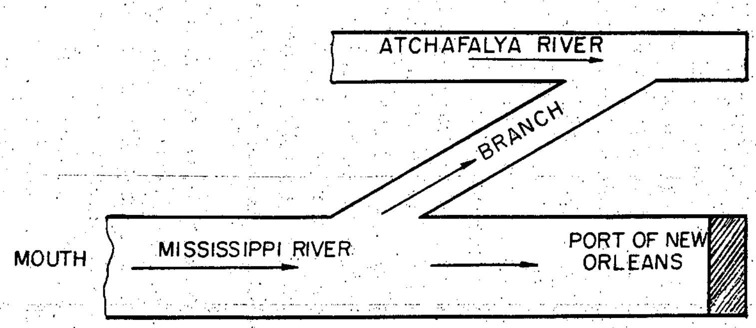


Fig. 4.18.

tendency of further increase. But, if the proportion is allowed to become rather higher and reaches to a value of the order of 40% or so, the process would become almost irreversible, and the whole flow may be rapidly diverted towards the Atchafalya river through this branch channel. If such a state arises, the port of New Orleans (situated at the join of Mississippi river with sea) could become redundant.

Example 4.12. *Determine the rate of bed load transport in a wide alluvial stream for the following data :*

Depth of flow = *4.5 m*

Velocity of flow = *1.3 m/s*

Bed slope $= 2.0 \times 10^{-4}$

Size distribution of the sediment :

d (mm)	*0.20*	*0.44*	*0.78*	*1.14*	*1.65*	*3.6*	*5.2*
% finer	*2*	*10*	*30*	*50*	*70*	*80*	*100*

Make use of Einstein's formula for bed load. Kinematic viscosity of water $= 1.01 \times 10^{-6}\ m^2/s$.

Solution. $V = \frac{1}{n} \cdot R^{2/3} \cdot \sqrt{S}$

where $R \approx D$ (for wide streams) = 4.5 m (given)

$V = 1.3$ m/s (given)

$S = 2 \times 10^{-4}$ (given)

$$\therefore \quad 1.3 = \frac{1}{n} \cdot (4.5)^{2/3} \cdot \sqrt{2 \times 10^{-4}}$$

or $$n = \mathbf{0.02965}$$

Also, $$n' = \frac{1}{24} \cdot (d_{50})^{1/6}$$

where $d_{50} = 1.14$ mm from the given table $= 1.14 \times 10^{-3}$ m

$$= \frac{1}{24} \cdot (1.14 \times 10^{-3})^{1/6} = \mathbf{0.0135}$$

$$\frac{n'}{n} = \frac{0.0135}{0.02965}\ \mathbf{0.454}$$

$$R' = R \cdot \left(\frac{n'}{n}\right)^{3/2} = 4.5\,(0.454)^{3/2} = \mathbf{1.377\ m}$$

Using equation (4.57), we have

$$w_0 = \text{fall velocity} = G \cdot \sqrt{gd\,(S_s - 1)}$$

$$\text{where } G = \sqrt{\frac{2}{3} + \frac{36\nu^2}{gd^3\,(S_s - 1)}} - \sqrt{\frac{36\,\nu^2}{gd^3\,(S_s - 1)}}$$

$$\text{where, } \frac{36\,\nu^2}{gd^3\,(S_s - 1)} = \frac{36 \times (1.01 \times 10^{-6})^2}{9.81 \times (1.14 \times 10^{-3})^3 \cdot 1.65}$$

$$= 1.5314 \times 10^{-3}$$

$$G = \sqrt{0.6667 + 1.5314 \times 10^{-3}} - \sqrt{1.5314 \times 10^{-3}}$$

$$= 0.817 - 0.039 = 0.778$$

$$\therefore \quad w_0 = 0.778\,\sqrt{9.81 \times (1.14 \times 10^{-3})\,(1.65)} = \mathbf{0.105\ m/s}$$

Using equation (4.60), we have

$$\frac{q_b}{w_0 \cdot d} = 40\left[\frac{\gamma_w R'S}{\gamma_w\, d\,(S_s - 1)}\right] = 40\left[\frac{R'S}{d\,(S_s - 1)}\right]^3 \qquad \because \; \tau_0 = \gamma_w R'S$$

To appreciate bed ripples, R' is used in place of R

$$\text{or} \quad \frac{q_b}{0.105 \times 1.14 \times 10^{-3}} = 40\left[\frac{1.377 \times 2.0 \times 10^{-4}}{1.14 \times 10^{-3}\,(1.65)}\right]^3$$

$\therefore$ Bed load by vol.

$$= q_b = 1.5027 \times 10^{-5}\ \text{m}^3/\text{sec/m width of channel.}$$

Bed load by weight (g_b)

$$g_b = q_b \cdot \gamma_w \cdot S_s$$

$$= 1.5027 \times 10^{-5} \times (9.81 \times 10^3) \times 2.65\ \text{N/sec/m} = \mathbf{0.39\ N/s/m} \quad \textbf{Ans.}$$

Example 4.13. *A wide irrigation channel is designed to have hydraulic mean depth of 3 m and bed slope equal to 1.6×10^{-4}. The bed sediment has an average median size of 0.3 mm. If the specific gravity of the bed soil is taken as 2.65 and the observed Manning's n to be 0.020, compute the rate of bed load transported by the channel in N/s/m width of channel.*

Also compute the suspended load concentrations at different depths to plot the sediment concentration curve for this channel. Make use of Einstein's formulas. Kinematic viscosity of water at given temperature may be taken to be $1.01 \times 10^{-6}\ m^2/s$.

Solution. Bed load transport rate as volume (q_b) is given by Einstein's equation (4.60) as :

$$\frac{q_b}{w_0 \cdot d} = 40\left[\frac{\gamma_w R'S}{\gamma_w \cdot d\,(S_s - 1)}\right]^3 = 40 \cdot \left[\frac{R'S}{d\,(S_s - 1)}\right]^3$$

To appreciate bed ripples, R' is used in place of R.

$$\text{where } w_0 = G \cdot \sqrt{gd\,(S_s - 1)} \qquad \text{...(4.57)}$$

$$\text{where } G = \sqrt{\frac{2}{3} + \frac{36 \cdot \nu^2}{gd^3\,(S_s - 1)}} - \sqrt{\frac{36\,\nu^2}{gd^3\,(S_s - 1)}}$$

where ν = kinematic viscosity of water at 20° C = 1.01×10^{-6}.

$$\frac{36\,\nu^2}{gd^3\,(S_s-1)} = \frac{36 \times (1.01 \times 10^{-6})^2}{9.81 \times (0.3 \times 10^{-3})^3\,(1.65)} = \frac{36 \times 1.01 \times 1.01 \times 10^{-12}}{9.81 \times (0.3)^3\,(1.65) \times 10^{-9}} = 0.084$$

$$\therefore \quad G = \sqrt{0.667 + 0.084} - \sqrt{0.084} = 0.867 - 0.290 = \mathbf{0.577}$$

$$w_0 = 0.577\sqrt{9.81 \times (0.3 \times 10^{-3})\,(1.65)} = \mathbf{0.04\ m/sec.}$$

Using equation (4.60) :

$$\frac{q_b}{0.04 \times (0.3 \times 10^{-3})} = 40\left[\frac{R'S}{d\,(S_s-1)}\right]^3$$

where $R' = R\left(\dfrac{n'}{n}\right)^{3/2}$

where $n' = \dfrac{1}{24} \cdot d^{1/6} = \dfrac{1}{24}\,(0.3 \times 10^{-3})^{1/6} = 0.01078$

$n = 0.020$ (given)

$\dfrac{n'}{n} = \dfrac{0.01078}{0.020} = 0.539$

$R = 3$ m (given)

$R' = 3 \times (0.539)^{3/2} = \mathbf{1.187\ m}$

$$\therefore \quad \frac{q_b}{0.040 \times (0.3 \times 10^{-3})} = 40\left[\frac{1.187 \times (1.6 \times 10^{-4})}{(0.3 \times 10^{-3})\,(1.65)}\right]^3 = 40\,(0.3837)^3 = 2.259$$

or $q_b = (2.259 \times 0.040 \times 0.3 \times 10^{-3})$

$= \mathbf{2.711 \times 10^{-5}\ m^3/sec/m}$ width of channel

$g_b = q_b \cdot \gamma_w \cdot S_s$

$= 2.711 \times 10^{-5}\,\dfrac{\text{m}^3}{\text{sec.m}} \times \left(9.81 \times 10^3\,\dfrac{\text{N}}{\text{m}^3}\right) \times 2.65$

$= 0.7$ N/s/m width of channel.

Hence, bed load transported per m width of channel

$= \mathbf{0.7\ N/s/m\ width.\ Ans.}$

For computing suspended load, we compute from Eq. (4.43),

$$c_a = c_{2d} = \frac{q_b}{23.2 \times V^{*\prime} \times d} \times 100\%$$

or

$$c_a = \frac{2.711 \times 10^{-5}}{23.2 \times V^{*\prime} \times (0.3 \times 10^{-3})} \times 100\%$$

where $V^{*\prime} = \sqrt{\dfrac{\tau_0'}{\rho_w}} = \sqrt{\dfrac{\rho_w g R' S}{\rho_w}} = \sqrt{gR'S}$

$= \sqrt{9.81 \times 1.187 \times 1.6 \times 10^{-4}} = \mathbf{0.043\ m/sec}$

$$\therefore \quad c_a = \frac{2.711 \times 10^{-5}}{23.2 \times 0.043 \times (0.3 \times 10^{-3})} \times 100\% = \mathbf{9.058\% = 90{,}580\ ppm.}$$

Now, using Eq. (4.40), we have

$$\frac{c}{c_a} = \left[\frac{a\,(D-y)}{y\,(D-a)}\right]^{\frac{w_0}{KV^*}}$$

where $V^* = \sqrt{gRS}$

$= \sqrt{9.81 \times 3 \times 1.6 \times 10^{-4}} = 0.069$ m/sec

$K = 0.4$ (constant)

$a = 2d = 2 \times 0.3 \times 10^{-3}$ m

$$\therefore \quad \frac{w_0}{KV^*} = \frac{0.04}{0.4 \times 0.069} = 1.457$$

$$\therefore \quad \frac{c}{9.058\%} = \left[\frac{(2 \times 0.3 \times 10^{-3})(3-y)}{y\,(3 - 2 \times 0.3 \times 10^{-3})}\right]^{1.457} = 2 \times 10^{-4}\left[\frac{3-y}{y}\right]^{1.457}$$

or
$$c = 1.812 \times 10^{-3}\left(\frac{3-y}{y}\right)^{1.457}$$

The different values of c at different values of y, say 0.05 m, 0.1 m, 0.15 m, 0.2 m, 0.5 m, 1.0 m, 2.0 m, 3.0 m are now computed by using the above equation, as :

$$c_{0.05m} = 1.812 \times 10^{-3}\left(\frac{3-0.05}{0.05}\right)^{1.457} = 0.6891\% = 6891 \text{ ppm}$$

$$c_{0.1\,m} = 1.812 \times 10^{-3}\left(\frac{3-0.1}{0.1}\right)^{1.457} = 0.2447\% = 2447 \text{ ppm}$$

$$c_{0.15\,m} = 1.812 \times 10^{-3}\left(\frac{3-0.15}{0.15}\right)^{1.457} = 0.1264\% = 1264 \text{ ppm}$$

$$c_{0.2\,m} = 1.812 \times 10^{-3}\left(\frac{3-0.2}{0.2}\right)^{1.457} = 0.0847\% = 847 \text{ ppm}$$

$$c_{0.5\,m} = 1.812 \times 10^{-3}\left(\frac{3-0.5}{0.5}\right)^{1.457} = 0.0188\% = 188 \text{ ppm}$$

$$c_{1.0\,m} = 1.812 \times 10^{-3}\left(\frac{3-1.0}{1.0}\right)^{1.457} = 0.0050\% = 50 \text{ ppm}$$

$$c_{2.0\,m} = 1.812 \times 10^{-3}\left(\frac{3-2.0}{2.0}\right)^{1.457} = 0.0005\% = 5 \text{ ppm}$$

$$c_{2.5\,m} = 1.812 \times 10^{-3}\left(\frac{3-2.5}{2.5}\right)^{1.457} = 0.00025\% = 2.5 \text{ ppm}$$

$$c_{3\,m} = 1.812 \times 10^{-3}\left(\frac{3-3}{3}\right)^{1.457} = \text{zero.}$$

DESIGN PROCEDURE FOR IRRIGATION CHANNELS

After having discussed the various theories for designing irrigation canals, we shall now discuss, as to how in actual practice, an irrigation canal is designed and constructed.

4.10. Cross-section of an Irrigation Canal

A typical and most desired section of a canal is shown in Fig. 4.19. This section is 'partly in cutting and partly in filling', and aims at balancing the quantity of earth work in 'excavation' with that in 'filling'.

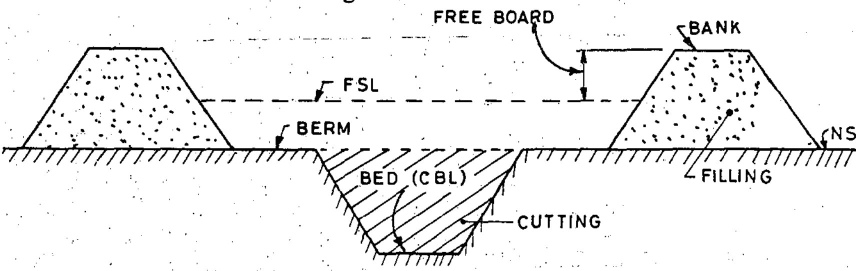


Fig. 4.19.

Sometimes, when the natural surface level (*i.e. NSL*) is above the top of the bank, the entire canal section will have to be in cutting, and it shall be called 'canal in cutting'. Similarly, when the *NSL* is lower than the Bed level of the canal, the entire canal section will have to be built in filling, and it is called 'canal in filling', or 'canal in banking'.

Side Slopes. The side slopes should be such that they are stable, depending upon the type of the soil. A comparatively steeper slope can be provided in cutting rather than in filling, as the soil in the former case shall be more stable.

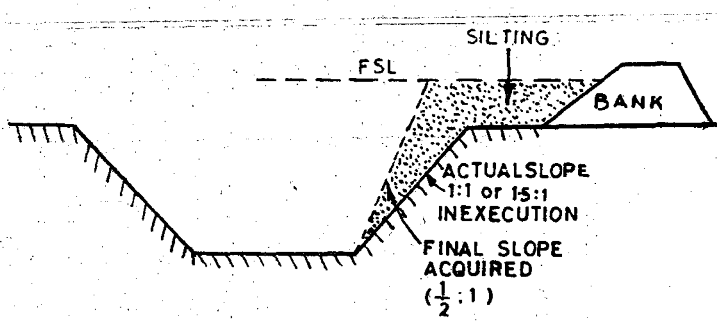


Fig. 4.20.

$1H : 1V$ (1 : 1) to $1\frac{1}{2}H : 1V$ ($1\frac{1}{2}$: 1) slope in cutting, and $1\frac{1}{2}H : 1V$ to $2H : 1V$ in filling, are generally adopted.

In case of channels with silt laiden water, the actual capacity of the channel is worked out with $\frac{1}{2}$: 1 side slopes, even though flatter slopes such as 1 : 1 ot $1\frac{1}{2}$: 1 may be constructed at the time of execution. This is because of the fact that the sides of such a channel gets silted up to a slope $\frac{1}{2}$: 1 with the passage of time, as shown in Fig. 4.20.

Berms. Berm is the horizontal distance left at ground level between the toe of the bank and the top edge of cutting.

The berm is provided in such a way that the bed line and the bank line remains parallel. If $s_1 : 1$ is the slope in cutting and $s_2 : 1$ in filling, then the initial berm width $= (s_2 - s_1)\, d_1$. Since ground level (NSL) fluctuates considerably, while canal bed level (CBL) varies very slightly, d_1 shall vary ; and, therefore, the berm width shall vary.

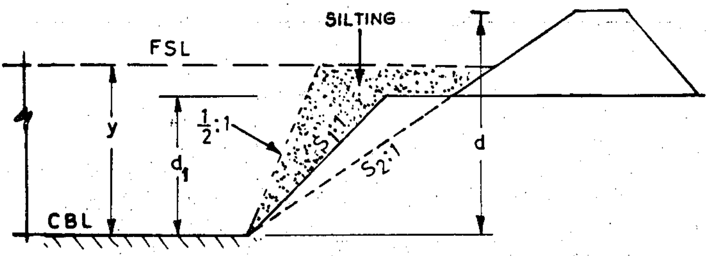


Fig. 4.21.

After the water flows in the channel for some time, the silt gets deposited on the sides giving them a slope of $\frac{1}{2}$: 1. The position of the berm, therefore, changes from ground level to FSL, as shown in Fig. 4.21 and its width becomes equal to $(s_2 - \frac{1}{2}) \cdot y$. If $s_2 = 1\frac{1}{2}$, then the final berm width = y, *i.e.* equal to the depth of the canal.

The berms when fully formed, serve the following purposes :

(*i*) The silt deposited on the sides is very fine and impervious. It, therefore, serves as a good lining for reducing losses, leakage and consequent breaches, etc.

(*ii*) They help the channel to attain regime conditions, as they help in providing a wider waterway, if required. Even fluctuations of discharge do not produce much fluctuations in depths because of wider waterway.

(*iii*) They give additional strength to the banks and provide protection against erosion and breaches.

(*iv*) The possibility of breaches gets reduced because the saturation line comes more in the body of the embankment.

(*v*) They protect the banks from erosion due to wave action.

(*vi*) They provide a scope for future widening of the canal.

(*vii*) Berms can be used as borrow pits for excavating soil to be used for filling.

Freeboard. The margin between FSL and bank level is known as freeboard. The amount of freeboard depends upon the size of the channel. The generally provided values of freeboard are given in Table 4.7.

Table 4.7. Values of Free-boards in Canals

Discharge in cumecs	*Extent of freeboard in metres*
1 to 5	0.50
5 to 10	0.60
10 to 30	0.75
30 to 150	0.90

Banks. The primary purpose of banks is to retain water. They can be used as means of communication and as inspection paths. They should be wide enough, so that a minimum cover of 0.5 metre is available above the saturation line, as shown in Fig. 4.22. High banks will have to be designed as earth dams.

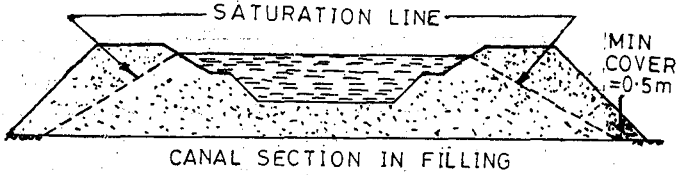


Fig. 4.22.

Service Roads. Service roads are provided on canals for inspection purposes, and may simultaneously serve as the means of communication in remote areas. They are provided 0.4 m to 1.0 m above FSL, depending upon the size of the channel.

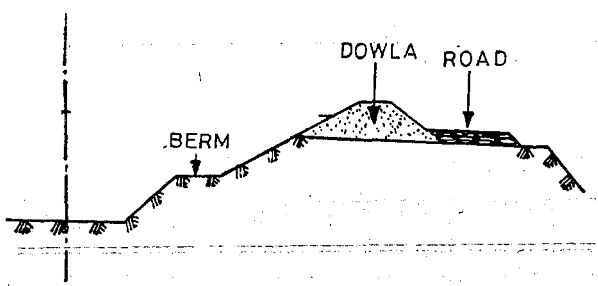


Fig. 4.23.

Dowlas. As a measure of safety in driving, dowlas 0.3 m high and 0.3 to 0.6 m wide at top, with side slopes of $1\frac{1}{2}:1$ to $2:1$, are provided along the banks, as shown in Fig. 4.23. They also help in preventing slope erosion due to rains, etc.

Back Berm or Counter Berms. Even after providing sufficient section for bank embankment, the saturation gradient line may cut the downstream end of the bank. In such a case, the saturation line can be kept covered at least by 0.5 metre with the help of counter berms, as shown in Fig. 4.24.

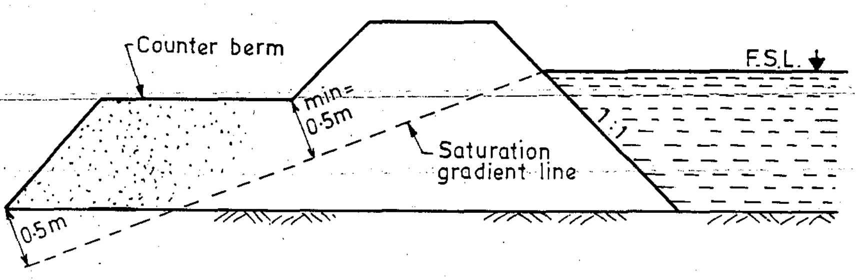


Fig. 4.24.

The straight saturation gradient line may be drawn with the following slopes.

Table 4.8. Assumed Values of Saturation Gradients in Different Soils

Type of soil	*Slope (H : V)*
Clay	1 in 4
Clayey Loam	1 in 6
Loam	1 in 8
Loamy sand	1 in 10
Sand	1 in 15

Spoil Banks. When the earthwork in excavation exceeds earthwork in filling, even after providing maximum width of bank embankments, the extra earth has to be disposed of economically. To dispose of this earth by mechanical transport, etc. may become very costly, and an economical mode of its disposal may be found in the form of collecting this soil on the edge of the bank embankment itself. The soil is, therefore, deposited in such a case, in the form of heaps on both banks or only on one bank, as shown in Fig. 4.25. These heaps of soil are discontinued at suitable intervals and longitudinal drains running by their sides are excavated for the disposal of rain water. Cross drains through the spoil banks may also be excavated, if needed.

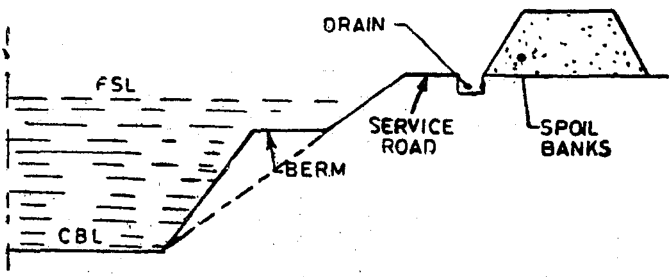
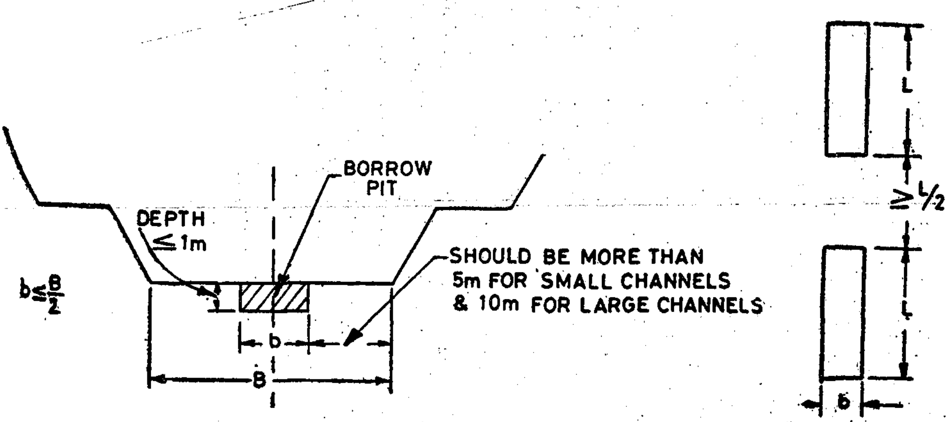


Fig. 4.25. Spoil bank.

Borrow Pits. When earthwork in filling exceeds the earthwork in excavation, the earth has to be brought from somewhere. The pits, which are dug for bringing earth, are known as borrow pits. If such pits are excavated outside the channel, they are known as *external borrow pits,* and if they are excavated somewhere within the channel, they



(*a*). Elevation. (*b*). Plan.

Fig. 4.26. Internal borrow pits.

are known as *internal borrow pits.* It is a very costly affair to bring soil from distances. Even in the nearby areas, these pits may cause mosquito nuisance due to collection of rain water in these pits, and hence, external borrow pits are not preferred.

Internal borrow pits are, therefore, excavated on the bed of the canal, as shown in Fig. 4.26, when needed.

The borrow pits should start from a point at a distance more than 5 m from the toe for small channels, and 10 m for large channels. The width of these pits *b,* should be less than half the width of the canal *B*, and should be dug in the centre. The depth of these pits should be equal to or less than 1 m.

Longitudinally, these pits should not run continuous, but a minimum space of $\frac{L}{2}$ should be left between two consecutive pits, (where L is the length of one pit) as shown in Fig. 4.26 (*b*).

4.11. Balancing Depth for Excavating Canals

It was pointed out earlier that the maximum economy can be achieved in canal construction, if the earthwork in excavation equals the earthwork in filling. Such a thing is possible when a canal is constructed partly in filling and partly in cutting, which mostly happens in practical life.

If this balance between cutting and filling can occur, then the need for spoil banks or borrow pits is entirely eliminated, and moreover, earthwork has to be paid only once in a single item.

For a given cross-section of a channel, there can be only one depth, for which such a balance between cutting and filling will occur. This depth is known as the Balancing depth. This depth can be worked out easily by equating the areas of cutting and filling, as illustrated in the following example.

Example 4.14. *Calculate the balancing depth for a channel section having a bed width equal to 18 m and side slopes of 1 : 1 in cutting and 2 : 1 in filling. The bank embankments are kept 3.0 m higher than the ground level (berm level) and crest width of banks is kept as 2.0 m.*

Solution. The channel section is shown in Fig. 4.27. Let d_1 be the balancing depth, *i.e.* the depth for which excavation and filling becomes equal.

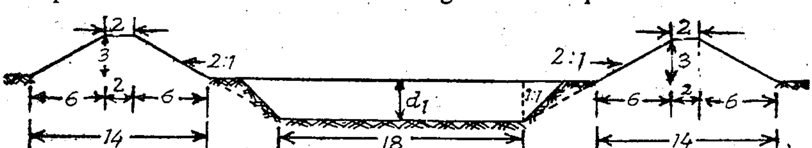


Fig. 4.27.

Area of cutting = $(18 + d_1)\, d_1$ sq. m.

Area of filling $= 2\left[\frac{2+14}{2} \times 3\right] = 48$ sq. m.

Equating cutting and filling, we get

$$(18 + d_1)\, d_1 = 48 \qquad \text{or} \qquad 18d_1 + d_1^2 = 48$$

or $\quad d_1^2 + 18d_1 - 48 = 0$

or $$d_1 = \frac{-18 \pm \sqrt{324+192}}{2}$$

$$= \frac{-18 \pm 22.7}{2}$$

Ignoring unfeasible – ve sign, we get

$$d_1 = \frac{-18+22.7}{2} = \frac{4.7}{2} = 2.35 \text{ m}$$

$\therefore$ Balancing Depth = **2.35 m. Ans.**

Example 4.15. *An irrigation channel having a full supply level of 4 m above the existing ground level is provided with banks 3 m wide at top. The side slopes are 2H : 1 V ; and the slope of the hydraulic gradient line through the bank soil is 5 : 1. Assuming a free board of 1 m, calculate the minimum width and height of counterberm needed to ensure that the seepage of water does not pose any problem for the safety of the canal banks.*

Solution. The given values are used in sketching the X-section of the Canal, as shown Fig. 4.28. The H.G. line is kept covered by 0.5 m (see $EF = HI = 0.5$ m).

The counter berm is started from the point E on the D/s slope, at a height h metre above the bed level. The width of the counter berm is kept as equal to EG ; say B metre. *Now, From H.G. line slope,* we have

$$NJ = 4 \times 5 = 20 \text{ m}$$

(1 : 5 slope of H.G. line starting at ht. = 4 m)

But $NJ = NM + ML + LK + KJ$

or $20 = 2 + 3 + 10 + KJ$

$\therefore$ $KJ = 5$ m.

But $B = EG = KH$ (In || gm $EKHG$)

or $B = KJ + JH$

or $B = 5 + JH$ ...(i)

where JH is determined from ΔHIJ ; as :

$$\frac{0.5}{JH} = \sin \alpha$$

where α is the angle which H.G. line makes with horizontal ; *i.e.*, $\tan \alpha = \frac{1}{5}$; or $\alpha = 11.31°$

$$\therefore \quad \frac{0.5}{JH} = \sin 11.31°$$

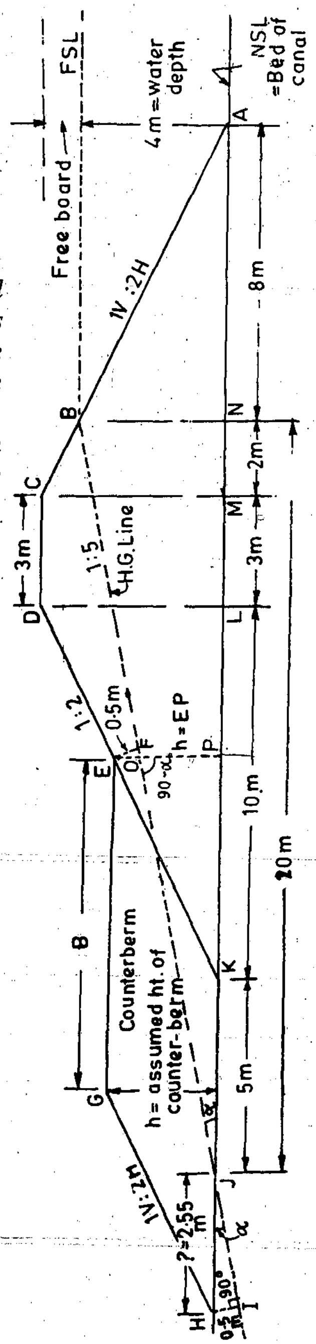


Fig. 4.28

$$\text{or } JH = \frac{0.5}{\sin 11.31°} = \mathbf{2.55\ m} \qquad ...(ii)$$

$\therefore \quad B = 5 + 2.55 = \mathbf{7.55\ m}$ **Ans.**

To determine ht of counter berm (h), we calculate as below :

$$h = EP = EO + OP$$

where EO is calculated from ΔEOF as

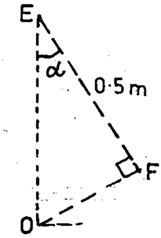


Fig. 4.28 (*a*)

$$\frac{EF}{EO} = \frac{0.5}{EO} = \cos \alpha$$

$$\text{or} \quad EO = \frac{0.5}{\cos \alpha} = \frac{0.5}{\cos 11.35°} = 0.51\ \text{m}$$

$$\therefore \quad OP = h - 0.51 \qquad ...(iii)$$

But $\dfrac{OP}{PJ} = \dfrac{1}{5}$ (slope of H.G. line)

$$\text{or} \quad \frac{h - 0.51}{PJ} = \frac{1}{5} \qquad ...(iv)$$

But $PJ = PK + KJ$

$= 2h + 5 \qquad ...(v)$

Putting in (*iv*)

$$\frac{h - 0.51}{2h + 5} = \frac{1}{5}$$

$$5h - 2.55 = 2h + 5$$

$$3h = 7.55$$

$$\mathbf{h = 2.52\ m.}\ \textbf{Ans.}$$

$$\left.\begin{array}{r}\text{Hence, width of counter berm} = 7.55\ \text{m} \\ \text{ht. of counterberm} = 2.52\ \text{m}\end{array}\right] \textbf{Ans.}$$

4.12. Fixing the L-Section of the Canal and Other Design Considerations

If a channel is designed according to Lacey's theory, it shall have a fixed slope and a fixed section for a given discharge and silt factor. But on the other hand, if the channel is designed on Kennedy's theory, it can have different sections for different slopes. In practice, it has been found that Lacey's slope equation gives excessive slopes. Lacey himself had changed his fixed slope equation afterwards, as pointed out earlier, making it flexible. *The slope of the channel is, however, fixed on available country slope consistent with economy.* A steeper slope governed by maximum permissible velocity, will be most economical, but it will lower the FSL, causing less irrigation. Hence, the maximum possible irrigation would indicate flatter slopes governed by minimum permissible velocity. A via-media between these two limits must be adopted for selecting a suitable bed slope for the channel.

If the chosen designed slope is found to be flatter than the natural available slope, the difference can be adjusted by providing suitably designed falls (explained in chapter 12). But if the designed slope is steeper than that available, then adjustments are made to change the design slope, so as to make it as near to the available slope as possible.

Since a change in depth causes non-uniform flow, it is desirable to change the depth as less as possible. For this reason, the channels in the upper reaches are generally designed with large bed width to depth ratio.

Moreover, in Kennedy's theory, there can be various combinations of width and depth for a given slope. The width and depth ratio can be controlled using the following empirical formulas ;

1. (*a*) *For channels upto 15 cumecs*

$$y = 0.5\sqrt{B} \quad \text{...(4.62)}$$

where y = Depth of water in channel
B = Base width of channel.

1. (*b*) For channels of 15 cumecs and above, depths of the following order may be provided :-

Table 4.9. Assumed Values of Water Depths for Canals

Discharge Q (cumecs)	*Depth y in metres*
15	1.7
30	1.8
75	2.3
150	2.6
300	3.0

2. C.W.C. has recommended a graphical relation giving B/y ratios for various discharges (0 to 300 cumecs) as shown in Fig. 4.29.

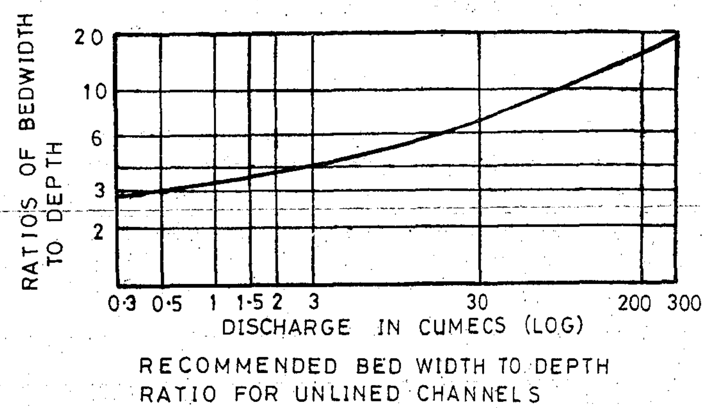


Fig. 4.29.

The limiting maximum permissible velocities for unlined channels in different soils are given in Table 4.10.

Table 4.10. Max. Permissible Velocities in Unlined Channels

Type of soil	*Maximum permissible velocity in m/sec*
Rock and gravel	1.5
Murum, hard soil, etc.	1.0 to 1.1
Sandy loam, black cotton soil etc.	0.6 to 0.9
Very light loose sand to average sandy soils	0.3 to 0.6
Ordinary soils	0.6 to 0.9

After marking the ground line, a trial bed line, and water depths, the FSL line on the *L*-section of the channel is checked, as it is governed by the following considerations:

(*i*) The FSL of the channel should be about 10 to 20 cm above the ground line for most of its length.

(*ii*) The water depth (y) or FSL governs the depth of excavation (d_1) and hence, attempts should be made to equal d_1 with balancing depth, so that the channel becomes in balanced earth work.

(*iii*) For branch canal, etc., where some distributaries have to take-off from it, the FSL of the channel should be kept 15 cm higher than the FSL of the off-taking channel, so as to allow for losses in head regulator.

Suitable adjustments in bed slope, depth, etc. can be made for fixing FSL on these considerations.

After fixing *L*-section of the channel, the cross-section can be fixed on the basis of various canal standards, given in Table 4.11.

Design Procedure

1. First of all, the longitudinal section of the existing ground along the proposed canal alignment, is plotted on a suitable scale (say 1 cm = 100 metres for horizontal scale, and 2 cm = 1 m for vertical scale).

2. A suitable channel slope is assumed from Lacey's diagrams for an approximate discharge and silt factor. It is made consistent with the existing slope.

3. A trial slope line is marked for drawing FSL line, keeping in view the guidelines already given. The depth and position of falls, etc. is decided tentatively.

4. The channel is then designed from its tail reach towards its head reach, kilometre to kilometre.

5. The discharge required in the channel in the given reach, for required irrigation potential, is worked out, and losses are added, so as to calculate the required discharge. The cumulative losses at a particular km of channel is the sum total of the losses occurring between the particular km and the next below it (*i.e.* for example at km 3, the total losses would be equal to losses between km 3 and km 4, plus, total losses below km 4).

Since the section at km 4 will be worked out earlier, the losses between km 3 and km 4 can be worked out on the wetted perimeter calculated on the basis of the channel section at km 4. The required discharge is increased by 10%, so as to obtain the design discharge.

6. The channel is now designed for this discharge and assumed bed slope, by Kennedy's theory, using Garret's diagrams. The bed width and depth ratios are kept within specified limits. Sometimes, even the slope may have to be changed for keeping them so.

7. The bed slope, FSL, falls, etc. are all adjusted using intelligence, judgement and knowledge. The bed levels, water depths, etc. are drawn on *L*-section. The *X*-sections at every km are then drawn, using canal standards.

Table 4.11. Canal Standards

S. No.	Item	*Size of canal*					
		Minor Distributaries		*Major Distributaries*		*Main and Branch Canals*	
		below 0.3 cumec	*0.3 to 1 cumec*	*1 to 5 cumecs*	*5 to 10 cumecs*	*10 to 30 cumecs*	*30 to 150 cumecs*
1.	Maximum width of the bank crest	1.0 m	1.5 m	2 m	2.25 m	2.5 m	3 m
2.	Width of roadway	Nil	3.5 m	3.5 m	5 m	5 m	6 m
3.	Free board	0.3 m	0.4 m	0.5 m	0.6 m	0.75 m	0.9 m
4.	Depth of earth cover over saturation gradient	0.5 m	0.5 m	0.5 m	0.5 m	0.8 m	1 m
5.	Width of berms	Maximum berm width = 0.6 m+ $\frac{1}{4}$th of the width of combined side slopes of cutting and embankment. Maximum berm width = 0.6 m+ $\frac{1}{2}$ the width of combined slopes					
6.	Width of land to be acquired (clear of banks) when canal cutting is deeper than the balancing depth.	Half the height of the bank above ground, subject to minimum of 1.5 m.					
7.	Width of land to be acquired (clear of banks) when canal cutting is lesser than the balancing depth.	Full height of bank above ground level plus 1.5.				Full height of bank+ 5 m.	Full height of bank+ 5 m

The calculations of discharge and channel dimensions, etc. can be carried out in a tabular form called "Schedule of Area Statistics and Channel Dimensions'. A specimen form of this table is given in table 4.12.

Tabel 4.12. Specimen Table for Schedule of Area Statistics and Channel Dimensions

Below km	*Gross commanded area*	*Culturable commanded area*	*Area to be irrigated*		
			Rabi	*Sugar*	*Rice*
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*
Outlet discharge factor	*Outlet discharge*	*Losses in reach*	*Total losses*	*Total discharge*	*Bed slope*
(7)	*(8)*	*(9)*	*(10)*	*(11)*	*(12)*
Bed width	*Water depth*	*Height of banks*	*Width of banks*	*Velocity*	*CVR* V/V_0
(13)	*(4)*	*(15)*	*(16)*	*(17)*	*(18)*

Example 4.16. *A distributary canal takes off from a branch canal having CBL at 204.0 m and FSL at 205.8 m. The gross commanded area at the head of the distributary is 30,000 hectares, and after each km it is reduced by 5,000 hectares. Out of this command, the culturable area is only 75%. The intensity of irrigation for the Rabi and Kharif seasons is 32% and 15% respectively. Design suitable channel sections for the first 3 km of this distributary, assuming the following data :*

(i) Total losses below km 3 = 0.44 cumec.

(ii) Channel losses occur @ 2 cumecs/million square metres of wetted perimeter.

(iii) Kor period for Rabi (wheat) = 4 weeks

(iv) Kor depth for Rabi = 14 cm

(v) Kor period for Kharif (Rice) = 2.5 weeks

(vi) Kor depth for Kharif = 20 cm.

(vii) Manning's n = 0.0225.

(viii) Critical velocity ratio = 0.95.

The ground levels at every 200 metres, along the line of the proposed alignment, have been obtained and are tabulated in table 4.13.

Table 4.13

Distance from head in metres	*Reduced level (G.T.S) in metres*
0	205.20
200	205.30
400	205.25
600	205.00
800	204.90
1,000	204.30
1,200	204.30
1,400	204.20
1,600	204.20
1,800	204.10
2,000	204.05
2,200	204.00
2,400	203.95
2,600	203.95
2,800	203.90
3,000	203.80

Solution. The channel is to be designed from its tail (where the losses are known) towards its head, km by km. The gross commanded areas and culturable commanded areas at various km are, first of all, worked out in Table 4.14.

Table 4.14

Below km (1)	*Gross commanded area in hectares* (2)	*Gross culturable area in hectares* (3)
0 (*i.e.* head)	30,000	22,500
1	25,000	18,750
2	20,000	15,000
3	15,000	11,250

Outlet discharges for the two crop seasons are determined as given below :

(*i*) *For Rabi*, $D = \dfrac{8.64\,B}{\Delta}$

where, $B = 4$ weeks $= 28$ days
$\Delta = 14$ cm $= 0.14$ m.

$$\therefore \quad D = \frac{8.64 \times 28}{0.14} = 1728 \text{ hectares/cumec.}$$

(*ii*) *For Kharif*, $D = \dfrac{8.64\,B}{\Delta}$

where, $B = 2.5$ weeks $= 2.5 \times 7$ days $= 17.5$ days
$\Delta = 20$ cm $= 0.20$ m.

$$\therefore \quad D = \frac{8.64 \times 17.5}{0.20} = 756 \text{ hectares/cumec}$$

Intensity of irrigation for Rabi = 32%
and Intensity of irrigation for Kharif = 15%.

If G is the gross culturable area at any point, then $0.32\,G$ is the Rabi area and $0.15G$ is the Kharif area.

$$\text{Discharge reqd. for this Rabi area} = \frac{0.32\,G}{\text{Outlet factor for Rabi}} = \frac{0.32\,G}{1728} = \frac{G}{5400}$$

$$\text{Similarly, discharge reqd. for Kharif area} = \frac{0.15\,G}{756} = \frac{G}{\mathbf{5040}}$$

Since the discharge required for Kharif crop is more than that required for Rabi crop, the outlet factor of Kharif crop becomes the controlling factor. Discharges needed at various kilometres for the given command are worked out in Table 4.15.

Table 4.15

Below km	*Gross culturable area from Table 4.14 col. (3)*	*Discharge required for Kharif crop in cumecs* $= \dfrac{\text{Col. (2)}}{5040}$
(1)	(2)	(3)
0	22,500	4.46
1	18,750	3.72
2	15,000	2.98
3	11,250	2.23

(*i*) **Design at km 3**

Losses below km 3 = 0.44 cumec (given)

Discharge reqd. for crop at this point (Table 4.15) = 2.23 cumecs.

Total discharge required $= 2.23 + 0.44 = 2.67$ cumecs
Design discharge = 10% more than required.
$= 1.1 \times 2.67 = 2.937$ cumecs, say **2.94 cumecs**
$Q = 2.94$ cumecs

$$\frac{V}{V_0} = \text{C.V.R.} = 1.0$$
$$n = 0.0225$$

Lacey's regime slope for this discharge and silt factor = 1, is approximately 22 cm per km. Let us kep the slope at 22.5 cm/km*:

Assume bed slope = 22.5 cm per km.

or $$S = \frac{0.225}{1000} = \frac{1}{4444}.$$

From Garret's diagrams [Plate 4.1 (*b*)], assuming $\frac{1}{2}$: 1 side slopes, the channel section is designed as shown in Table 4.16.

Table 4.16

Discharge	*S*	*B* *m*	*y* *m*	$A = \left(B + \frac{y}{2}\right)y$ m^2	$V = \frac{Q}{A}$ *m/sec*	V_0 *m/sec*	$\frac{V}{V_0}$	*Remarks*
2.94 cumecs	$\frac{1}{4444}$	5.0	0.98	5.38	0.55	0.53	1.04	much larger than 0.95
		4.5	1.05	5.28	0.56	0.58	0.96	O.K.

Hence, adopt $B = 4.5$ m
$y = 1.05$ m
$S = \frac{1}{4444}$ (*i.e.* 22.5 cm per km.)

These dimensions, quite nearly satisfy the bedwidth-depth relationship, given by $y = 0.5\sqrt{B}$; and hence, the assumed slope is all right and can be adopted.

(*ii*) **Design at km 2**

Outlet discharge required below km 2, from Table 4.15, col. (3) = 2.98 cumecs.
Losses below km 3 = 0.44 cumec.

Losses in channel between km 3 to km 2 : For the calculation of these, the perimeter of the section at km 3 shall be taken, as the section at km 2 is not known so far.

Wetted perimeter $= B + \sqrt{5} \cdot y = 4.5 + \sqrt{5} \times 1.08 = \mathbf{6.92\ m}$

Loss @ 2 cumecs/million sq. m. $= 2 \times \left[\frac{6.92 \times 1000}{10^6}\right] = \mathbf{0.014\ cumec.}$

Total Losses below km 2

= Losses below km 3 + Losses between km 3 and km 2
= 0.44 + 0.014 = 0.454 cumec

Total discharge required at km 2 = 2.98 + 0.454 = 3.434 cumecs
Design discharge = 1.1 × 3.434 = 3.7874 ; say **3.79 cumecs**

Use the same slope of 22.5 cm in 1 km, *i.e.* $\frac{1}{4444}$.

* The bed slope is generally kept in multiples of 2.5 cm/km, to obtain bed levels in fraction of 0.005 m, *i.e.* the fractions up to which levels can be read on levelling staff.

Using Garret's diagrams [Plate 4.1 (*b*)], we design the channel section as shown in Table 4.17.

Table 4.17

Q	S	B m	y m	$A=\left(B+\frac{y}{2}\right)y\ m^2$	$\frac{Q}{A}=V$ m/sec	V_0 m/sec	$\frac{V}{V_0}$	Remarks
3.79 cumecs	$\frac{1}{4444}$	6.0	1.05	6.85	0.55	0.58	0.95	O.K.

Hence, adopt $B = 6.0$ m

$y = 1.05$ m

$$S = \frac{1}{4444}$$

(*iii*) **Design at km 1**

Outlet discharge required below km 1, from Table 4.15, col. (3) = 3.72 cumecs.

Losses below km 2 worked out earlier = 0.454 cumec.

Losses between km 2 and km 1 : To work out these losses, the perimeter of the section at km 2 shall be taken, as the section at km 1 is not known so far.

$$\therefore \text{ Wetted perimeter} = B + \sqrt{5}y$$
$$= 6.0 + \sqrt{5} \times 1.08 = 6.0 + 2.42 = \mathbf{8.42\ m.}$$

Losses @ 2 cumecs/million sq. m. (*i.e.* in length of 1 km, *i.e.* 1000 m)

$$= 2 \times \left(\frac{8.42 \times 1000}{10^6}\right) = 0.017 \text{ cumec.}$$

Total Losses below km 1

$$= 0.454 + 0.017 = 0.471 \text{ cumec}$$

Total discharge required at km 1

$$= 3.72 + 0.471 = 4.191 \text{ cumecs}$$

Design discharge $= 1.1 \times 4.191 = \mathbf{4.61\ cumecs.}$

Let us adopt a slope of 20 cm in 1 km, *i.e.* $S = \frac{1}{5000}$

Using Garret's diagrams [Plate 4.1 (*c*)], the required channel section is designed as shown in Table 4.18.

Table 4.18

Q	S	B m	y m	$A=\left(B+\frac{y}{2}\right)y$, m^2	$\frac{Q}{A}=V$ m/sec	V_0 m/sec	$\frac{V}{V_0}$	Remarks
4.61 cumecs	$\frac{1}{5000}$	6.0	1.2	7.92	0.582	0.615	0.947	O.K.

Hence adopt $B = 6.0$ m

$y = 1.2$ m

$S = \frac{1}{5000}$ (20 cm in 1 km.)

(*iv*) **Design at 0 km.**

Outlet discharge required at 0 km, from Table 4.15, col. (3) = 4.46 cumecs

Losses below km 1, as worked out earlier = 0.471 cumec.

Losses between km 0 to 1 : To work out these losses, the perimeter of the section at km 1 shall be taken, as the section at km 0 is not known so far.

Now, wetted perimeter

$$= B + \sqrt{5}\cdot y$$
$$= 6.0 + \sqrt{5} \times 1.2 = 6 + 2.68 = 8.68 \text{ m}$$

Losses @ 2 cumecs/million sq. m. in a length of 1 km

$$= 2\left[\frac{8.68 \times 1000}{10^6}\right] = 0.01736 = \text{say } 0.017 \text{ cumec.}$$

Total losses below km 0

$$= 0.471 + 0.017 = 0.488 \text{ cumec}$$

Total discharge required at 0 km

$$= 4.46 + 0.488 = 4.948 \text{ cumecs.}$$

Design discharge $= 1.1 \times 4.948 = \mathbf{5.44}$ **cumecs.**

Let us adopt a slope of 20 cm in 1 km, *i.e.* $S = \frac{1}{5000}$

Using Garret's diagrams [Plate 4.1 (*c*)], the required channel section is worked out as shown in Table 4.19.

Table 4.19

Q	S	B m	y m	$A = \left(B + \frac{y}{2}\right)y$ m^2	$V = \frac{Q}{A}$ m/sec	V_0 m/sec	$\frac{V}{V_0}$	*Remarks*
5.44 cumecs	$\frac{1}{5000}$	7.2	1.20	9.36	0.58	0.615	0.944	O.K.

Hence, adopt $B = 7.2$ m

$y = 1.2$ m

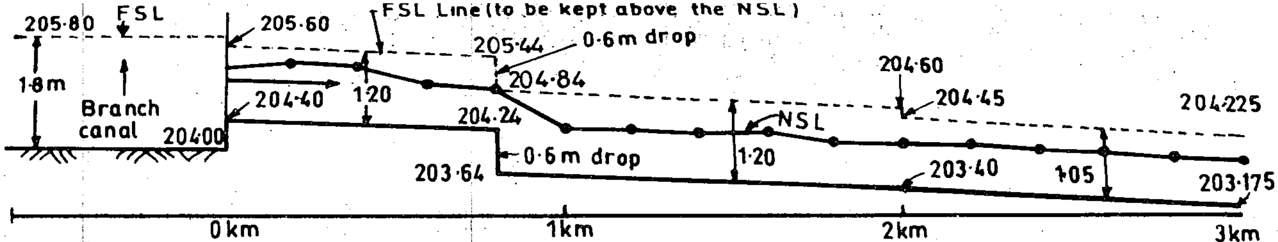
$S = \frac{1}{5000}$ (20 cm in 1 km)

All the data worked out above, has been entered at their proper places in the 'schedule of area statistics and channel dimensions' (Table 4.20). The table has been completed with the help of Canal Standards already given in Table 4.11. The L-section of the distributary is drawn, as shown in Fig. 4.30, starting from the head (*i.e.* 0 km) by keeping its FSL at head at 0.2 m below the FSL of the branch channel. The cross-sections at various km are drawn in Figs. 4.31 (*a*), (*b*), (*c*) and (*d*), by assuming $1\frac{1}{2} : 1$ slopes in filling and 1 : 1 slopes in cutting. The dowel level is kept 0.15 m higher than the bank level, and road level at 0.15 m below the bank level.

Note. The slopes of 1 : 1 shall afterwards become $\frac{1}{2} : 1$, due to silting, and that is why in design calculations, $\frac{1}{2}$ H : 1 V slopes are taken.

Table 4.20
Schedule of Area Statistics and Channel Dimensions

Below km	*Gross commanded area (hectares)*	*Culturable commanded area (hectares)*	*Controlling area to be irrigated (15%) in hectares*	*Controlling outlet discharge factor*	*Losses in reach (cumecs)*	*Total losses (cumecs)*	*Design discharge (cumecs)*	*Bed slope (cm/km)*	*Bed width (m)*	*Water depth (metres)*	*Freeboard from Table 14.11 in metres*	*Ht. of bank above ground decided after drawing L-section*	*Width of bank from Table 14.11*	*Velocity in m/sec*	$\frac{V}{V_0} = m$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
0	30,000	22,500	3,775	756	0.017	0.488	5.44	20	7.2	1.2	0.6	1.00	2.25	0.58	0.944
1	25,000	18,750	2,812.5	756	0.010	0.471	4.61	20	6.0	1.2	0.5	1.00	2.0	0.582	0.947
2	20,000	15,000	2,250.0	756	0.014	0.454	3.79	22.5	6.0	1.05	0.5	0.90	2.0	0.55	0.95
3	15,000	11,250	1,687.5	756	—	0.440	2.94	22.5	4.2	1.05	0.5	0.925	2.0	0.56	0.96



Cutting depth	0.80	0.94	0.93	0.72 0.66	1.26	0.70	0.74	0.68	0.72	0.76	0.65	0.644	0.638	0.782	0.678	0.62
Designed discharge	5.44 m^3/s					4.61 m^3/s					3.79 m^3/s					2.94 m^3/s
Bed slope	20 cm/km					20 cm/km					22.5 cm/km					22.5 cm/km
Depth	1.20 m					1.20 m					1.05 m					1.05 m
Bed width	7.2 m					6.0 m					6.0 m					4.2 m
Designed FSL (m)	205.60			205.44	204.84	204.80				204.60	204.45					204.255
Ground level available (m)	205.20	205.30	205.25	205.0	204.90	204.30	204.30	204.20	204.20	204.10	204.05	204.00	203.95	203.95	203.90	203.80
Designed bed level (m)	204.40	204.36	204.32	204.28 204.24	203.64	203.60	203.56	203.52	203.48	203.44	203.40	203.355	203.310	203.265	203.220	203.175
Chainage in metres	0	200	400	600	800	1000	1200	1400	1600	1800	2000	2200	2400	2600	2800	3000
	0 km					(1 km)					(2 km)					(3 km)

Fig. 4.30. L-Section of the Distributary.

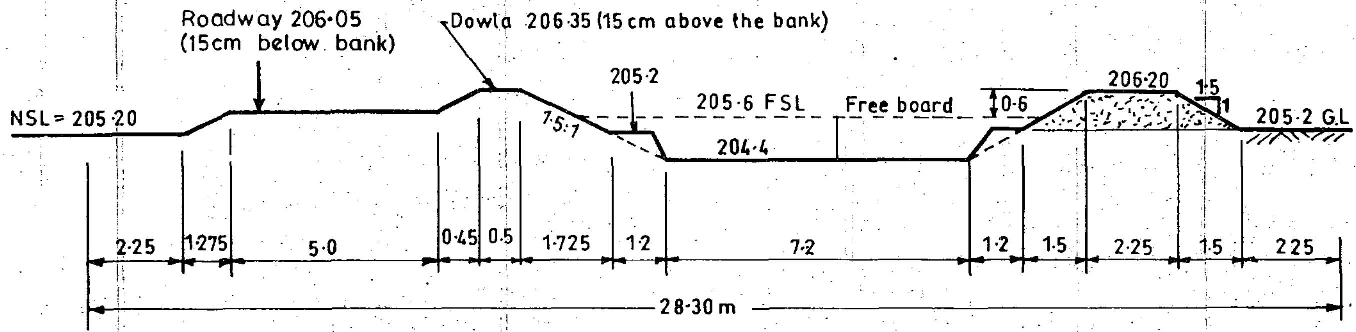


Fig. 4.31. (*a*) Section at km–0.

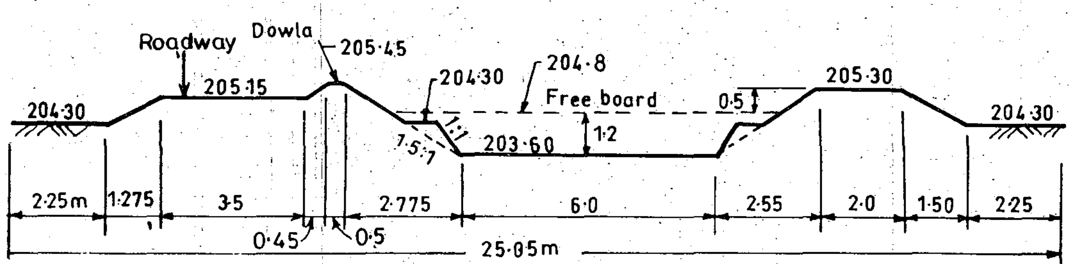


Fig. 4.31. (*b*) Section at km–1.

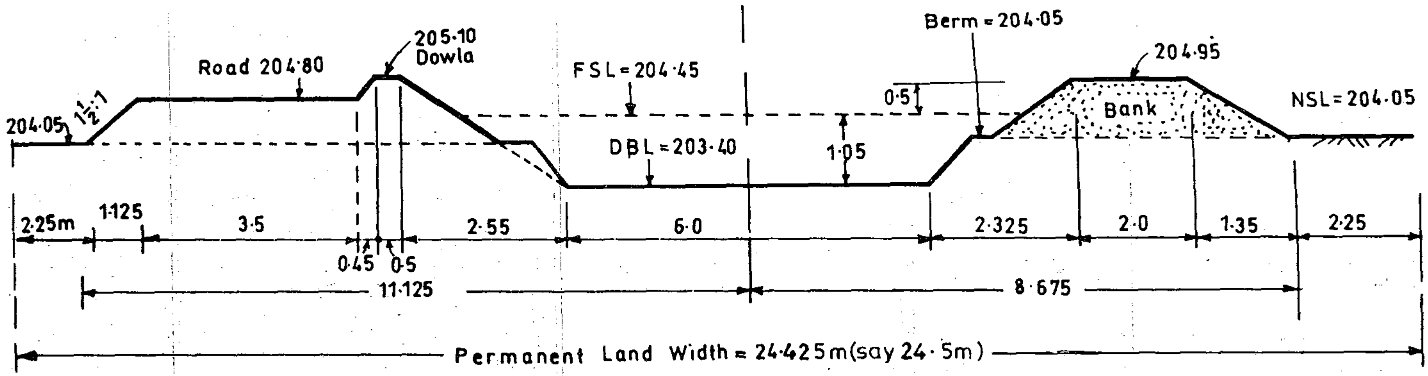


Fig. 4.31. (*c*) Section at km–2.

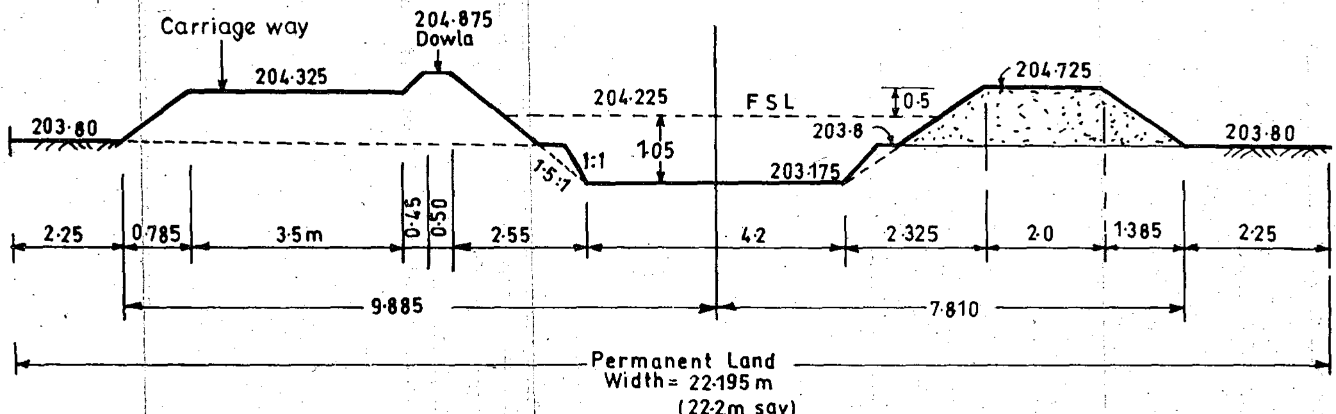


Fig. 4.31. (*d*) Section at km–3.

Example 4.17. *Design a reservoir fed canal to carry a discharge of 5 cumecs. Given Manning's N = 0.025, and the soil is of compacted silty clay with recommended maximum value of permissible velocity = 0.65 m/s.*

Solution. $Q = 5$ cumecs ; $N = 0.025$; $V_{max} = 0.65$ m/s.

From Manning's formula

$$V = \frac{1}{N} \cdot R^{2/3} \cdot S^{1/2}. \text{ and } Q = A \cdot V$$

or $$V = \frac{Q}{A}$$

$\therefore$ $$V_{max} = \frac{Q}{A_{min}}$$

or $$0.65 = \frac{5}{A_{min}}$$

$$A_{min} = \frac{5}{0.65} \text{ m}^2 = 7.69 \text{ m}^2$$

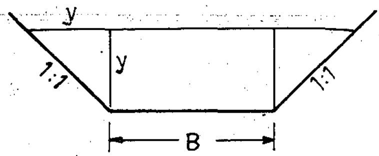


Fig. 4.32.

Since CWC's chart (Fig. 4.29) will generally not be supplied in examinations, we may solve this problem by using the relation between B and y, as given by eqn. (4.53), *i.e.*,

$$y = 0.5 \sqrt{B} \text{ (for } Q < 15 \text{ cumecs)}$$

or $$y = \tfrac{1}{2} \sqrt{B}. \quad \text{or} \quad B = 4y^2.$$

Now, using a trapezoidal channel section with 1 H : 1 V side slopes, we have

$$A = (B + y) \cdot y. = (4y^2 + y)\, y$$

or $$4y^3 + y^2 = 7.69$$

Solve by Hit and Trial

Let $y = 1$ m L.H.S. = 4 + 1 = 5

Let $y = 1.2$ m L.H.S. = 8.35

Let $y = 1.165$ m L.H.S. = 7.68 ≈ 7.69 (*i.e.* R.H.S.)

Hence, $y \approx 1.165$

The above calculations give only approximate value, we may hence choose depth y as anything like 1.165 m. So let us choose

$$y = 1.2 \text{ m}$$

$$A = (B + y)\, y$$

or $$7.69 = (B + 1.2)\, 1.2.$$

or $$B = \frac{7.69}{1.2} - 1.2 = \mathbf{5.21\ m.}$$

Hence, use $\left.\begin{aligned} B &= 5.21 \text{ m} \\ y &= 1.2 \text{ m} \end{aligned}\right\}$ **Ans.**

Now, bed slope of channel can be determined as :

$$Q = \frac{1}{N} \cdot A \cdot R^{2/3} \cdot \sqrt{S}$$

where $A = (5.21 + 1.2)\, 1.2 = 7.69$

$$R = \frac{A}{P} = \frac{7.69}{(B + 2\sqrt{2} \cdot y)} = \frac{7.69}{5.21 + 2\sqrt{2} \times 1.2} = 0.894 \text{ m}$$

$\therefore$ $$5 = \frac{1}{0.025} \times 7.69 \times (0.894)^{2/3} \cdot \sqrt{S}$$

or $5 = 285.45\sqrt{S}$ or $\sqrt{S} = \frac{1}{57.08}$

or $S = \frac{1}{3260}$ **Ans.**

4.13. Maintenance of Irrigation Canals

Irrigation canals are nothing but earthwork constructions, and as such, very much susceptible to damage. They, therefore, require a lot of maintenance, upkeep, watch and ward, etc. as to ensure their continuous efficient functioning. Various problems which are posed by the irrigation canals during their use, and as such need constant attention, are : (*i*) silting of canals ; (*ii*) weed and plant growth, (*iii*) failure of weaker banks ; (*iv*) hollows created by burrowing animals, crabs, etc. in the filled up sections, causing seepage and piping ; (*v*) canal breaches due to piping, overflowing of canal, etc. Some of these factors, such as the (*i*) and (*ii*) reduce the efficiency of the canals, whereas the others may result in complete stoppage of canal supplies to the irrigated fields. All these causes of troubles and their remedial measures are discussed below :

(*i*) **Silting of Canals.** Irrigation canals normally get silted during their course of flow. Whenever the flow velocity in the channel reduces, the silt carried by the water in suspension gets deposited on the bed and sides of the canal. The silt so deposited reduces the effective canal cross-section and the carrying capacity of the channel.

In order to prevent too much of silt deposition, irrigation channels, (particularly those which are constructed in alluvial soils) must be properly designed, so as to ensure a velocity which neither causes any silting nor scouring in the channel. These design principles have already been discussed. However, even in such channels, the discharge may sometimes be less than the designed; and in that eventuality, silting may take place in the channels.

Moreover, attempts should be made to remove the silt from the water entering the canal itself. From the headworks, for this purpose, almost silt-free water is to be admitted in the canal. However, inspite of using all these measures, certain amount of silt will definitely enter the canal, and certain amount may be added to it, by some scouring from within the channel. This silt often gets deposited on the bed and sides of the canal.

The deposited silt normally deforms the shape of a channel, and reduces its carrying capacity. It must, therefore, be removed periodically by *desilting operations.*

The desilting can be carried out by actually excavating the silt either manually (when the canal is dry) or by dredgers (when the canal is either dry or running). However, if the canal functions properly and is in regime and taking its fully supply, it is not necessary to clear the silt to the theoretical cross-section. But if the canal is not functioning properly, desilting is required, but only to the extent of clearing a portion of it as to get the canal back into efficient working order.

The extent of silting that has taken place in a canal can be found from measuring the silt depth over bed bars.*

The excavated silt from the canals should be disposed of suitably depending upon its quality. Moreover, the excavated silt should not be deposited above the canal banks, as in that case, the same silt may find its way back into the desilted channel.

* Bed bars are the masonry walls partly extending into the canal bed and flush with it, and partly flush with the sides of the channel as shown in Fig. 13.23. For larger canals, a bed bar may consist of concrete block with its upper face flush with theoretical bed level of the canal as shown in Fig. 13.24. They serve as permanent marks of reference to indicate the correct alignment and theoretical bed levels of the channel.

(*ii*) **Weed and Plant growth.** Irrigation canals normally get infested with aquatic weed growth like bushes, hyacinth, etc. Various types of aquatic weeds and plants get grown in canals. The silted canals provide excellent base for weed growth. The sun rays also help in promoting weed growth. The weed growth hinders the flow of water, reduces the carrying capacities of channels, and thereby impairing their efficiencies. The weed growth may also sometimes impart harmful qualities to irrigation water.

Weed growths may be controlled by maintaining higher velocities in canals. Moreover, many a times, the weed growths will have to be physically removed from the canals in order to improve their efficiencies.

In case of newly constructed canals, regular inspections should be undertaken to locate any spots where the weed growth has set in. Weeds from such spots should be removed complete, so that the infestation does not spread. In the case of old canals, where aquatic weed growth is profuse, suitable mechanical or chemical methods may be employed, at as early stage as possible.

(*iii*) **Failure of Weaker Banks.** The banks of the canal should be made in full designed width (as per I.S. provisions), and should be maintained as such. Due to constant use by men and animals, the canal banks get eroded at various places. They must be repaired, and all cuts and breaches filled up with suitable soil and proper tamping.

Moreover, grass or turfing should not be scrapped while smoothening the bank surfaces, as it helps in stabilising the soil and thus preventing its erosion. However, long grasses will have to be cut as far as necessary to smoothen the surface of the bank and to avoid holes being hidden under high grasses.

Sometimes, when a canal-reach runs in filling, the banks are subjected to water pressure. In such a case, there exists every possibility of damage occurring to the banks. In such cases, the banks must be given extra strength by increasing their sections. In order to keep the cost of construction low, additional soil for increasing the bank section may be obtained by the natural silting process. For this purpose, favourable conditions are created to cause silting internally. In this system, sufficiently wider section is provided, and a part of it later gets silted, forming the additional bank as shown in Fig. 4.33. Such a method is possible only for new channels. To accelerate silting, low submergible spurs projecting from the banks into the channel may be constructed.

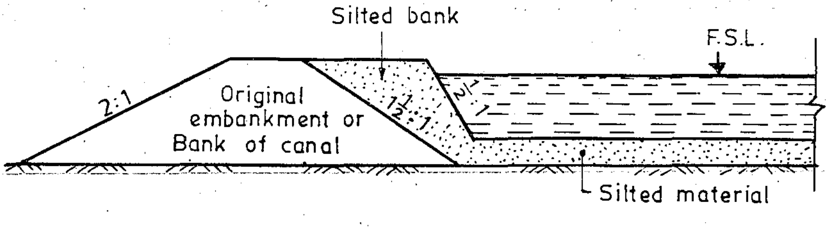


Fig. 4.33. Internal silting method for widening canal banks in canals constructed in filling.

(*iv*) **Canal Breaches.** Canal breach is, infact, an opening or a gap developed in the canal bank due to erosion of some portion of the bank. When once water starts flowing out of such a breach, it starts becoming deeper & wider, unless remedial measures are taken immediately to plug the breach. The breaches in canal banks may be caused due to various reasons, such as :

(*a*) breach due to faulty design or construction of the canal banks ;

(*b*) breach due to overflow of the canal ;

(*c*) breach due to seepage or piping ;

(*d*) breach due to intentional cuts made by cultivators.

These reasons are briefly summarised below :

(*a*) **Breach due to faulty design or construction of the banks.** If the width of a canal bank is not kept sufficient, or if its height is not as per the correct design requirements, the bank may fail to hold the water properly. Similarly, if the soil material used at site is poor and not up to the specifications or if the rolling and watering, etc. has not been properly done at the time of construction, the banks may not be so strong as were expected to be, and hence fail to hold the canal water without being damaged. This continuous recurring damage to the bank may ultimately lead to a full-fledged cut and breach within the bank length.

(*b*) **Breach due to overflow of canal.** Sometimes, the discharge in the canal may exceed its design capacity, and water may overtop the canal bank(s). The overflow will definitely damage the bank(s) considerably.

(*c*) **Breach due to piping.** Sometimes, the banks may not be overtopped, but the water depth may go above the full supply depth due to increased discharge or reduced silted canal section. In such a case, the hydraulic gradient line within the 'filled' canal bank section will also rise (as shown in Fig. 4.34). And if the provided cover over the hydraulic gradient line is not sufficient, it may pass out of the banks, thus dislodging the soil particles from the outer slopes of the banks. It will consequently weaken and erode the banks, which may ultimately lead to a complete breach within the bank, and water may rush out of the canal.

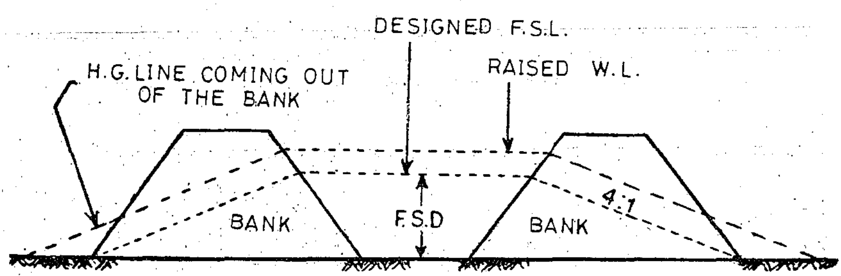


Fig. 4.34.

Sometimes, the water may start leaking through the holes created within the body of the bank, by insects or burrowing animals. The leaking water will go on removing the sand particles alongwith ; and if went unchecked, the holes may go on increasing in size, and ultimately resulting in a rush of water through the canal bank, leading to sinking of the bank soil, and formation of a full fledged breach. Normally, however, these holes are smaller in size and do not pose a serious problem, if timely inspection and remedial plugging is done. As a preventive measure, sometimes, a sand core may be provided within the bank embankment section, which will settle and fill the holes. In such a case, of course, the banks will settle down, but at least breaches will be avoided.

(*d*) **Breach due to intentional cuts made by cultivators.** Sometimes, when the area gets flooded due to excessive rain or poor drainage, the cultivators cut the canal bank in order to pass the drainage water to the other side. This sometimes proves rather detrimental to their expectations, as the canal supplies may also enter the low lying area, thus further aggravating the water congestion in the area.

Sometimes, similarly, the cultivators cut the banks to obtain illegal water supplies from the canal. These small cuts may widen and take the shape of big breaches in a short time, consequently damaging the land and crops.

Closure of breaches. In case of *small minors and distributaries,* a breach may be closed by dumping huge quantities of earth instantaneously from both sides of the gap. As the water of the breach spreads on the adjoining land, there is usually no outside nearby place to borrow earth for closing the breach. The earth has, therefore, to be collected either by cutting the outer slope of the existing bank, or from the spoil banks (if existing), or berms of the canal. It is essential to store huge quantities of earth on both sides of the gap before closing the breach. Moreover, the closing process should be started from both sides of the breach, at the same time, as pointed out earlier.

In case of *bigger canals,* such as a major distributary or a branch canal, the above procedure cannot be adopted, as the huge discharge from such a canal may completely wash off the dumped soil. In such a case, it is, therefore, necessary to, first of all, reduce the flow through the breach. This should be done by driving a double line of stakes or

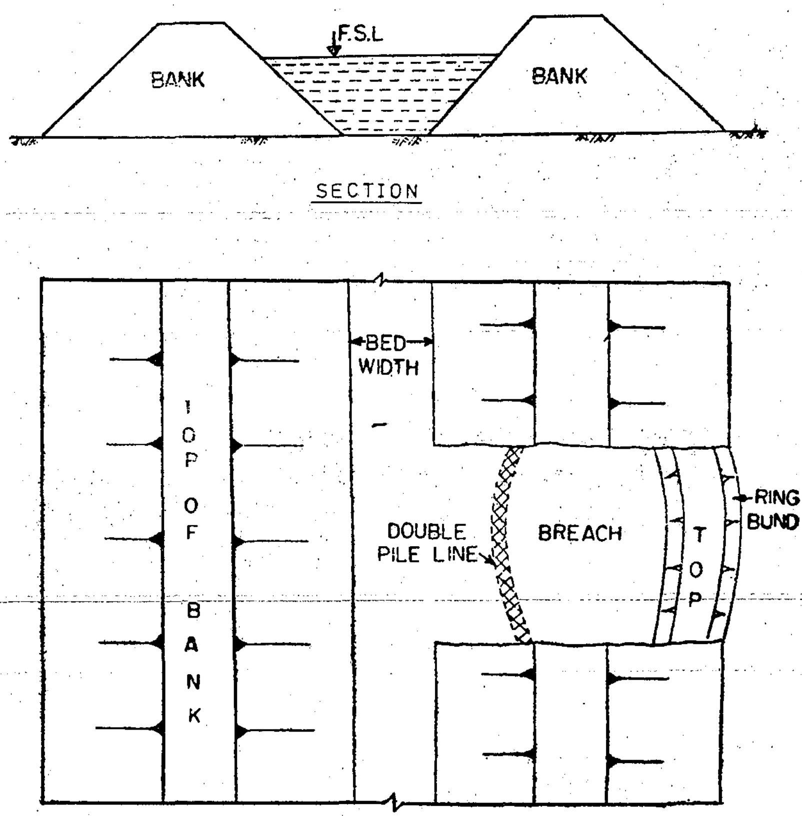


Fig. 4.35. Closure of a breach in a canal bank.

wooden piles (ballies) in the opening of the breach, as shown in Fig. 4.35. The space between the pile lines is filled with planks or bushes, etc. The filler material can be secured by placing sand bags on the top. If the breach is very wide, another line of defence may be provided.

This will reduce the outflow from the breach. No dumping of earth in the breach should progress before the flow through the breach has been arrested to some extent in this fashion. Meanwhile earth should be piled up on the bank on both sides of the breach.

This deposited earth should then be dumped instantaneously from both sides to form a *ring bund* on the outer side of the breach as shown. The opening is then properly filled with suitable earth in layers, each layer being properly compacted. All jungle from the ring bund site should be removed before earth work progresses. Moreover, the earth should be freed from grasses or bushes, etc., before dumping.

4.14. Modern Simplified Equations for Optimal Designs of Alluvial Canals

While discussing the design procedure by Kennedy's theory, we have illustrated that the alluvial canals are normally designed to satisfy known values of discharge (Q), rugosity coefficient (n), and silt grade (m). The value of longitudinal slope (S) is also usually predecided on the basis of ground considerations. Thus, the design is usually carried out for known values of Q, n, m & S to satisfy Kutter's equation as well as Kennedy's equation, both. Such a design will give us dimensions of a channel which will be hydraulically adequate, (indicated by Kutter's Eqn.) and will also be non-silting-non scouring (indicated by Kennedy's eqn.). *But infact, such a channel may not behave satisfactorily from considerations of silt transportation.* This is verified by experience in the field. Evidently, some guidance for fixing a suitable bed width - depth ratio $\left(\frac{b}{y}\ ratio\right)$ is required. These ratios were fixed in various areas on the basis of experience, or by empirical equations and charts like the ones given in eqn. (4.62) and Fig. 4.29. The resulting solution obtained for channel dimensions (with given values of Q, n, m *and* S) is finally compared with the recommended values of $\frac{b}{y}$ from Fig. 4.29, or Eq. (4.62). If the two values differ significantly, suitable modification in the bed slope (S) would become necessary. Since the slope is decided on the basis of ground profile, it will limit the range of slope. Within this range of slope, one can obtain different combinations of b and y satisfying Kennedy's and Kutter's equations and the curve in Fig. 4.29. All such computations are cumbersome and time consuming. To avoid time consumption, Garret's diagrams are used, which provide graphical solution to Kennedy's and Kutter's equations. However, this method involves interpolation errors. Considering these drawbacks of the age old design method, a need was being felt to develop explicit type of equations, which will satisfy both Kennedy's and Kutter's equations, without involving trial and interpolation errors.

Some modern equations have, therefore been developed to overcome the above shortcomings of the old age method, and are given below.

Considering a trapezoidal channel section with $\frac{1}{2}H : 1V$ side slope, we have from Kennedy's Eqn.

$$V = 0.55\, m \cdot y^{0.64} \qquad \text{...}(i)$$

Also from continuity eqn., we have

$$Q = A \cdot V = y\left(b + \frac{1}{2} \cdot y\right) \cdot V \qquad \text{...(ii)}$$

Eliminating V between (*i*) and (*ii*), we get

$$\begin{aligned} Q &= y(b + 0.5y)\, 0.55 \cdot m \cdot y^{0.64} \\ &= 0.55m \cdot y^{1.64}(b + 0.5y) \\ &= 0.55 \cdot m \cdot y^{1.64} \cdot y\left(\frac{b}{y} + 0.5\right) \end{aligned}$$

or
$$Q = 0.55\, m \left(\frac{b}{y} + 0.5\right) y^{2.64}$$

Substituting $\frac{b}{y} = r$, we get

$$Q = 0.55\, m \cdot (r + 0.5)\, y^{2.64}$$

or
$$y = \left[\frac{Q}{0.55 \cdot m\,(r + 0.5)}\right]^{\frac{1}{2.64}}$$

or
$$y = \left[\frac{1.818\, Q}{(r + 0.5)\, m}\right]^{0.3788} \qquad \text{...(4.63)}$$

$$\text{where } \frac{b}{y} = r \qquad \text{...(4.64)}$$

One can compute both b and y from Eq. (4.63) & (4.64), if a suitable explicit equation written symbolically as :

$$r = f(Q, n, m, S) \qquad \text{...(iii)}$$

is available, which satisfies both Kennedy's and Kutter's equations. A large number of canal designs can be obtained by varying the parameters Q, n, m and S in their practical ranges and a suitable correlation can be established between r, Q, n, m and S.

By varying the parameters r, Q, n, m and S, in their practical ranges, 1296 number of canal designs were obtained by satisfying both the Kennedy's and Kutter's equations. Analysing all the resulting solutions obtained earlier, it is found that a unique relation could be established between r, Q, n, m and S. By a method of curve fitting, the following empirical formula has been obtained :

$$\boxed{r = \left[\frac{1.607\, S^{1.63} \cdot Q^{0.033}}{n^{3.26} \cdot m^{3.293}} - 0.258\right]^{-0.915}} \qquad \text{...(4.65)}$$

For known values of Q, n, m and S, value of r can be computed from the above eqn. With this value of r, the value of y can be calculated from Eq. (4.63), and then the value of b can be calculated by Eqn. (4.64). The resulting solution may finally be compared with the curve of Fig. 4.29. If the matching differs widely, a modification in S value would be necessary and may be done in the following manner.

The curve of Fig. 4.29 can be represented by an equation :

$$r = \left[15 + 6.44\, Q\right]^{0.382} \qquad \text{...(4.66)}$$

Eq. (4.66) can be considered as the optimality condition for the value of r. On combining Eqns. (4.65) and (4.66), we get

$$S=\frac{n^2 \cdot m^{2.02}}{1.338 \cdot Q^{0.02}}\left[0.258+(15+6.44\,Q)^{-0.417}\right]^{0.6135} \qquad \text{...(4.67)}$$

Eq. (4.67) will give optimal value of S, which will satisfy the curve of Fig. 4.29. However, if a particular value of S is to be maintained, the design may be carried out by relaxing the requirement of r.

Design Steps

For given values of Q, n and m the proposed design steps are :

(*i*) Compute the optimal value of S-from Eqn. (4.67).

(*ii*) Find the required value of r from Eq. (4.66).

(*iii*) Find y from Eq. (4.63).

(*iv*) Find b from Eq. (4.64).

If the slope is to be maintained at a given value, then the computation steps will be as follows :

(*i*) Find r-from Eq. (4.65)

(*ii*) Find y-from Eq. (4.63)

(*iii*) Find b-from Eq. (4.64).

Example 4.18. *Design an irrigation channel to carry 50 cumecs of Discharge. The channel is to be laid at a slope of 1 in 4000. The critical velocity ratio (m) for the soil is 1.1. use Kutter's rugosity coefficient (n) as 0.023. Solve by using the modern equations.*

Note. *This problem is the same as was there in Example 4.6, which was solved by old methods.*

Solution. $Q = 50\ \text{m}^3/\text{s}$; $S=\frac{1}{4000}$

$m = 1.1$; $n = 0.023$

(*i*) *Find r by using Eq. (4.65) as :*

$$r=\left[\frac{1.607 \cdot S^{1.63} \cdot Q^{0.033}}{n^{3.26} \cdot m^{3.293}}-0.258\right]^{-0.915}$$

Substituting the given values of S, Q, n and m, we get

$$r=\left[\frac{1.607 \cdot \left(\frac{1}{4000}\right)^{1.63} \cdot (50)^{0.033}}{(0.023)^{3.26} \cdot (1.1)^{3.293}}-0.258\right]^{-0.915}$$

$$=\left[\frac{1.607 \times 1.3447 \times 10^{-6} \times 1.1378}{4.5628 \times 10^{-6} \times 1.36869}-0.258\right]^{-0.915}$$

$$=(0.3937-0.258)^{-0.915}=(0.1357)^{-0.915}$$

$$=\frac{1}{(0.1357)^{0.915}}=6.219$$

(*ii*) *Now find y, by using Eq. (4.63) as :*

$$y = \left[\frac{1.818\,Q}{(r+0.5)\,m}\right]^{0.3788} = \left[\frac{1.818 \times 50}{(6.219+0.5)\,1.1}\right]^{0.3788} = 2.587 \text{ m ; say } 2.59 \text{ m.}$$

(*iii*) *Find b from Eq. (4.64) as :*

$$\frac{b}{y} = r \qquad \therefore \qquad b = yr = 2.59 \times 6.219 = 16.1 \text{ m}$$

The dimensions of the designed channel will hence be :

$$\left.\begin{aligned} B &= \text{bed width } (b) = 16.1 \text{ m} \\ y &= \text{water depth} = 2.59 \text{ m} \end{aligned}\right] \textbf{ Ans.}$$

Note 1. Compare this design with the design done in Example 4.6, where we obtained $B = 14.14$ m and $y = 2.7$ m

Note 2. These modern equations can be easily used to design canals, which would not only satisfy Kutter's & Kennedy's Eqns. but also provide desired $\frac{B}{y}$ ratios.

Note 3. Such equations can be easily used in digital computer programmings for design of irrigation canals.

Example 4.19. *Design an irrigation channel to carry 40 cumecs of discharge with an optimum value of Bedwidth/Depth ratio. The value of rugosily coefficient n = 0.023, and critical velocity ratio, m = 1.*

Solution. (*i*) Use Eq. (4.67) to determine optimal value of bed slope *S*, as

$$S = \frac{n^2\, m^{2.02}}{1.338\, Q^{0.02}}\left[0.258 + (15 + 6.44\,Q)^{-0.417}\right]^{0.6135}$$

$$\text{where } n = 0.023$$
$$m = 1$$

Substituting values, we get

$$S = \frac{(0.023)^2\,(1)^{2.02}}{1.338\,(40)^{0.02}}\left[0.258 + (15 + 6.44 \times 40)^{-0.417}\right]^{0.6135}$$

$$= 3.6725 \times 10^{-4}\left[0.3544\right]^{0.6135} = \mathbf{1.944 \times 10^{-4}}$$

(*ii*) Find *r* from Eq. (4.66)

$$r = \left[15 + 6.44\,Q\right]^{0.382} = \left[15 + 6.44 \times 40\right]^{0.382} = \mathbf{8.519}$$

(*iii*) Find *y* from Eq. (4.63) as :

$$y = \left[\frac{1.818\,Q}{(r+0.5)\,m}\right]^{0.3788}$$

$$= \left[\frac{1.818 \times 40}{(8.519+0.5)\,1}\right]^{0.3788} = 2.20 \text{ m}$$

(*iv*) Find *b* from Eq. (4.64) as :

$$\frac{b}{y} = r$$

or $\qquad b = yr = 2.20 \times 8{,}519 = 18.78 \text{ m}$

The designed dimensions of the channel are :

$$\left.\begin{array}{l}\text{bed width, } b = 18.78 \text{ m} \\ \text{water depth, } y = 2.20 \text{ m} \\ \text{Bed slope, } S = 1.944 \times 10^{-4} = \dfrac{1}{5144}\end{array}\right] \textbf{Ans.}$$

Please compare the results which were obtained for this data in Example 4.7 as

$b = 8.35$ m, $y = 3.34$ m, $S = 1$ in 3700].

Example 4.20. *Make an alternative design for the canal at km 3 in Example 4.16 by using Modern Equations in place of using Gerret's Diagrams as were used in Table 4.16.*

Solution. The data relating to canal design at km 3, was :

$$Q = 2.94 \text{ m}^3/\text{s} ; \qquad n = 0.0225$$

$$S = \frac{1}{4444} = 0.225 \times 10^{-3}$$

Side slopes $= \frac{1}{2} : 1$; $\qquad$ CVR $= m = 1$

Now, instead of designing this canal section by Garret's diagrams, we can design it by using modern equations of optimal canal design, as follows :

(*i*) Find r from Eq. (4.65) as :

$$r = \left[\frac{1.607\,(0.225 \times 10^{-3})^{1.63} \cdot (2.94)^{0.033}}{(0.0225)^{3.26} \cdot (1)^{3.293}} - 0.258\right]^{-0.915} = 4.659$$

(*ii*) Find y from Eq. (4.63) as :

$$y = \left[\frac{1.818 \times 2.94}{(4.659 + 0.5) \cdot 1}\right]^{0.3788} = \mathbf{1.01}$$

(*iii*) Find b from Eq. 4.64

$$b = y \cdot r = 1.013 \times 4.659 = \mathbf{4.72\ m}$$

Hence, adopt $\left.\begin{array}{l}B = 4.72 \text{ m} \\ y = 1.01 \text{ m}\end{array}\right]$ Compare these figures with the values worked out in Table 4.16 by Garret's diagrams as :

$$\left.\begin{array}{l}B = 4.5 \text{ m} \\ y = 1.05 \text{ m}\end{array}\right]$$

Note. In this manner, the use of Garret's diagrams can be completely avoided.

PROBLEMS

1. What is sediment ? How is sediment transported in streams ? What are dunes and antinodes ? Write an equation giving the relation between total sediment transport and streamflow. Explain how would you estimate the life of a reservoir.

2. What is the difference between suspended load and bed load ?

State the better empirical bed load formulae. Give briefly the theory of distribution and transportation of suspended matter. What is Einstein's approach ?

How is suspended sediment measured and expressed ?

3. (*a*) Explain the following terms connected with sediment transport phenomenon:

(*i*) Suspended load (*ii*) Bed load

(*iii*) Regime channel (*iv*) Lacey's silt factor

(*v*) Threshold motion of the sediment.

(*b*) In the design of a trapezoidal canal, the following dimensions are obtained :

side slopes = 2 vertical to 1 horizontal
bed width = 21 m
depth = 1.5 m
bed slope = 2.25×10^{-3}

Manning's friction factor for the canal was $n = 0.022$. Using Kennedy's equation on silt theory, given by $V_0 = 0.55, D^{0.64}$ with usual notations, find whether the chosen canal section is satisfactory.

4. (*a*) Discuss the mechanics involved in Sediment transport.

(*b*) Explain in details, the Shield's method for design of channels with pitched slopes.

(*c*) An irrigation channel is to be constructed in coarse alluvium gravel with 4 cm size. The channel has to carry 5 cumecs of discharge and the longitudinal slope is 0.04. The banks of the channel will be pitched against scouring. Find the minimum width of the channel.

[**Hint :** Follow example 4.1.]

5. (*a*) Derive an expression for the average tractive force per unit of wetted area that is generated in a trapezoidal channel section of given R and S. How does this shear stress distribution at banks differs from that at bed ?

(*b*) Also derive an expression to prove that the shear stress required to move a grain on the bank of a channel is lesser than the shear stress required to move the grain on the channel bed.

6. (*a*) Discuss critically the statement "the banks of an unlined canal are more susceptible to erosion than its bed, and hence the stability of the banks and not of its bed is the governing factor in unlined canal designs".

(*b*) A canal is to be designed to carry a discharge of 600 cumecs. The bed slope is kept as 1 in 1600. The soil is coarse alluvium having a grain size of 5 cm. Assuming the canal to be unlined with unprotected banks and of a trapezoidal section, determine a suitable section for the canal ; ϕ for the soil may be taken as 37°.

[**Hint :** Follow example 4.3]

7. What is meant by "regimen of a river" ?

Compare briefly the silt theories of Kennedy and Lacey.

Design a regime channel to carry a discharge of 50 cumecs. Assume silt factor as 1.0.

8. When do you classify the channel as having attained regime condition ?

Describe briefly the observations of Lacey on the regime of river.

Design a channel using Lacey's theory to carry a discharge of 100 cumecs. Assume silt factor as 1.

9. Describe briefly the two recognised silt theories. Explain how one theory is an improvement over the other.

What are defects in both these theories.

10. What is meant by 'regime' ? Differentiate between regime in natural rivers and in artificial channels.

Design an irrigation channel section for the following data :

Discharge = 40 cumecs
Silt factor = 1.0
Side slopes = $\frac{1}{2} : 1$;

Determine the longitudinal slope also.

11. Discuss the salient features of Kennedy's theory for the design of earth channels based on the critical velocity concept, and mention its limitations.

Also, design an earth canal section to carry 50 cumecs discharge at a slope of 0.25 m/km, having been given that $N = 0.0225$, and $m = 1.00$, where the symbols have their usual meaning.

12. Explain what is meant by 'unlimited incoherent alluvium' in the context of Lacey's theory and discuss the concept of regime embodied in this theory for the design of earth channels in alluvium.

Design an irrigation channel to carry 40 cumecs at a slope of 1 in 5000 with $N = 0.0225$ (Manning's) and Kennedy's $m = 0.9$.

13. (*a*) Differentiate between 'Initial regime' and 'Final regime'.

(*b*) Design an irrigation channel section for the following data :

Discharge = 30 cumecs ; Silt factor = 1.0 ; Side slopes = $\frac{1}{2}$: 1.

Draw the complete channel cross-section assuming it to be in part cutting and part filling.

14. (*a*) Discuss briefly the problems that arise and the methods which are adopted while designing the sections for irrigation canals in India.

(*b*) An area of 40,000 ha. has to be irrigated by a canal for growing wheat ; water requirement for which is 10 cm per month. Design and draw a suitable canal section with the data given below :

Mean slope of the ground = 1 in 3400,

Manning's roughness coefficient = $N = 0.025$,

Side slopes = 1.1.

Use Kennedy's formula. Try a depth equal to 2 m.

$$\left(\textbf{Hint:}\quad Q = \frac{40.000 \times 10^4 \times 0.1}{30 \times 24 \times 60 \times 60} \text{ cumecs}\right)$$

15. Write detailed notes on any two of the following :

(*i*) Importance of sediment transport in designing earthen irrigation canals.

(*ii*) Kennedy's and Lacey's silt theories for designing irrigation canals in India.

(*iii*) Comparison of Kennedy's and Lacey's silt theories, and further improvements over Lacey's theory.

(*iv*) Use of Garret's Diagrams for designing irrigation canals.

(*v*) Suspended load and its measurement.

(*vi*) Bed load and its measurement.

(*vii*) Popular Methods of designing irrigation canals in western countries.

16. (*a*) What is meant by 'Suspended load' in an irrigation canal ?

(*d*) Derive an expression for measuring the suspended load concentration at any distance *y* from the bed of the channel, provided the concentration at a known distance is already known.

[**Hint :** Derive equation 4.34 and 4.40]

17. (*a*) Define bed load, and discuss Meyer-Peter's formulae for determining bed load in irrigation canal.

(*b*) Design a channel which has to carry 30 cumecs of discharge with a bed load concentration of 40 p.p.m. by weight. The average grain diameter of the bed material may be taken as 0.35 mm. Use Lacey's regime perimeter and Meyer-Peter's formulas.

[**Hint :** Follow example 4.10]

18. Derive Einestein's formula for bed load transport in a form which can help in deducing that the 'silt carrying capacity of a river channel increases during floods'. What other interesting conclusions can be drawn from this formula ?

[**Hint :** Derive Equation 4.65]

19. (*a*) Draw a typical canal cross-section which is partly constructed in cutting and partly in filling. Discuss briefly its various components, such as : side slopes, berms, banks, service road, dowla, spoil banks, etc.

(*b*) What is meant by 'Balancing depth and how is it determined' ?

(*c*) The following data refer to an irrigation canal

Bed width	= 1.0 m
Side slopes	= 2 : 1 (in filling) and 1 : 1 (in cutting)

Top width of embankment on either side of canal = 3 m

Full supply depth	= 5 m.
Free board	= 1 m.

Determine the balancing depth. Draw a neat cross-section of the canal illustrating the various dimensions and the levels.

(Take ground level as 100.00 m)

20. Write short notes on :

(*i*) Free board in canals. (*ii*) Berms and counter berms.

(*iii*) Requirements of permanent land widths for constructing canals.

(*iv*) Spoil Banks. (*v*) Borrow pits.

(*vi*) Balancing depth.

(*vii*) Fixing bed width ratio for an irrigation canal.

(*viii*) Canal standards. (*ix*) Schedule of area statistics.

21. Describe briefly as to what data you will collect and how you will proceed for fixing the L-section of an irrigation canal. Also discuss step by step, the procedure that you will adopt for designing the entire length of the channel section, if the culturable commanded area and the details of the crops along the canal are given.