20

Earthen Dams and Rock Fill Dams

20.1. Introduction

Earthen dams and earthen levees are the most ancient type of embankments, as they can be built with the natural materials with a minimum of processing and primitive equipment. But in ancient days, the cost of carriage and dumping of the dam materials was quite high. However, the modern developments in earth moving equipments have considerably reduced the cost of carriage and laying of the dam materials. The cost of gravity dams on the other hand, has gone up because of an increase in the cost of concrete, masonry, etc. Earthen dams are still cheaper as they can utilise the locally available materials, and less skilled labour is required for them.

Gravity dams and arch dams require sound rock foundations, but earthen dams can be easily constructed on earth foundations. However, earth dams are more susceptible to failure as compared to rigid gravity dams or arch dams. Before the development of the subject of Soil-Mechanics, these dams were being designed and constructed on the basis of experience, as no rational basis for their design was available. This led to the failure of various such earthen embankments. However, in these days, these dams can be designed with a fair degree of theoretical accuracy, provided the properties of the soil placed in the dam, are properly controlled. This condition makes the design and construction of such dams, thoroughly interdependent. Continuous field observations of deformations and pore water pressures have to be made during the construction of such dams. Suitable modifications in the design, are then made during construction, depending upon these field observations.

20.2. Types of Earthen Dams

The earthen dam can be of the following three types :

1. *Homogeneous Embankment type*
2. *Zoned Embankment type*
3. *Diaphragm type.*

These three types of dams are described below :

(1) **Homogeneous Embankment Type.** The simplest type of an earthen embankment consists of a single material and is homogeneous throughout. Sometimes, a blanket of relatively impervious material may be placed on the upstream face. A purely homogeneous section is used, when only one type of material is economically or locally available. Such a section is used

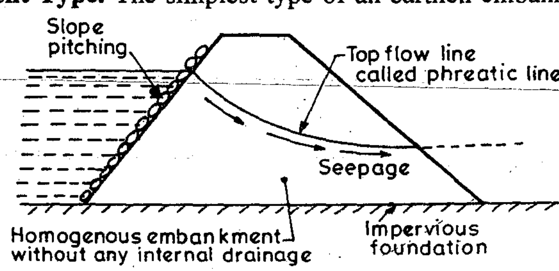


Fig. 20.1 (*a*). Homogeneous type embankment.

for low to moderately high dams and for levees. Large dams are seldom designed as homogeneous embankments.

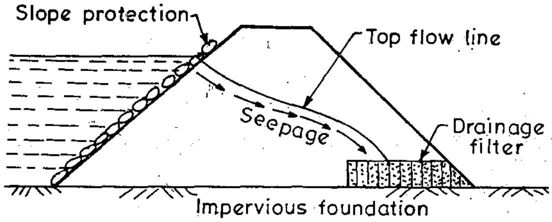


Fig. 20.1 (*b*). Homogeneous embankment provided with a drainage filter.

A purely homogeneous section poses the problems of seepage, and huge sections are required to make it safe against piping, stability, etc. Due to this, a homogeneous section is generally added with an internal drainage system : such as a horizontal drainage filter [Fig. 20.1 (*b*)], rock toe, etc. The internal drainage system keeps the phreatic line (*i.e.* top seepage line) well within the body of the dam, and steeper slopes and thus, smaller sections can be used. the internal drainage is, therefore, always provided in almost all types of embankments.

(2) **Zoned Embankment Type.** Zoned embankments are usually provided with a central impervious core, covered by a comparatively pervious transition zone, which is finally surrounded by a much more pervious outer zone (Fig. 20.2).

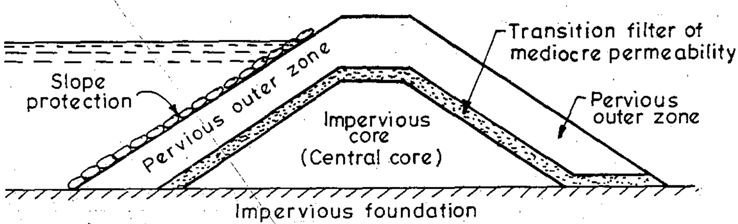


Fig. 20.2. Zoned type embankment.

The central core checks the seepage. The transition zone prevents piping through cracks which may develop in the core. The outer zone gives stability to the central impervious fill and also distribute the load over a large area of foundations.

This type of embankments are widely constructed and the materials of the zones are selected depending upon their availabilities. *Clay, inspite of it being highly impervious, may not make the best core, if it shrinks and swells too much. Due to this reason, clay is sometimes mixed with fine sand or fine gravel, so as to use it as the most suitable material for the central impervious core. Silts or silty clays may be used as the satisfactory central core materials. Freely draining materials, such as coarse sands and gravels, are used in the outer shell.* Transition filters are provided between the inner zone and the outer zone, as shown in Fig. 20.2. This type of transition filters are always provided, whenever there is an abrupt change of permeability from one zone to the other.

(3) **Diaphragm Type Embankments.** Diaphragm type embankments have a thin impervious core, which is surrounded by earth or rock fill. The impervious core, called diaphragm, is made of impervious soils, concrete, steel, timber or any other material. It acts as a water barrier to prevent seepage through the dam. The diaphragm may be placed either at the centre as a central vertical core or at the upstream face as a blanket. The diaphragm must also be tied to the bed rock or to a very impervious foundation material, if excessive under-seepage through the existing previous foundations has to be avoided (Fig. 20.3).

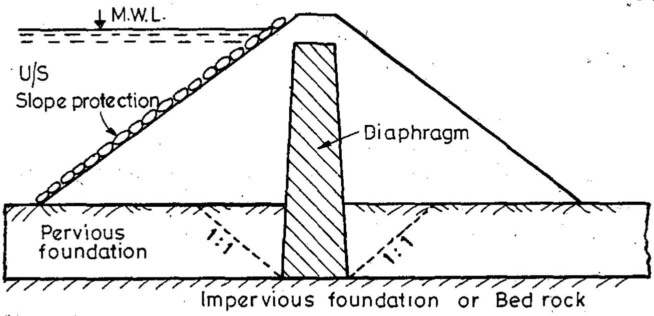


Fig. 20.3. Diaphragm type embankment.

The diaphragm type of embankments are differentiated from zoned embankments, depending upon the thickness of the core. If the thickness of the diaphragm at any elevation is less than 10 metres or less than the height of the embankment above the corresponding elevation, the dam embankment is considered to be of 'Diaphragm Type'. But if the thickness equals or exceeds these limits, it is considered to be of zoned embankment type.

20.3. Methods of Construction

There are two methods of constructing earthen dams :

(1) *Hydraulic-fill Method* ; and

(2) *Rolled-fill Method.*

(1) **Hydraulic-fill Method.** In this method of construction, the dam body is constructed by excavating and transporting soils by using water. Pipes called flumes, are laid along the outer edge of the embankment. The soil materials are mixed with water and pumped into these flumes. The slush is discharged through the outlets in the flumes at suitable intervals along their lengths. The slush, flowing towards the centre of the bank, tends to settle down. The coarser particles get deposited soon after the discharge near the outer edge, while the fines get carried and settle at the centre, forming a zoned embankment having a relatively impervious central core.

Since the fill is saturated when placed, high pore pressures develop in the core material, and the stability of the dam must be checked for these pressures. This type of embankment is susceptible to settlement over long periods, because of slow drainage from the core.

Hydraulic-fill method is, therefore, seldom adopted these days. Rolled-fill method for constructing earthen dams is, therefore, generally and universally adopted in these modern days.

(2) **Rolled-fill Method.** The embankment is constructed by placing suitable soil materials in thin layers (15 to 30 cm) and compacting them with rollers. The soil is brought to the site from burrow pits and spread by bulldozers, etc. in layers. These layers are thoroughly compacted by rollers of designed weights. Ordinary road rollers can be used for low embankments (such as for levees or bunds) ; while power-operated rollers are to be used for dams. The moisture content of the soil fill must be properly controlled. The best compaction can be obtained at a moisture content somewhere near the optimum moisture content. (The optimum moisture content is the moisture required for obtaining optimum density in the fill). Compaction of coarse gravels cannot be properly done by rolling and is best done by vibrating equipment. Detail of rolling and compactiing different types of soils are available in "Soil Mechanics and Foundation Engineering" by the same author.

20.4. Shearing Strength of Soils

Before we describe the causes of failures of earthen dams and the criteria for their safe design ; we shall review a few important conceptions of soil-mechanics and their importance in the design of earth dams.

Coulomb's Law. The resistance offered by a soil mass against the shearing forces, is known as the shear strength of the soil. In order to ensure that the foundation soil and the soil of the body of the dam are stable, we must ensure that the shearing strength of this soil mass is sufficiently greater than the shear stresses developed within the soil mass by external forces. According to Coulomb's Law, the shear strength of any soil mass is provided by two factors, *i.e.*

(*i*) Cohesion between the soil grains.

(*ii*) Internal friction between the soil particles.

Thus, the unit shear strength (τ_f) of a soil is given by

$$\tau_f = c + \sigma' \cdot \tan \phi \text{ (kN/m}^2\text{)} \quad \text{...(20.1)}$$

where c = Unit cohesion of soil in (kN/m^2)

σ' = intergranular compressive stress, called the effective vertical stress (kN/m^2)

ϕ = Angle of internal friction of soil.

The constant c and ϕ depend upon the type of soil and the moisture content in it. Sands and gravels are cohesionless soils and $c \approx 0$ for them ; while ϕ is approximatrely zero for saturated cohesive impervious clays.

20.5. Various Kinds of Densities and Their Relations

Every soil has pores or voids in it, which remain filled either with air or with water or with both, depending upon the void ratio (e) and the moisture content (w) of the soil. The void ratio, porosity, moisture content, etc. of a soil are defined as follows :

$$\text{Void ratio } (e) = \frac{\text{Volume of voids in a soil mass}}{\text{Volume of solids in the soil mass}} = \frac{V_v}{V_s}$$

$$\text{Porosity of soil } (n) = \frac{\text{Volume of voids}}{\text{Total volume of soil}} = \frac{V_v}{V}$$

$$= \frac{V_v}{V_s + V_v} = \frac{\dfrac{V_v}{V_s}}{1 + \dfrac{V_v}{V_s}} = \frac{e}{1+e}$$

or

$$e = \frac{1+e}{n}$$

$$\text{or Water content of soil } (w) = \frac{\text{Mass of moisture in soil mass}}{\text{Mass of solids in soil mass}}$$

$$= \frac{M_w}{M_s}$$

where M_w = Mass of water in the soil mass.

M_s = Mass of solids in the soil mass.

The unit weight of soil (γ) called bulk unit weight, is defined as the weight per unit volume of soil mass. If W is the total weight of soil, then

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_v} \text{ (air does not have any weight)}$$

or $$\gamma = \frac{W_s\left(1 + \frac{W_w}{W_s}\right)}{V_s\left(1 + \frac{V_v}{V_s}\right)} \qquad ...(A)$$

Now $\frac{W_s}{V_s} = \gamma_s = S_s \cdot \gamma_w$

where γ_s = unit weight of solids

γ_w = unit weight of water, and

S_s = specific gravity of solid soil particles $= \frac{\gamma_s}{\gamma_w}$

Substituting in (*A*), we get

$$\therefore \quad \gamma = \frac{W_s}{V_s}\left[\frac{1 + \frac{W_w}{W_s}}{1 + \frac{V_v}{V_s}}\right] = S_s \cdot \gamma_w \left[\frac{1+w}{1+e}\right]$$

Hence, $$\gamma = S_s \cdot \gamma_w \cdot \left[\frac{1+w}{1+e}\right] \qquad ...(20.2)$$

If the soil mass is dry, $w = 0$

$$\therefore \quad \gamma_{dry} = \frac{S_s \cdot \gamma_w}{1+e} \qquad ...(20.3)$$

$$\therefore \quad \gamma = \gamma_{dry}(1+w)$$

Degree of saturation (S) is defined as

$$S = \frac{V_w}{V_v} = \frac{\text{Volume of water}}{\text{Volume of voids}}$$

Now $$e = \frac{V_v}{V_s} = \frac{V_v}{V_s} \times \frac{V_w}{V_w} = \frac{V_v}{V_w} \times \frac{V_w}{V_v}$$

$$= \frac{1}{S}\left[\frac{\frac{M_w}{\gamma_w}}{\frac{M_s}{S_s \cdot \gamma_w}}\right] = \frac{1}{S} \cdot \frac{M_w \cdot S_s}{M_s} = \frac{1}{S} \cdot w \cdot S_s$$

$$\boxed{e = \frac{wS_s}{S}} \qquad ...(20.4)$$

For saturated soils $S = 1$,

$$\therefore \quad e = w \cdot S_s$$

$$\therefore \quad \boxed{\text{Moisture content, } w = \frac{e}{S_s} \text{ for saturated soil}} \qquad ...(20.5)$$

Now, if unit weight of saturated soil is designated as γ_{sat}, then

$$\gamma = \frac{S_s\gamma_w(1+w)}{(1+e)} \qquad \text{from Eq. (20.2)}$$

$$\gamma_{sat} = \frac{S_s \cdot \gamma_w \left(1 + \frac{e}{S_s}\right)}{1+e} = \frac{S_s \cdot \gamma_w \, (S_s + e)}{S_s \, (1+e)}$$

or
$$\boxed{\gamma_{sat} = \frac{\gamma_w \, (S_s + e)}{(1+e)}} \qquad \text{...(20.6)}$$

Submerged unit weight of soil is defined as the unit weight of saturated soil minus the unit weight of water, and is designated as $\gamma_{sub.}$

$$\gamma_{sub} = \gamma_{sat} - \gamma_w = \frac{\gamma_w \, (S_s + e)}{1+e} - \gamma_w$$

$$\gamma_{sub} = \gamma_w \left[\frac{S_s - 1}{1+e}\right] \qquad \text{...(20.7)}$$

20.6. Pore-Water Pressure and its Significance in the Design of Earth Dams

We know that every soil has some voids or pores which are partly or fully filled with water. Let us consider a soil mass below the water-table *BB* in Fig. 20.4.

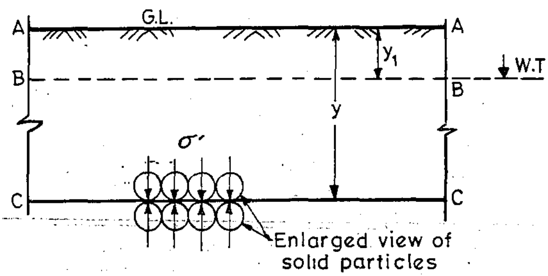


Fig. 20.4

The soil below *BB* is fully saturated and all its pores between the solid particles of soil, are full of water. The soil below the water-table is, therefore, subjected to hydrostatic uplift. Hence, at any level (say *CC*), the total downward normal pressure (σ) exerted by the weight of the soil above this level, shall be supported partly by the intergranular pressure (σ' or σ_{eff}) developed between dry particles of the soil, and partly by the hydrostatic pressure (u) due to the water present in the pores. Hence

$$\sigma = \sigma' + u \qquad \text{...(20.8)}$$

where, σ is the total normal pressure on soil.

σ' = Total effective pressure, *i.e.* the intergranular pressure, or the pressure which is transmitted from grain to grain of soil.

u = Pore water pressure or *neutral pressure.* (It is the hydrostatic pressure due to presence of water in the soil pores.)

In Fig. 20.4, if the pressures are considered at a level say *CC* at a depth y from the ground, we can easily write,

Total normal pressure (σ) at level *CC*

$$= \gamma_{dry} \, y_1 + \gamma_{sat} \, (y - y_1)$$

where y_1 is the depth of watertable below the ground level.

$$\therefore \quad \sigma = \gamma_{dry}\, y_1 + \gamma_{sat.}\,(y - y_1)$$

Pore water pressure $= u = \gamma_w\,(y - y_1)$

Hence, the net effective pressure (σ') is given by

$$\sigma' = \sigma - u = \gamma_{dry}\, y_1 + \gamma_{sat}\,(y - y_1) - \gamma_w\,(y - y_1)$$
$$= \gamma_{dry}\, y_1 + \gamma_{sub}\,(y - y_1)$$

It, therefore, becomes evident that the effective normal stress is much less than the total normal stress, as a part of the total stress gets consumed by water as pore pressure. The effective stress is dependent on the submerged unit weight of soil.

The water pressure or pore pressure acts equally in all directions. It does not press the soil grains against one another, and, therefore, does not lead to compression of the soil or an increase in its frictional resistance, that is why, it is called a 'neutral stress'.

When the pore pressure is considered, the Coulomb's law will become

Unit Shear Strength of soil $(\tau_f) = c + \sigma' \cdot \tan\phi$

or $$\boxed{\tau_f = c + (\sigma - u)\tan\phi} \quad ...(20.9)$$

This clearly indicates that the shearing strength of a soil gets reduced due to the presence of pore pressure. The pore water pressure gets developed in the body of the earthen dam when the seepage takes place through the body of the dam, thus reducing the shear strength of the soil.

The shear stress (τ) developed at any plane in an earth structure is given by

$$\tau = \frac{\sigma_1 - \sigma_2}{2} \cdot \sin 2\theta \quad ...(20.10)$$

where σ_1 = major principal stress

σ_2 = minor principal stress

θ = is the angle between the plane considered for shear and the plane on which σ_1 acts.

It is apparent from the Equation (20.10), that the value of shearing stress remains unaltered, whether σ_1 and σ_2 are used or their effective components $[(\sigma_1 - u)$ and $(\sigma_2 - u)]$ are used. But the shear strength of the soil gets reduced when effective component $(\sigma - u)$ is used in place of σ.

Hence, the stability of the dam against shear failure must be checked when the maximum pore water pressure is present

Consolidation. The development of pore pressure is important, even during the time of construction of an earth dam. When the fully or partly saturated soil is placed and rolled in the body of the dam, the entire applied external load is taken up by the water immediately, and transferred to the soil afterwards. The pore water thus gets compressed, and if it is unable to drain out freely (due to low permeability of the soil), the pore pressure rises. *This rise in pore pressure during compaction is known as* **'hydrostatic excess' pressure in pore water.** It further reduces the shearing strength of the soil and hence, the stability of the soil. As the excess water drains out, more and more consolidation will take place, as this pressure will be transferred to soil grains and the shear strength will tend to achieve its normal value. *Hence, the pore pressure temporarily reduces the shear strength of the soil during compaction* by preventing full compaction. But the shear strength gets recovered after the compaction is over, as the pore water is ultimately squeezed out.

In highly compressible soils, having low coefficient of permeability and moisture content above its optimum moisture content, this condition of 'hydrostatic excess' becomes very serious. Such soils are, therefore, more liable to fail during construction.

Similarly, appreciable consolidation of soil may takes place in fine grained compressible soils like clay, even after the construction is over, though sufficient compaction was done during construction.

Due to these reasons, pore pressure observations are often made during the construction period of an earth dam. If the hydrostatic excess of pore pressure rises to a dangerous amount, the construction may be stopped for some time till the excess water drains out and full natural compaction takes place. The construction may be restarted after this 'excess' is either fully dissipated or reduced to a safe value. The shear failure of the soil of the dam or its foundation is, therefore, very much connected with the development of pore pressure in the body of the dam and in the foundation, and must be properly checked and accounted for.

20.7. Causes of Failure of Earthen Dams

Earth dams are less rigid and hence more susceptible to failure. Every past failure of such a dam has contributed to an increase in the knowledge of the earth dam designers. Earthen dams may fail, like other engineering structures, due to improper designs, faulty constructions, lack of maintenance, etc. The various causes leading to the failure of earth dams can be grouped into the following three classes :

(1) Hydraulic failures
(2) Seepage failures
(3) Structural failures.

These causes are described below in details :

20.7.1. Hydraulic failures. About 40% of earth dam failures have been attributed to these causes. The failure under this category, may occur due to the following reasons:

(*a*) *By over topping*. The water may overtop the dam, if the design flood is underestimated or if the spillway is of insufficient capacity or if the spillway gates are not properly operated. Sufficient freeboard should, therefore, be provided as an additional satety measure.

(*b*) *Erosion of upstream face*. The waves developed near the top water surface due to the winds, try to notch-out the soil from the upstream face and may even, sometimes, cause the slip of the upstream slope. Upstream stone pitching or riprap should, therefore, be provided to avoid such failures.

(*c*) *Cracking due to frost action*. Frost in the upper portion of the dam may cause heaving and cracking of the soil with dangerous seepage and consequent failure. An additional freeboard allowance upto a maximum of say 1.5 m should, therefore, be provided for dams in areas of low temperatures.

(*d*) *Erosion of downstream face by gully formation*. Heavy rains falling directly over the downstream face and the erosive action of the moving water, may lead to the formation of gullies on the downstream face, ultimately leading to the dam failure. This can be avoided by proper maintenance, filling the cuts from time to time especially during rainy season, by grassing the slopes and by providing proper berms at suitable heights (Fig. 20.5), so that the water has not to flow for considerable distances. The proper drainage arrangements are made for the removal of the rain water collected on the horizontal berms. Since the provision of berms ensures the collection and removal of water before it acquires high downward velocities, the consequent erosion caused by the moving water (run off) is considerably reduced.

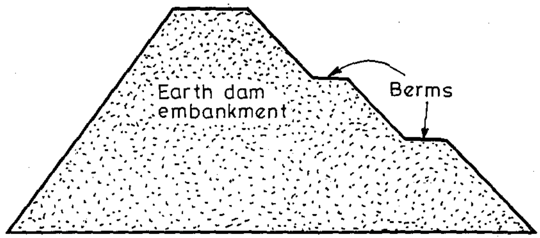


Fig. 20.5

(*e*) *Erosion of the d/s toe.* The d/s toe of the earth dam may get eroded due to two reasons, *i.e.* (*i*) the erosion due to cross currents that may come from the spillway buckets; and (*ii*) the erosion due to tail water. This erosion of the toe can be avoided by providing a downstream slope pitching or a riprap up to a height slightly above the normal tail water depth. Side walls of the spillway (called diaphragm walls) must be of sufficient height and length, as so to prevent the possibility of the cross flow towards the earthen embankment.

20.7.2. Seepage Failures. Controlled seepage or limited uniform seepage is inevitable in all earth dams, and ordinarily it does not produce any harm. However, uncontrolled or concentrated seepage through the dam body or through its foundation may lead to piping or sloughing and the subsequent failure of the dam. Piping is the progressive erosion and subsequent removal of the soil grains from within the body of the dam or the foundation of the dam. Sloughing is the progressive removal of soil from the wet downstream face. More than 1/3rd of the earth dams have failed because of these reasons.

(*a*) *Piping through foundations.* Sometimes, when highly permeable cavities or fissures or strata of coarse sand or gravel are present in the foundation of the dam, water may start seeping at a huge rate through them (Fig. 20.6). This concentrated flow at a high gradient, may erode the soil. This leads to increased flow of water and soil, ultimately resulting in a rush of water and soil, thereby creating hollows below the foundation. The dam may sink down into the hollow so formed, causing its failure.

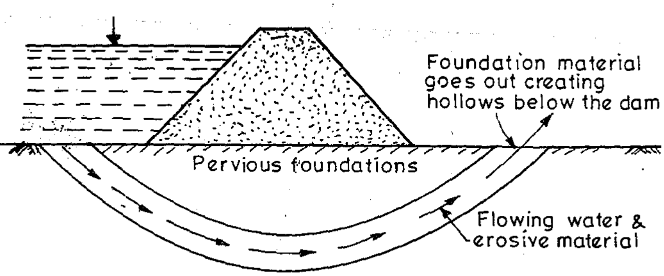


Fig. 20.6. Piping through the dam foundation.

(*b*) *Piping through the dam body.* When the concentrated flow channels get developed in the body of the dam, (Fig. 20.7) soil may be removed in the same manner as was explained in foundation piping, leading to the formation of hollows in the dam body, and subsequent subsidence of the dam. These flow channels may develop due to faulty construction, insufficient compaction, cracks developed in embankment due to foundation settlement, shrinkage cracks, animal burrows, etc. All these causes can be removed by better construction and better maintenance of the dam embankments.

Piping through the dam body, generally get developed near the pipe conduits passing through the dam body. Contact seepage along the outer side of conduits may either develop into piping, or seepage through leaks in the conduits may develop into piping. This can be avoided by thoroughly and properly compacting the soils near the outlet conduits and by preventing the possibilities of leakage through conduits, but preventing the formation of cracks in the conduits. These cracks in the conduits are caused by differential settlement and by overloading from the embankment. When these factors are controlled, automatically, the possibility of piping due to leakage through the conduits is reduced.

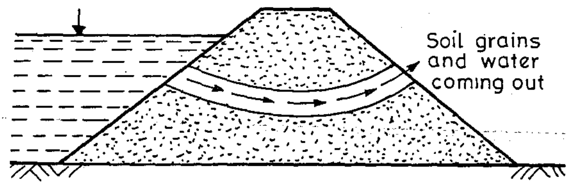


Fig. 20.7. Piping through the dam body.

(*c*) *Sloughing of D/S Toe.* The process behind the sloughing of the toe is somewhat similar to that of piping. The process of failure due to sloughing starts when the downstream toe becomes saturated and get eroded, producing a small slump or a miniature slide. The miniature slide leaves a relatively steep face which becomes saturated by the seepage from the reservoir and slumps again, forming a more unstable surface. The process continues till the remaining portion of the dam is too thin to withstand the horizontal water pressure, leading to the sudden failure of the dam.

20.7.3. Structural failures. About 25% of the dam failures have been attributed to structural failures. Structural failures are generally caused by shear failures, causing slides.

(*a*) *Foundation slide.* (*i.e.* overall stability of the dam). When the foundation of the earth dams are made of soft soils, such as fine silt, soft clay, etc., the entire dam may slide over the foundation. Sometimes, seams of fissured rocks, shales or soft clay, etc. may exits under the foundation, and the dam may slide over some of them, causing its failure. In this type of failure, the top of embankment gets cracked and subsides, the lower slope moves outward forming large mud waves near the heel, as shown in Fig. 20.8.

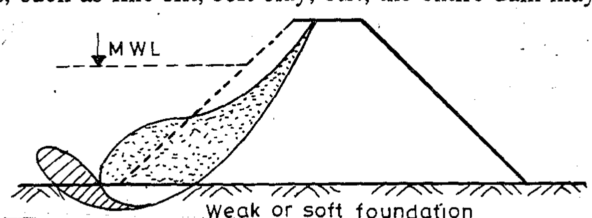


Fig. 20.8. Sliding due to soft or weak foundation.

Excessive pore water pressure in confined seams of sand and silt, artesian pressure in abutments, or hydrostatic excess developed due to consideration of clay seams embedded between sands or silts, etc. may reduce the shear strength of the soil, until it becomes incapable of resisting the induced shear stresses, leading to the failure of the dam foundation whithout warning. Loose sand foundations may fail by the liquefaction or flow slides.

(*b*) *Slide in Embankments.* When the embankment slopes are too steep for the strength of the soil, they may slide causing dam failure.

The most critical condition of the slide of the u/s slope is the sudden draw-down of the reservoir (Fig. 20.9); and the d/s slope is most likely to slide, when the reservoir is full (Fig. 20.10). The u/s slope failures seldom lead to catastrophic failures, but the d/s slope failures are very serious. These failures, generally occur due to development of excessive unaccounted pore pressures which may reduce the shearing strength of the soils as explained in the previous article. Many embankments may fail during the process of consolidation, at the time of construction or after the construction.

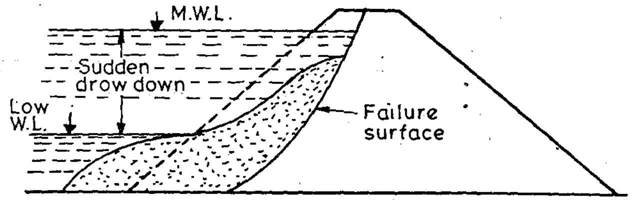


Fig. 20.9. U/S slope slide due to sudden draw-down.

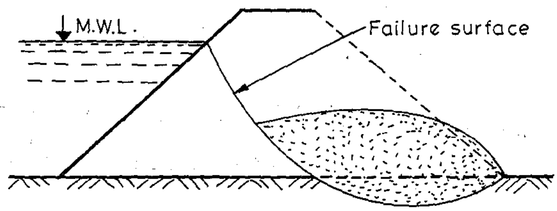


Fig. 20.10. D/S. slope slide during full reservoir condition.

20.8. Design Criteria for Earth Dams

(1) A fill of sufficiently low permeability should be developed out of the available materials, so as to best serve the intended purpose with minimum cost. Borrow pits should be as close to the dam site as possible, so as to reduce the leads.

(2) Sufficient spillway and outlets capacities should be provided so as to avoid the possibility of overtopping during design flood.

(3) Sufficient freeboard must be provided for wind set-up, wave action, frost action and earthquake motions.

(4) The seepage line (*i.e.* phreatic line) should remain well within the downstream face of the dam, so that no sloughing of the face occurs.

(5) There is little harm in seepage through a flood control dam. if the stability of foundations and embankments is not impaired, by piping, sloughing, etc. : but a conservation dam must be as watertight as possible.

(6) There should be no possibility of free flow of water from the upstream to the downstream face.

(7) The upstream face should be properly protected against wave action, and the downstream face against rains and against waves upto tail water. Provisions of horizontal berms at suitable intervals in the d/s face may be thought of, so as to reduce the erosion due to flow of rain water. Ripraps should be provided on the entire u/s slope and also on the d/s slope near the toe and up to slightly above the tail water so as to avoid erosion.

(8) The portion of the dam, downstream of the impervious core, should be properly drained by providing suitable horizontal filter drain, or toe drain, or chimney drain, etc.

(9) The upstream and downstream slopes should be so designed as to be stable under worst conditions of loading. These critical conditions occur for the u/s slope during sudden drawdown of the reservoir, and for the d/s slope during steady seepage under full reservoir.

(10) The u/s and d/s slope should be flat enough, as to provide sufficient base width at the foundation level, such that the maximum shear stress developed remains well below the corresponding maximum shear strength of the soil, so as to provide a suitable factor of safety.

(11) We know that the consolidation of the soil does not take place instantaneously when the compaction is done by external leadings. It takes place slowly as the excess pore water goes out and the load is transferred to the soil grains. In coarse gravels, the void openings are large enough so as to permit rapid escape of confined water and air, and full compaction may occur before the construction is over. But in fine grained impervious soils, the consolidation is slow. It, therefore, becomes necessary in such cases, as to provide an additional height of the fill. After consolidation, the the embankment will be of the desired height. Hence, a suitable allowance in the height of embankment (between 2 to 3% of dam height, determined by laboratory tests) must be made in fine grained soils so as to account for the consolidation that may take place upto years after construction. Dewatering the foundations may sometimes be used to accelerate the process of consolidation.

(12) Since the stability of the embankment and foundation is very critical during construction or even after construction (*i.e.* during the period of consolidation), due to development of excessive pore pressures and consequent reduction in shear strength of soil, the embankment slopes must remain safe under this critical condition also.

All the above criteria must be satisfied and accounted for, in order to obtain a safe design and construction of an earth dam.

20.9. Selecting a Suitable Preliminary Section for an Earth Dam

The preliminary design of an earth dam is done on the basis of existing dams of similar characteristics and the design is finalised by checking the adequacy of the selected section for the worst loading conditions. Empirical rules are frequently used in these designs.

A few recommendations, for selecting suitable values of top width, free board, u/s and d/s slopes, drainage arrangements, etc. are given below for preliminary designs :

Freeboard. Freeboard or minimum freeboard is the vertical distance between the maximum reservoir level and the top of the dam (*i.e.* the crown or crest of dam). The vertical distance between normal pool level or spillway crest and the top of the dam is termed as normal freeboard.

The minimum height of the freeboard for wave action is generally taken to be equal to 1.5 h_w, where h_w is given by the equations (19.11) and (19.12). Most of the hydraulic failures of earth dams have occurred due to overtopping of dams. Hence, the freeboard must be sufficient enough, as to avoid any such possibility of overtopping. Values of freeboard, for various heights, recommended by U.S.B.R. are given in table 20.1.

Table 20.1. U.S.B.R. Recommendations for Freeboard for Earth Dams

Spillway Type	*Height of Dam*	*Minimum freeboard Over MWL*
Uncontrolled (*i.e.* Free) Spillway	Any height	Between 2 m to 3 m
Controlled spillway	Height less than 60 m	2.5 m above top of gates
Controlled spillway	Height more than 60 m	3 m above top of gates

An additional freeboard upto 1.5 m should be provided for dams situated in areas of low temperatures for frost action.

Width. The top width of large earthen dams should be sufficient to keep the seepage line well within the dam, when reservoir is full. It should also be sufficient to withstand earthquake shocks and wave action. For small dams, this top width is generally governed by minimum roadway width requirements.

The top width (A) of the earth dam can be selected as per the following recommendations :

$$A = \frac{H}{5} + 3 \text{ for very low dams} \qquad \ldots(20.11)$$

$$A = 0.55\sqrt{H} + 0.2\,H \text{ for dams lower than 30 m} \qquad \ldots(20.12)$$

$$A = 1.65\,(H + 1.5)^{1/3} \text{ for dams higher than 30 m} \qquad \ldots(20.13)$$

where H is the height of the dam.

Upstream and Downstream slopes. The side slopes depend upon various factors such as the type and nature of the dam, and foundation materials, height of dam, etc. etc. The recommended values of side slopes as given by Terzaghi are tabulated in Table 20.2.

Table 20.2. Terzaghi's Side Slopes for Earth Dams

Type of Material	*U/S slope (H : V)*	*D/S slope (H : V)*
Homogeneous well graded	2.5 : 1	2 : 1
Homogeneous course silt	3 : 1	2.5 : 1
Homogeneous silty clay		
(*i*) Height less than 15 m	2.5 : 1	2 : 1
(*ii*) Height more than 15 m	3 : 1	2.5 : 1
Sand or Sand and gravel with a central clay core	3 : 1	2.5 : 1
Sand or Sand and gravel with R.C. diaphragm	2.5 : 1	2 : 1

The various dimensions of low earth dams for their preliminary sections, may sometimes be selected from the recommendations of Strange, as given in Table 20.3.

Table 20.3. Preliminary Dimensions of Low Earth Dams (Strange's recommendations)

Height of dam in metres	*Maximum freeboard in metres*	*Top width (A) in metres*	*U/S slope (H : V)*	*D/S slope (H : V)*
Up to 4.5	1.2 to 1.5	1.85	2 : 1	1.5 : 1
4.5 to 7.5	1.5 to 1.8	1.85	2.5 : 1	1.75 : 1
7.5 to 15	1.85	2.5	3 : 1	2 : 1
15 to 22.5	2.1	3.0	3 : 1	2 : 1

SEEPAGE ANALYSIS

Seepage occurs through the body of all earthen dams and also through their pervious foundations. The amount of seepage has to be controlled in all conservation dams and the effects of seepage (*i.e.* position of phreatic line) has to be controlled for all dams, in order to avoid their failures.

The seepage through a pervious soil material, for two dimensional flow, is given by Laplacian equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad ...(20.14)$$

where $\phi = K \cdot h$ = Velocity potential

K = Permeability of the soil

h = Head causing flow.

The above equation is based on the following assumptions :

(*i*) Water is incompressible.

(*ii*) The soil is incompressible and porous. The size of the pore space do not change with time regardless of water pressure.

(*iii*) The quantity of water entering the soil in any given time is the same as the quantity flowing out of the soil.

(*iv*) Darcy's law is valid for the given soils.

(*v*) The hydraulic boundary conditions at the entry and exit are known.

A graphical solution of the above equation, (*i.e.* Eq. 20.11) suggests that the flow through the soil, following the above assumptions, can be represented by a *flow-net* ; which consists of two sets of curves, known as '*Equipotential lines*' (*i.e.* lines of equal energy) and '*stream lines*' (*i.e.* flow lines), mutually perpendicular to each other, as shown in Fig. 20.11.

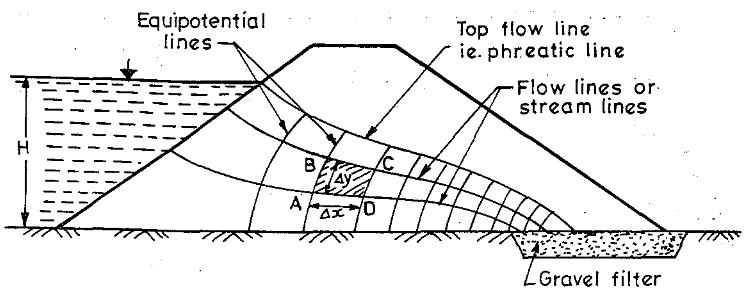


Fig. 20.11. Flow Net.

20.10. Seepage Discharge Through the Isotropic Soils

The amount of seepage can be easily computed from the flownet. Let us assume that the soil is isotropic, *i.e.* its permeability is constant in all directions, or $K_H = K_V$. (*i.e.* horizontal permeability is equal to the vertical permeability). The dam section is first of all plotted to a given scale (Same scale for horizontal and vertical directions). The flow net is drawn by free hand sketching the making suitable adjustments and corrections until the flow lines and equipotential lines intersect at right angles. It is convenient to draw only a limited number of flow lines and equipotential lines, such that the rate of flow between each pair of flow lines (called flow channels) is equal and the energy drop between any two successive potential lines is the same. The distance between the flow lines is made equal to the distance between the potential lines, thus forming a series of squares. Where the flow lines are curved, the squares formed will be distorted, but they will be more perfect as the number of lines is increased.

The seepage rate (q) can be computed from the flow net, using Darcy's Law. Applying the principle of continuity between each pair of flow lines, it is evident that the velocity must vary inversely with the spacing. Assuming the dam cross-section of Fig. 20.11 to have a unit width, we have :

The flow through the square $ABCD$ (called field) or through the flow channel containing this square

$$= \Delta q = K \cdot iA$$

$$= K\left(\frac{\Delta H}{\Delta x}\right)(\Delta y \times 1)$$

where ΔH is the energy drop between the two equi-potential lines bounding the square $ABCD$ and Δx and Δy are defined in Fig. 20.11.

$$\therefore \qquad \Delta q = K\left(\frac{\Delta H}{\Delta x}\right)\Delta y$$

But
$$\Delta H = \frac{\text{Total drop, } i.e. \text{ total head causing flow}}{\text{Number of increments into which the total drop is equally divided}}$$

or
$$\Delta H = \frac{H}{N_d}$$

where N_d = Total number of drops in the complete flow-net.

$$\Delta q = K\frac{H}{N_d}\left(\frac{\Delta y}{\Delta x}\right)$$

$$= \frac{K \cdot H}{N_d} \text{ (since } \Delta y = \Delta x)$$

The total flow through all the channels, *i.e.* the total flow through the unit width of the dam

$$= q = \Sigma \cdot \Delta q$$

$$= \left(\frac{K \cdot H}{N_d}\right) \times \text{number of flow channels}$$

or
$$\boxed{q = \frac{K \cdot H}{N_d} N_f} \qquad \text{...(20.15)}$$

This is the required expression, representing discharge passing through a flow net and is applicable only to isotropic soils (*i.e.* soils for which $K_H = K_V$).

20.11. Seepage Discharge for Non-isotropic Soils

If the permeability of the soil is different in the horizontal direction than that in the vertical direction ; the flow net is drawn in the same manner as was explained earlier for isotropic soils, with the only difference that the dam section shall be drawn to the same vertical scale *but to a transformed horizontal scale.* All horizontal dimensions shall be reduced by multiplying them by a factor equal to $\sqrt{\frac{K_V}{K_H}}$. Flow net and squares will be drawn in the same manner, and number of flow channesl (N_f) and number of drops (N_d) shall be counted. The discharge can then be computed by the equation

$$q = \sqrt{K_H \cdot K_V} \; \frac{H \cdot N_f}{N_d} \qquad \text{...(20.16)}$$

20.12. Line of Seepage or Phreatic Line in Earth Dams

Line of seepage or phreatic line or saturation line is defined as the line within the dam section below which there are positive hydrostatic pressures in the dam. The hydrostatic pressure on the phreatic line is equal to the atmospheric pressure and hence, equal to zero. Above the phreatic line, there is a zone of capillary saturation called capillary fringe, in which the hydrostatic pressures are negative. The appreciable flow through the dam body below the phreatic line, reduces the effective weight of this soil, and thus reduces the shear strength of the soil due to pore pressure. But on the other hand, the insignificant flow through the capillary fringe, leads to greater shear strength, because the capillary tension in water leads to increased intergranular pressure. The effects of the capillary fringe are thus on a slightly safer side and hence neglected.

It is, therefore, absolutely essential to determine the position of the phreatic line, as its position will enable us to determine the following things :

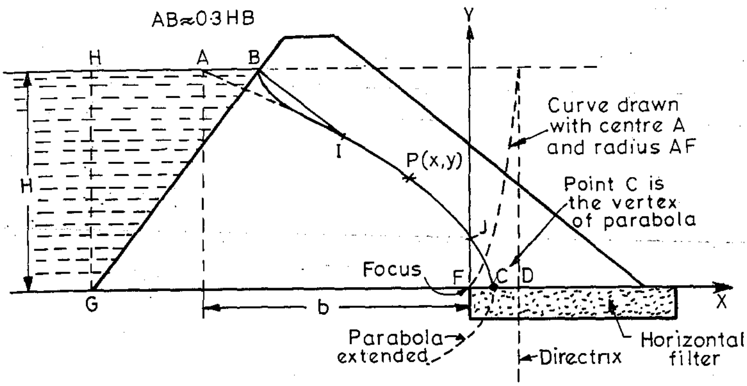
(*i*) It gives us a divide line between the dry (or moist) and submerged soil. The soil above the seepage line will be taken as dry and the soil below the seepage line shall be taken as submerged for computation of shear strength of soil.

(*ii*) It represents the top streamline and hence, helps us in drawing the flow net.

(*iii*) The seepage line determination, helps us to ensure that it does not cut the downstream face of the dam. This is extremely necessary for preventing softening or sloughing of the dam.

20.12.1. Determination of Phreatic Line when the Dam section is Homogeneous and Provided with a Horizontal Filter. It has been found by experiments that the seepage line is pushed down by the filter and it is very nearly parabolic except near its junction with the u/s face. Since the u/s face of the dam (*i.e.* *GB* in Fig. 20.12) becomes an equipotential line when fully covered with water (water shall rise up to the same piezometric head at every point of this line), the seepage line shall be perpendicular to this face near its junction point *B*. Let a base parabola with focus at *F* is drawn and produced so as to intersect the water surface at a point *A*. Cassagrande has shown that for dams with reasonably flat upstream slopes, $AB \approx 0.3\,HB$, where *H* is the projection of the point *G* (*i.e.* the end point of u/s slope) on the water surface. Knowing the point *A*, the base parabola *AIJC* can be drawn with its focus at *F*. It can then be corrected for the curve *BI* such that *BI* is perpendicular to *GB*, thus, *BIJC* will finally represent the seepage line.

Equation of the base parabola. The equation of the parabola can be determined from the basic property of the parabola *viz., the distance of any point P (x, y) on the parabola from its focus is the same as the distance of that point P (x, y) from a line*



Point C is the vertex of the parabola
Fig. 20.12

called directrix. Taking the focus (F) as the origin, the equation of the parabola can be written as

$$\sqrt{x^2+y^2}=x+FD$$

where the vertical line through D is the directrix.

ED is the distance of the focus from the directrix, called focal distance and is generally represented by S.

The equation of the parabola, then becomes

$$\sqrt{x^2+y^2}=x+S \quad \text{...(20.17)}$$

If the horizontal distance between the already determined point A and the focus (F) is taken as say b, then (b, H) represents the coordinates of the point A on the parabola. Substituting in equation (20.17), we get

$$\sqrt{b^2+H^2}=b+S$$

or
$$S=\sqrt{b^2+H^2}-b \quad \text{...(20.18)}$$

S can be calculated from this equation.

The focal distance S can also be measured by drawing an arc FH with centre A and radius equal to AF so as to intersect the Horizontal water surface AB produced at H. The vertical line HD through H will then represent the directrix. FD will be equal to S. The centre point (C) of FD will be the vertex of the parabola.

Also from equation (20.17) : when $x=0$, $y_{(0)}=S$. Hence, the vertical ordinate FJ at F will be equal to S. Hence, S is many a times, represented by $y_{(0)}$. Knowing the points A, C, J and working out a few more points from the equation (20.17), the parabola can be easily drawn and corrected for the curve BI, so as to get the seepage line $BIJC$.

The discharge can also be calculated easily from the equation of the seepage line, without taking the trouble of drawing a flow net, as explained below :

Consider a unit width of the dam. Let q be the seepage discharge per unit width of the dam. Then, according to Darcy's Law, $q=KiA$. When steady conditions have reached, the discharge crossing any vertical plane across the dam section will be the same. Hence, the values of i and A can be taken for any point on the seepage line

$$\therefore \quad i = \frac{dy}{dx}$$

$$A = y \times 1 \text{ (i.e. saturated depth} \times \text{width)}$$

$$\therefore \quad q = K\frac{dy}{dx} \cdot y \qquad ...(20.19)$$

But the equation of the parabola is

$$\sqrt{x^2 + y^2} = x + S \qquad ...(20.17)$$

or $$x^2 + y^2 = (x+S)^2$$

or $$y^2 = (x+S)^2 - x^2$$

or $$y = \sqrt{S^2 + 2xS}.$$

Equation (20.19) becomes

$$q = K\left[\frac{1}{2} \cdot (S^2 + 2xS)^{\frac{1}{2}-1} \cdot 2S\right] \cdot \left[\sqrt{S^2 + 2xS}\right]$$

$$= K \cdot S \cdot (S^2 + 2xS)^{-\frac{1}{2}} \sqrt{S^2 + 2xS}$$

$$= K \cdot S \cdot \frac{\sqrt{S^2 + 2xS}}{\sqrt{S^2 + 2xS}} = K \cdot S.$$

or $$\boxed{q = KS} \qquad ...(20.20)$$

The coefficient of permeability K and the focal distance (S) are known ; the discharge q can be easily computed. This is an important equation. Strictly speaking, this equation is applicable only to dams with horizontal drainage but is used for other type of section also and gives quite close values of discharge.

20.12.2. Determination of Phreatic Line when the Dam Section is Homogeneous (without Filter). The phreatic line can be determined on the same principles as was done for dam with a filter case. The focus (F) of the parabola, in this case, will be the lowest point of the downstream slope as shown in Fig. 20.13. The base parabola $BIJC$ will cut the downstream slope at J and extend beyond the dam toe up to the point C (*i.e.* the vertex of the parabola).

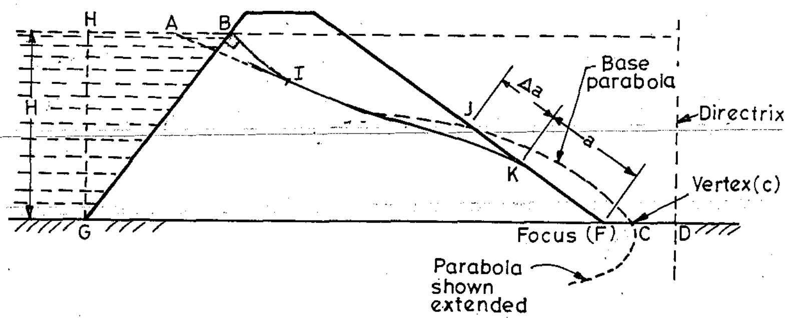


Fig. 20.13

The seepage line will, however, emerge out at K, meeting the downstream face tangentially there. *The portion KF is known as discharge face and always remains saturated.* The correction JK (say Δa) by which the parabola is to be shifted downward can be determined as follows :

(A) Graphical general solution. Cassagrande has given a general solution to evaluate Δa for various inclinations of discharge face. Let a be the angle which the discharge face makes with the horizontal. The various values of $\frac{\Delta a}{a+\Delta a}$ have been given by Cassagrande, as shown in table 20.4.

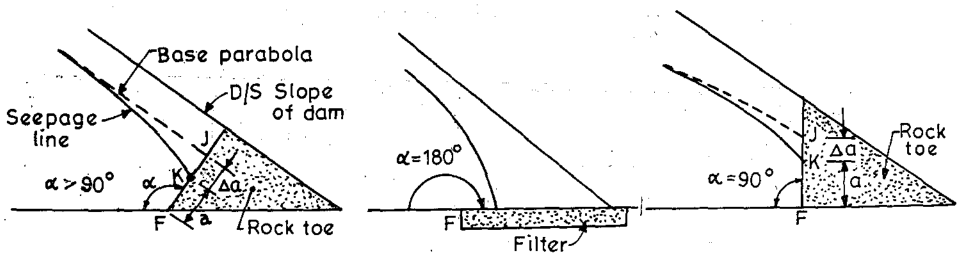
Table 20.4

α *in degrees*	$\frac{\Delta a}{a+\Delta a}$	*Remarks*
30°	0.36	*Note.* Intermediate values can be interpolated, or read out from a graph between α and $\frac{\Delta a}{a+\Delta a}$ plotted with the values given here.
60°	0.32	
90°	0.26	
120°	0.18	
135°	0.14	
150°	0.10	
180°	0.0	

$(a+\Delta a)$ is the distance FJ (*i.e.* the distance of the focus from the point where the parabola cuts the d/s face) and is known. Δa can then be evaluated. a and Δa can be connected by a general equation.

$$\Delta a = (a+\Delta a)\left[\frac{180° - \alpha}{400°}\right] \qquad ...(20.21)$$

The value of α will be equal to 180° for a horizontal filter case and may be equal to or more than 90° in case a rock toe is provided at the downstream end, as shown in Fig. 20.14 (*a*). α will be less then 90° when no drainage is provided.



(*a*) (*b*) (*c*)

Fig. 20.14. Various types of discharge faces.

(B) Analytical solutions for determining the position of point k, *i.e.* the point at which the seepage line intersects the d/s slope.

Case (*a*) when $\alpha < 30°$

Schaffernak and Van Iterson have derived an equation for determining the value of 'a' (and thus fixing the position of point K) in terms of H, b' and α. Their final equation is

$$a = \frac{b'}{\cos\alpha} - \sqrt{\frac{b'^2}{\cos^2\alpha} - \frac{H^2}{\sin^2\alpha}} \qquad ...(20.22)$$

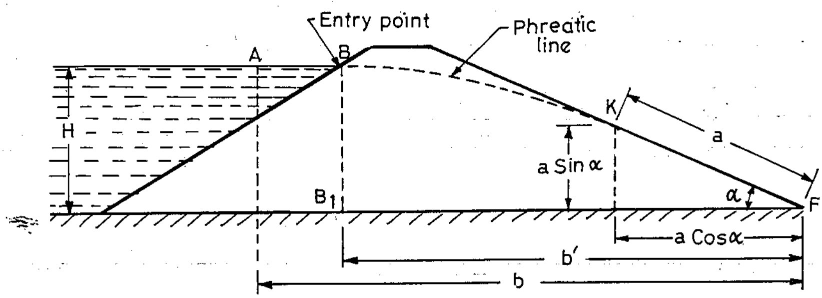


Fig. 20.15

The above equation has been obtained on the assumption that the hydraulic gradient (i) is equal to the slope of the phreatic line. This assumption is nearly true so long as the downstream slope is sufficiently flat (*i.e.* $\alpha < 30°$).

This equation can be easily derived on the basis of the above assumption as follows:

$$i = \text{Hydraulic gradient} = \frac{dy}{dx}$$

$$A = y \cdot 1$$

$$q = KiA = K \cdot \left(\frac{dy}{dx}\right) \cdot (y \cdot 1)$$

$$\text{But } \frac{dy}{dx} = \tan\alpha$$

$$\text{and} \quad y = a\sin\alpha$$

$$\therefore \quad q = K \cdot \tan\alpha \cdot a \cdot \sin\alpha$$

$$= K \cdot a \cdot \sin\alpha \cdot \tan\alpha$$

or $$K \cdot \frac{dy}{dx}(y) = K \cdot a \cdot \sin\alpha \tan a$$

or $$\frac{dy}{dx}(y) = a\sin\alpha\tan\alpha$$

or $$dy \cdot y = a\sin\alpha\tan\alpha\, dx$$

or $$y \cdot dy = a\sin\alpha\tan\alpha\, dx.$$

Integrating both sides between the limits

$$x = a\cos\alpha \quad \text{to} \quad x = b'$$

and $$y = a\sin\alpha \quad \text{to} \quad y = H,$$

we get $$\int_{y=a\sin\alpha}^{y=H} y \cdot dy = a\sin\alpha \cdot \tan\alpha \int_{x=a\cos\alpha}^{x=b'} dx$$

or $$\left|\frac{y^2}{2}\right|_{a\sin\alpha}^{H} = a\sin\alpha\tan\alpha \left| x \right|_{a\cos\alpha}^{b'}$$

or $$\frac{H^2 - a^2\sin^2\alpha}{2} = a\sin\alpha\tan\alpha\left[b' - a\cos\alpha\right]$$

or $$\frac{H^2}{2} - \frac{a^2}{2}\sin^2\alpha = ab'\sin\alpha\tan\alpha - a^2 \cdot \sin\alpha\cos\alpha \cdot \tan\alpha$$

or $$\frac{H^2}{2} - \frac{a^2}{2}\sin^2\alpha = ab'\sin\alpha\tan\alpha - a^2\cdot\sin^2\alpha$$

or $$\frac{H^2}{2} = ab'\sin\alpha\tan\alpha - \frac{a^2}{2}\sin^2\alpha$$

or $$\frac{a^2}{2}\cdot\sin^2\alpha - ab'\sin\alpha\tan\alpha + \frac{H^2}{2} = 0$$

or $$a^2\sin^2\alpha - 2ab'\sin\alpha\tan\alpha + H^2 = 0$$

or $$a^2 - \frac{2ab'}{\cos\alpha} + \frac{H^2}{\sin^2\alpha} = 0$$

or $$a = \frac{\frac{2b'}{\cos\alpha} \pm \sqrt{\left(\frac{2b'}{\cos\alpha}\right)^2 - \frac{4H^2}{\sin^2\alpha}}}{2}$$

Ignoring unfeasible +ve sign, we have

$$\boxed{a = \frac{b'}{\cos\alpha} - \sqrt{\frac{b^2}{\cos^2\alpha} - \frac{H^2}{\sin^2\alpha}}} \quad ...(20.22)$$

This is the required equation.

Case (*b*)·When α lies between 30° and 60°

Cassagrande has derived an equation for determining the value of 'a' in terms of b, H and α. His final equation is

$$a = \sqrt{b^2 + H^2} - \sqrt{b^2 - H^2\cot^2\alpha} \quad ...(20.23)$$

where b is defined in Fig. 20.16, H is the head causing flow and α is the angle which the d/s face makes with the horizontal (clockwise) as defined earlier.

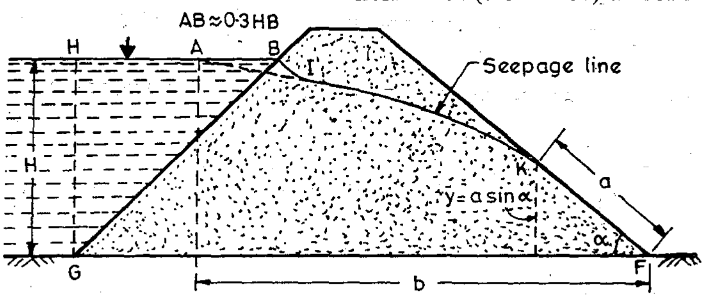


Fig. 20.16

Equation (20.23) gives satisfactory results for values of α less than 30°, but for steeper slopes, the deviations from the correct values become quite high and for such cases, Cassagrande has suggested the use of sin α in place of tan α.

In other words, the hydraulic gradient (i) is given by $\frac{dy}{ds}$ (*i.e.* sin α) of the seepage line and not by $\frac{dy}{dx}$ (*i.e.* tan α) as was taken in the previous case.

Therefore, $q = K\cdot i\cdot A$

$$\left(\frac{\not{\ }}{ds}\right) y \cdot 1 = K\,(\sin\alpha)\,(a\sin\alpha) \qquad \because \frac{dy}{ds} = \sin\alpha$$

$$K \cdot a \cdot \sin^2\alpha$$

$$y = a\sin^2\alpha \cdot ds.$$

ween the limits

$$y = a\sin\alpha, \qquad s = a$$

$$y = H \qquad s = S_0$$

where S_0 is the total length of parabola from the point A to the point F.

or
$$\int_{y=a\sin\alpha}^{y=H} y \cdot dy = a \cdot \sin^2\alpha \int_{s=a}^{s=S_0} ds$$

or
$$\left|\frac{y^2}{2}\right|_{a\sin\alpha}^{H} = a\sin^2\alpha \cdot |s|_{s=a}^{s=S_0}$$

or
$$\frac{H^2 - a^2\sin^2\alpha}{2} = a\sin^2\alpha\,[S_0 - a]$$

or
$$\frac{H^2}{2} - \frac{a^2}{2}\sin^2\alpha = a\sin^2\alpha \cdot S_0 - a^2\sin^2\alpha$$

or
$$\frac{a^2}{2}\sin^2\alpha - S_0\sin^2\alpha \cdot a + \frac{H^2}{2} = 0$$

or
$$a^2 - 2S_0 \cdot a + \frac{H^2}{\sin^2\alpha} = 0$$

or
$$a = \frac{2 \cdot S_0 \pm \sqrt{4 \cdot S_0^2 - 4 \cdot \dfrac{H^2}{\sin^2\alpha}}}{2}$$

Ignoring +ve sign, we get

$$a = S_0 - \sqrt{S_0^{\,2} - \frac{H^2}{\sin^2\alpha}} \qquad \text{...(20.24)}$$

The total length of the parabola S_0 can be approximately taken to be equal to $\sqrt{b^2 + H^2}$, then

$$S_0 = \sqrt{b^2 + H^2}$$

Substituting, we get

$$a = \sqrt{b^2 + H^2} - \sqrt{b^2 + H^2 - \frac{H^2}{\sin^2\alpha}}$$

or
$$a = \sqrt{b^2 + H^2} - \sqrt{b^2 - H^2\left(\frac{1}{\sin^2\alpha} - 1\right)}$$

$$\boxed{a = \sqrt{b^2 + H^2} - \sqrt{b^2 - H^2\cot^2\alpha}}$$ which is the required eqn. (20.23).

Example 20.1. *An earth dam made of a homogeneous material has the following data :*

Coefficient of permeability of dam material	$= 5 \times 10^{-4}$ *cm/sec.*
Level of top of dam	*= 200.0 m.*
Level of deepest river bed	*= 178.0 m*
H.F.L. of reservoir	*= 197.5 m.*
Width of the top of dam	*= 4.5 m.*
Upstream slope	*= 3 : 1.*
Downstream slope	*= 2 : 1.*

Determine the phreatic line for this dam section and the discharge passing through the dam.

Solution. Taking the focus (*F*) at the d/s toe of the dam as the origin, the equation of the base parabola is given by $\sqrt{x^2+y^2} = x + S$

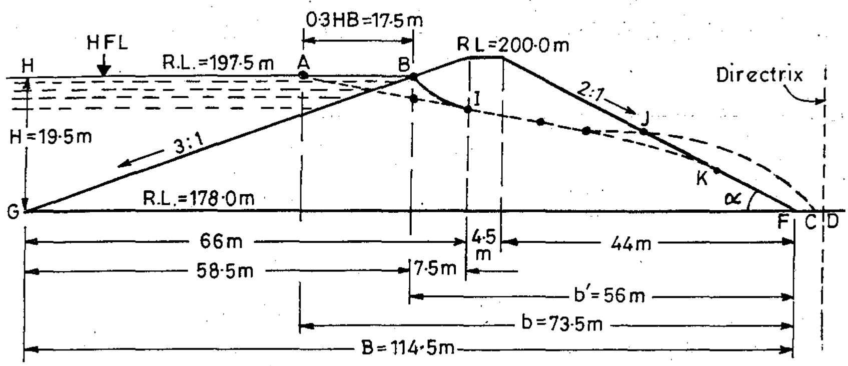


Fig. 20.17

where *S* is the distance of the point (*x*, *y*) from the directrix, called focal distance. Select the point *A*, in such a way that

$$AB \approx 0.3\, HB$$

or $$AB \approx 0.3 \times 58.5 = 17.5 \text{ m.}$$

Plot the point *A* at a distance 17.5 m from *B* (Fig. 20.17). This is the start-point of the parabola. Now the coordinates of the point *A* w.r. to *F* as origin are (73.5 m, 19.5 m). Substituting these ordinates in the equation of the base parabola, we get

$$\sqrt{(73.5)^2 + (19.5)^2} = 73.5 + S.$$

or $$\mathbf{S = 2.54 \text{ m.}}$$

The vertex (*C*) of the base parabola shall be situated at a distance equal to $\frac{S}{2}$ or 1.27 m from *F*, beyond the d/s toe of the dam, as shown in Fig. 20.17.

A few more coordinates of the base parabola at known distances (*x*) are worked out in table 20.4 using,

$$x^2 + y^2 = x^2 + S^2 + 2xS$$

or $$y = \sqrt{S^2 + 2xS}$$

Table 20.4

x	$y^2 = 2xS + S^2 = 5.08x + 6.45$	$y = \sqrt{S^2 + 2xS}$
0	6.45	2.54
10	57.25	7.68
20	108.05	10.39
30	158.85	12.61
40	209.65	14.48
44	229.97	15.16
48.5	252.83	15.92
56	290.93	17.04
60	311.25	17.64
70	362.05	19.02
73.5	380.00	19.50

The base parabola with all these ordinates, is then drawn.

Now, this parabola has to be corrected at entry and exit as explained earlier. At entry, the phreatic line is started from the point B in such a way that it becomes at right angles to the u/s face GB of the dam. A reverse curvature BI is, therefore, given as shown in Fig. 20.17.

At exit, the point K at which the phreatic line intersects the d/s face can be easily obtained by using the Eq. (20.21) as :

$$\Delta a = (a + \Delta a)\left(\frac{180 - \alpha}{400}\right)$$

where $\tan \alpha = \frac{1}{2}$; or $\alpha = 26° \cdot 54$

$(a + \Delta a)$ = Distance FJ, *i.e.* the distance of the focus from the point at which the base parabola intersects the d/s face, and is measured from Fig. 20.17 ≈ 25.6 m.

$$\therefore \quad \Delta a = 25.6\left[\frac{180 - 26.54}{400}\right] = 25.6 \times \frac{153.46}{400} = 9.84 \text{ m} \text{ ; say } \mathbf{9.8\ m.}$$

$$a = 25.6 - 98 = 15.8 \text{ m.}$$

Knowing 'a', the point K is plotted and the phreatic line BIK is completed.

Discharge through the dam section can be obtained from the equation (20.20) as :

$$q = K \cdot S, \qquad \text{where } K = 5 \times 10^{-4} \text{ cm/sec} = 5 \times 10^{-6} \text{ m/sec.}$$

$$S = 2.54 \text{ m}$$

$$\therefore \quad q = 5 \times 10^{-6} \times 2.54 \text{ m}^3\text{/m run/sec.}$$

$$= 12.7 \times 10^{-6} \text{ cumes/m length of dam}$$

Hence, $q = 12.7 \times 10^{-6}$ cumecs/m length of dam. **Ans.**

Note. The value of 'a' can also be determined from Eq. (20.22), if $(a + \Delta a)$ is not to be measured, as the dam section is not to be plotted to scale.

In that case $a = \dfrac{b'}{\cos a} - \sqrt{\left(\dfrac{b'}{\cos \alpha}\right)^2 - \left(\dfrac{H}{\sin \alpha}\right)^2}$

where $b' = 56$ m

$H = 19.5$ m

$\sin 26.56° = 0.447$

$\cos 26.54° = 0.894.$

$$\therefore \quad a = \frac{56}{0.894} - \sqrt{\left(\frac{56}{0.894}\right)^2 - \left(\frac{19.5}{0.447}\right)^2}$$

$$= 62.6 - \sqrt{3{,}920 - 1{,}900} = 62.6 - 44.9 = \mathbf{17.7\ m.}$$

Hence, $a = 17.7$ m which is slightly above the value obtained from Eq. (20.21) and is thus on a safer side.

Example 20.2. *A flow net is plotted for a homogeneous earthen dam of height 22 m and freeboard 2.0 m. The results obtained are,*

Number of potential drops = 10

Number of flow channels = 4.

The dam has a horizontal filter of 30 m length at the downstream end and the coefficient of permeability of the dam material is 5×10^{-4} cm/sec. Calculate the discharge per m run of the dam.

Solution. The discharge through a dam section is approximately given by the Eq. (20.15) as :

$$q = K.H.\frac{N_f}{N_d}$$

where $K = 5 \times 10^{-4}$ cm/sec $= 5 \times 10^{-6}$ m/sec.

$H = 22 - 2 = 20$ m.

$N_f = 4$

$N_d = 10$

$$\therefore \quad q = 5 \times 10^{-6} \times \frac{20 \times 4}{10}$$

$$= \mathbf{4.0 \times 10^{-6}}$$ **cumecs/m run of dam. Ans.**

Example 20.3. *For the dam section given in example 20.1, draw the seepage line if a horizontal filter of length equal to 25 m is provided inward from the downstream toe of the dam.*

Solution. The dam section is plotted from the dimensions given in example 20.1. The horizontal filter is also provided. Now, the left end of the filter will act as a focus and is designated as F and is taken as origin.

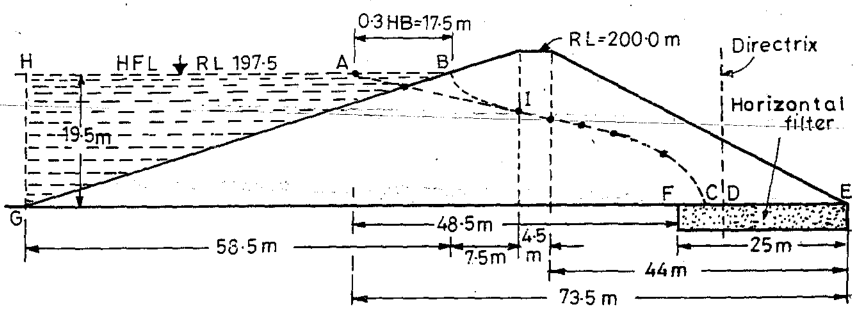


Fig. 20.18

The coordinates of any point (x, y) on a parabola of equation

$$\sqrt{x^2+y^2}=x+S$$

will now be for the point A.

The point A is determined as in example 20.1. Its coordinates w.r. to F as origin are (48.5 m, 19.5 m)

$\therefore$ $$\sqrt{(48.5)^2+(19.5)^2}=48.5+S$$

or $$\sqrt{2{,}352+380}=48.5+S$$

or $$\mathbf{S=3.77\ m.}$$

The vertex (C) of the parabola shall be situated at a distance equal to $\frac{S}{2}$, *i.e.* 1.83 m beyond the point F and directrix shall be at a distance 3.77 m from F.

At point F, $x=0$

$\therefore$ $y=S=3.77$ m.

A few more coordinates of the base parabola are worked out in table 20.5, using

$$x^2+y^2=x^2+S^2+2xS$$

or $$y=\sqrt{2xS+S^2}$$

Table 20.5

x	$y^2=(2xS+S^2)=7.54x+14.21$	$y=\sqrt{2xS+S^2}$
0	14.21	3.77
10	89.61	9.47
15	127.31	11.27
19	157.47	12.54
23.5	191.40	13.85
31	247.95	15.75
40	315.81	17.78
48.5	380.00	19.5

The base parabola is drawn and correction at the entry point for the curve BI is made by free hand sketching, in such a way that the phreatic line becomes ⊥ to u/s face GB of the dam. The exit point should also be corrected such that the phreatic line meets ⊥ to base line FCD. The final phreatic line BIC is thus drawn as shown in Fig. 20.18.

20.12.3. Phreatic Line for a Zoned Section. In case of a zoned section having a central impervious core, such as shown in Fig. 20.19 ; the effects of the outer zone can be neglected altogether.

The focus of the base parabola will, therefore, be located at the d/s toe of the core, as shown in Fig. 20.19. The phreatic line can then be drawn as usual with free hand corrections required at suitable places.

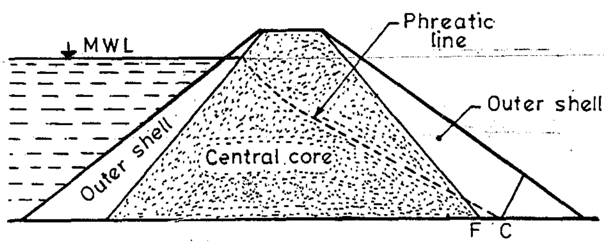


Fig. 20.19

20.13. Stability of Earthern Slopes

An earth embankment usually fails, because of the sliding of a large soil mass along a curved surface. It has been established by actual investigation of slides of railway embankments in Sweden, that the surface of slip is usually close to cylindrical, *i.e.* an arc of a circle in cross-section. The method which is described here and is generally used for examining the stability of slopes of an earthen embankment is called the *Swedish Slip Circle Method* or *The Slices Method.* The method thus assumes the condition of plane strain with failure along a cylindrical arc.

The location of the centre of the possible failure arc is assumed. The earth mass is divided into a number of vertical segments called slices. These verticals are usually equally spaced, though it is not necessary to do so. Depending upon the accuracy desired, six to twelve slices are generally sufficient.

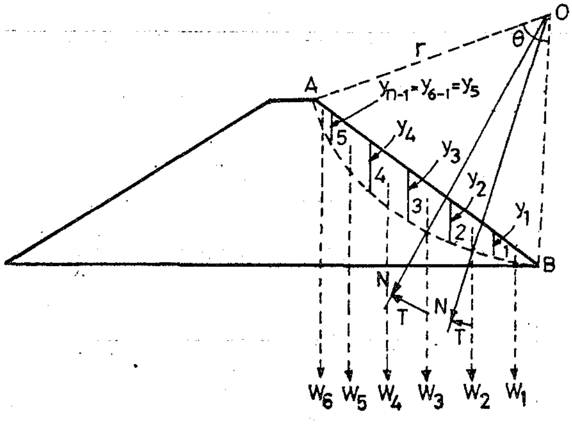


Fig. 20.20

Let O be the centre and r be the radius of the possible slip surface as shown in Fig. 20.20. Let the total arc AB be divided into slices of equal width say b metres each. The width of the last slice will be something different say let it be $m \cdot b$ metres.

Let these slices be numbered as 1, 2, 3, 4, ... and let the weight of these slices be $W_1, W_2, W_3, W_4 \ldots$

The forces between these slices are neglected and each slice is assumed to act independently as a vertical column of soil of unit thickness (⊥ to paper) and width b. The weight W of each slice is assumed to act at its centre. The weight W of each slice can be resolved into two components ; say a normal component (N) and a tangential component (T) such that

$$N = W \cos \alpha$$
$$T = W \sin \alpha$$

where α is the angle which the slope makes with the horizontal.

The normal component (N), will pass through the centre of rotation (O) and hence does not create any moment on the slice. However, the tangential component (T) causes a disturbing moment equal to ($T \cdot r$), where r is the radius of the slip circle. The tangential components of a few slices may create resisting moments; in that case T is considered as negative.

The total disturbing moment (M_d) will be equal to the algebraic sum of all the tangential moments, *i.e.*

$$M_d = \Sigma T \cdot r = r \cdot \Sigma T$$

where $\Sigma T = (T_1 + T_2 + T_3 + \ldots)$

The resisting moment is supplied by the development of shearing resistance of the soil along the arcual surface AB. The magnitude of shear strength developed in each slice will depend upon the normal component (N) of that slice. Its magnitude will be

$$= c \cdot \Delta L + N \tan \phi$$

where c is the unit cohesion, ΔL is the curved length of the slice and ϕ is the angle of internal friction of soil.

This shear resistance is acting at a distance r from O and will provide a resisting moment

$$= r\,[c \cdot \Delta L + N \tan \phi]$$

The total resisting moment over the entire arc AB

$$M_r = r\,[\Sigma C \cdot \Delta L + \Sigma N \cdot \tan \phi]$$
$$= r\,[c \Sigma\, \Delta L + (\Sigma N) \tan \phi]$$
$$= r\,[c \cdot \widehat{AB} + (\Sigma N) \tan \phi]$$

where ΣN is the sum of all the normal components.

$$= N_1 + N_2 + N_3 + \ldots$$

Length $\widehat{AB}$ of slip circle

$$= \widehat{AB} = \left[\frac{2\pi \cdot r}{360°}\right] \times \theta$$

where θ is the angle in degrees, formed by the arc AB at centre O.

Hence, the factor of safety against sliding

$$= \text{F.S.} = \frac{\text{Resisting moment}}{\text{Disturbing moment}} = \frac{M_r}{M_d}$$

or

$$= \frac{r\,[c \cdot \widehat{AB} + (\tan \phi)\, \Sigma N]}{r \cdot \Sigma T}$$

or

$$\text{F.S.} = \frac{[c \cdot \widehat{AB} + (\tan \phi)\, \Sigma N]}{(\Sigma T)} \qquad \text{...(20.24)}$$

Eq. (20.24) can be worked out by working out ΣT and ΣN separately. This evaluation of ΣN and ΣT can be simplified as explained below :

If $y_1, y_2, y_3, \ldots$ are the vertical extreme ordinates (boundary ordinates) of the slices 1, 2, 3... respectively, then the respective weights can be written as

$$W_1 = \left(\frac{0 + y_1}{2}\right) \cdot b \cdot \gamma \cdot 1$$

where γ is the unit weight of soil and unit width of the slice is considered.

$$W_2 = \left(\frac{y_1 + y_2}{2}\right) b \cdot \gamma$$

$$W_3 = \left(\frac{y_2 + y_3}{2}\right) b \cdot \gamma$$

$$W_4 = \left(\frac{y_3 + y_4}{2}\right) b \cdot \gamma$$

....

....

$$W_n = \left(\frac{y_{n-1} + 0}{2}\right) m \cdot b \cdot \gamma$$

where n is the total number of slices.

$$\therefore \quad \Sigma W = W_1 + W_2 + W_3 + \ldots + W_n$$

$$= \left[y_1 b + y_2 b + y_3 b + \ldots \frac{y_{n-1}}{2} b + \frac{y_{n-1}}{2} m \cdot b \right] \gamma$$

$$= \left[y_1 + y_2 + y_3 + \ldots y_{n-1} \left(\frac{1+m}{2} \right) \right] \gamma \cdot b$$

Now $\quad \Sigma N = N_1 + N_2 + N_3 + \ldots$

$$= W_1 \cos \alpha + W_2 \cos \alpha + W_3 \cos \alpha + \ldots$$

$$\therefore \quad \Sigma N = \cos \alpha \, [W_1 + W_2 + W_3 + \ldots]$$

or $\quad \Sigma N = \cos \alpha \, (\Sigma W).$

Similarly $\Sigma T = \sin \alpha \, (\Sigma W)$ if all T's are +ve.

ΣW can be evaluated by adding all the vertical boundary ordinates of all the slices. (The last ordinate should be multiplied by $\frac{m+1}{2}$ before adding) and then multiplying this summation (say Σy) by $b \cdot \gamma$.

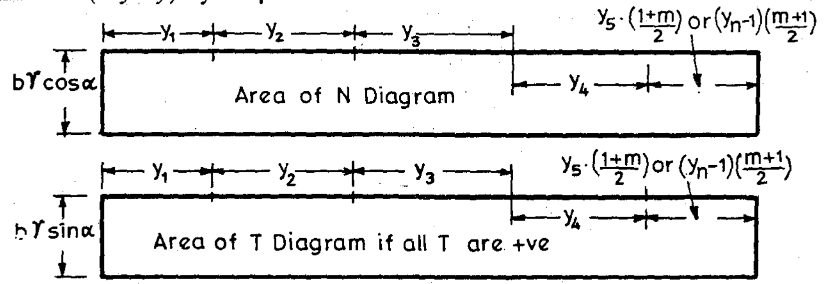


Fig. 20.21

The area of N diagram will represent ΣN and that of T diagram will represent ΣT.

As a general case, the value of ΣN and ΣT can be worked out in a tabular form as shown below in table 20.6.

Table 20.6

Slice number	*Wt. of each slice*	$N = W \cos \alpha$	$T = W \sin \alpha$	$c \cdot \Delta L$
1	W_1	N_1	T_1	
2	W_2	N_2	T_2	
3	W_3	N_3	T_3	
⋮				
n	W_n	N_n	T_n	
		ΣN	ΣT	$\Sigma c \cdot \Delta L$

The F.S. is then calculated from equation

$$\text{F.S.} = \frac{\Sigma c \cdot \Delta L + (\Sigma N - \Sigma U) \tan \phi}{\Sigma T} \qquad \ldots(20.25a)$$

20.13.1. Location of the Centre of the Critical Slip Circle. In order to find out the worst case, numerous slip circles should be assumed and factor of safety (F.S.)

calculated for each circle, as explained earlier. The minimum factor of safety will be obtained for the critical slip circle. In order to reduce the number of trials, Fellenius has suggested a method of drawing a line (*PQ*), representing the locus of the critical slip circle.

The determination of this line *PQ* for the d/s slope of an embankment is shown in Fig. 20.22(*a*) and similarly, the determination of this line *PQ* for u/s slope is shown in Fig. 20.22(*b*). The point *Q* is determined in such a way that its coordinates are $(4.5H, H)$ from the toe, as shown. The point *P* is obtained with the help of directional angles α_1 and α_2 as shown. The value of α_1 and α_2 for various slopes, are tabulated in Table 20.7.

Table 20.7

Slope	*Directional angles*	
	α_1 *in degrees*	α_2 *in degrees*
1 : 1	27.5	37
2 : 1	25	35
3 : 1	25	35
4 : 1	25	35
5 : 1	25	35

After determining the locus of the critical slip circle, the critical slip circle can be drawn, keeping in view the following few points :

(*i*) Except for very small values of ϕ, the critical arc passes through the toe of the slope.

(*ii*) If a hard stratum exists at a shallow depth under the dam, the critical arc cannot cross this stratum, but can only be tangential to it.

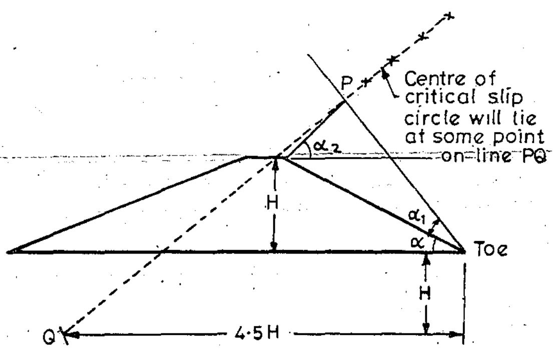


Fig. 20.22 (*a*). Locus of critical circle for d/s slope.

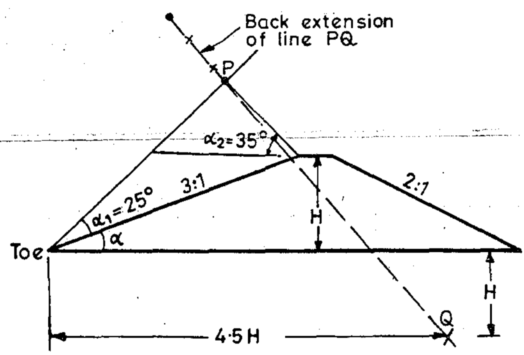


Fig. 20.22 (*b*) Locus of critical circle for u/s slope.

(*iii*) For very small values of ϕ (0 to 15°), the critical arc passes below the toe of the slope if the inclination of the slope is less than 53° (which is generally the case). The centre of the critical arc in such a case is likely to fall on a vertical line drawn through the centre of the slope, as shown in Fig. 20.23.

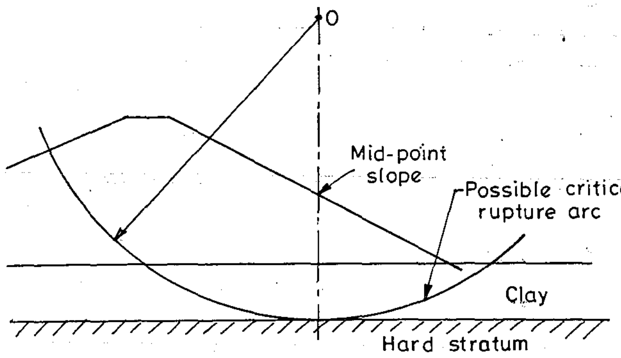


Fig. 20.23.

20.13.2. Determination of Pore Pressures from Flow Net. It was stated earlier that the soil is composed of voids which are filled with water and air. Whenever, any load is applied in the form of consolidation, it is taken up by water and gradually transferred to soil grains as the excess water drains out. The shear strength of the soil thus goes on increasing.

Immediately after construction, sizable pore pressures may be present, which may gradually dissipate. But as soon as the reservoir is filled, water enters the pores of the dams and a new pattern of pore pressure gets developed.

Under steady state seepage, the pore pressure at any point in a dam is equal to the hydrostatic head due to water in the reservoir minus the head loss in seepage through the dam up to that point. The pore pressure at any point within a dam section can easily be found from the flow net.

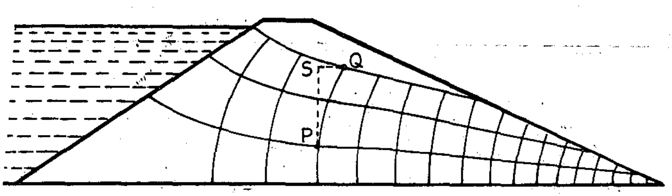


Fig. 20.24

For example, in Fig. 20.24, the pore pressure at any point say *P* will be equal to the difference in elevation between *P* and *Q*, where *Q* is the point of intersection of the phreatic line with the equipotential line through the point *P*.

Hence, if a piezometer is installed at *F*, the water shall rise up to an elevation of *Q*. If *QS* is a horizontal line through *Q*, *PS* will be the pore pressure at *P*.

20.13.3. Stability of Downstream Slope during Steady Seepage. The most critical condition for which the stability of the d/s slope must be examined, occurs, when the reservoir is full and the seepage is taking place at full rate.

The seeping water below the phreatic line, exerts a pore pressure on the soil mass which lies below the phreatic line. Hence, if the slices of the critical arc, happen to include this submerged soil, [Fig. 20.25 (*a*)], the shear strength developed on those slices shall be correspondingly reduced. The net shear strength on such a slice shall be $= c\,\Delta L + (N - U)\tan\phi$, where U is the pore pressure.

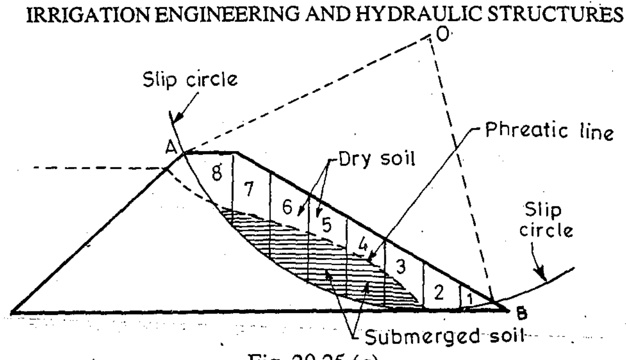


Fig. 20.25 (*a*)

The factor of safety (S.F.) for the entire slip circle is then given by the equation.

$$\text{F.S.} = \frac{c \cdot \widehat{AB} + \tan\phi\,(\Sigma N - \Sigma U)}{\Sigma T} \qquad \ldots(20.25)$$

where ΣU is the total pore pressure on the slip circle.

The pore pressure at a point is represented by the piezometric head at that point as explained earlier. The variation of the pore pressure along a failure arc is, therefore, obtained as explained below :

First of all, draw a flow net and thus determine the points of intersections of equipotential lines with the failure arc. At each point of intersection, measure the vertical ordinate from that intersection to the level at which that particular equipotential line cuts the phreatic line. The pore pressures represented by the vertical heights so obtained, are then plotted to a scale in a direction perpendicular to the sliding surface at the respective points of intersection.

The pore pressure distribution is thus shown in Fig. 20.25(*b*) (shaded area). The area of this diagram can be measured by a planimeter. The area of this diagram can also be calculated by ordinate method as was done for N and T cases taking the unit weight of water as 9.81 kN/m^3 ($\approx$ 10 kN/m^3). Knowing ΣN, ΣU and ΣT, F.S. can be calculated easily by using equation 20.25.

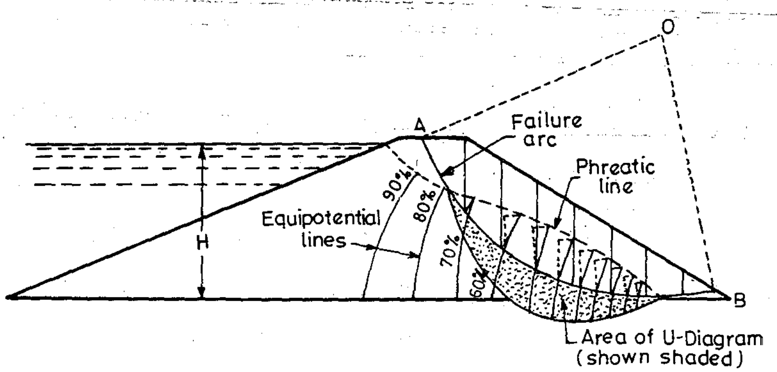


Fig. 20.25 (*b*)

Approximate method. In the absence of a flow net, the normal components, which are responsible for the shear strength of the soil, can be calculated on the basis of submerged unit weight of soil (*i.e.* $\gamma_{sub} = \gamma_{sat} - \gamma_w$). On the other hand, the values of tangential components *i.e.* T's which are responsible for creating disturbing moments, should be calculated on the basis of saturated unit weight of soil.

In such a case, the width of N rectangle which was taken as $b \cdot \gamma \cdot \cos \alpha$ will become $b \cdot \gamma_{sub} \cos \alpha$ (Fig. 20.21) and the width of T rectangle will remain $b \cdot \gamma_{sat} \cdot \sin \alpha$. The new N diagram is a $(N - U)$ diagram assuming the entire soil to be submerged and can be called N' diagram. Equation (20.25) can be written as

$$\text{F.S.} = \frac{c \cdot \widehat{AB} + \tan \phi \cdot (\Sigma N')}{\Sigma T} \qquad \text{...(20.26)}$$

20.13.4. Stability of Upstream Slope During Sudden Drawdown. When the reservoir is full, the critical region is near the downstream face. If no drainage arrangement is made and the d/s slope is also steep, the phreatic line may intersect the d/s slope creating serious conditions there. This can be avoided by providing drainage filter or drainage toe, etc., or by broadening the base of the dam so that the head loss is great enough to bring the line of saturation beneath the d/s toe of the dam.

For the u/s slope, the critical condition can occur, when the reservoir is suddenly emptied. In such a case, the water level within the soil will remain as it was when the soil pores were full of water. The weight of this water within the soil, now tends to slide the u/s slope along a circular arc.

The tangential component of the saturated soil lying over the arc, will create a disturbing force ; while the normal component minus the pore pressure shall supply the shear strength of the soil. High pore pressures shall be developed in this case and although a true solution can be obtained from the flow net and pressure net, an approximate solution can be easily obtained, by considering the soil resting over the failure arc as saturated, for calculating T's ; and as submerged for calculating N's.

The factor of safety (F.S.) is finally obtained from the equation

$$\text{F.S.} = \frac{c \cdot \widehat{AB} + \tan \phi \, \Sigma N'}{\Sigma T}$$

N's represent normal components on submerged density and T's represent tangential components on saturated unit weight of soil. The maximum factor of safety obtained for the critical slip circle should be 1.5, for safe designs.

20.13.5. Stability of the u/s slope portion of the dam, during sudden drawdown, from the consideration of horizontal shear developed at base under the u/s slope of the dam. It is an approximate method for checking the stability of the u/s slope against sudden drawdown. It is based on the simple principle that a horizontal shear force (say P_u) is exerted by the saturated soil (*i.e.* by the soil as well as by water contained within the soil). The resistance to this force (say R_u) is provided by the shear resistance

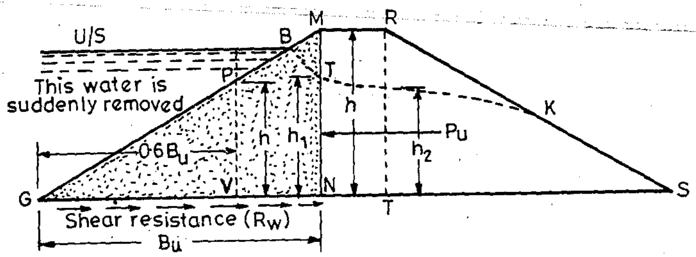


Fig. 20.26

developed at the base of the soil mass, contained within the u/s triangular shoulder *GMN* (Fig. 20.26).

Considering a unit length of the dam, the horizontal force P_u is given by the equation

$$P_u = \left[\frac{\gamma_1 h^2}{2}\tan^2\left(45 - \frac{\phi}{2}\right) + \gamma_w \cdot \frac{h_1^2}{2}\right] \text{(say in kN)} \quad ...(20.27)$$

where γ_1 is the weighted density at the centre of triangular shoulder upstream and is given by

$$\gamma_1 = \frac{\gamma_{sub} \cdot h_1 + \gamma_{dry} \cdot (h - h_1)}{h} \quad ...(20.28)$$

Shear resistance (R_u) of u/s slope portion of the dam developed at base GN is given by

$$R_u = C + W \tan\phi \quad ...(20.29a)$$

where W = the weight of the u/s triangular shoulder of dam

C = The total cohesive force developed at base *GN*.

If c is the unit cohesion of the dam soil, then $C = c \times (B_u \times 1)$ where B_u = length *GN*.

The triangular profile of the u/s slope portion of dam has an area *GBTN* as the submerged soil (*i.e.* the soil below the seepage line) and an area equal to *BMT* as a dry area. The correct weight W will be equal to ($\gamma_{sub} \times$ Area $GBTN + \gamma_{dry} \times$ Area BMT). These areas can be measured by a planimeter. If the measuring of the areas is to be avoided, the entire area may be taken as submerged. By so doing, the weight W will be slightly reduced, and thus $W \tan\phi$ or R_u or F.S. will be slightly reduced. Hence, the results obtained will be on a safer side.

In such a case,

$$W = [\text{Area of } \Delta\, GMN] \cdot \gamma_{sub} = \gamma_{sub} \cdot (\tfrac{1}{2} \cdot Bu \cdot h)$$

or

$$R_u = C + W \tan\phi$$
$$= c \cdot (B_u \times 1) + (\gamma_{sub} \cdot \tfrac{1}{2} \cdot B_u \cdot h) \tan\phi \quad ...(20.29)$$

Now P_u and R_u are known, the factor of safety against sliding can be easily calculated, using

$$\text{F.S.} = \frac{R_u}{P_u} \quad ...(20.30)$$

It should be more than 1.5.

The factor of safety calculated above is w.r. to average shear (τ_{av}), which will be equal to

$$\tau_{av} = \left(\frac{P_u}{B_u \times 1}\right) \quad ...(20.31)$$

It has been found by photoelastic studies, that the maximum intensity of shear stress occurs at a distance 0.6 B_u from the heel (*i.e.* 0.4 B_u from the top shoulder) and is equal to 1.4 times the average shear intensity.

$\therefore$ Maximum shear stress induced

$$= \tau_{max} = 1.4\left(\frac{P_u}{B_u}\right) \quad ...(20.32)$$

which is developed at the point V (Fig. 20.26) such that $GV = 0.6\, B_u$.

The unit shearing resistance developed at this point V is give by

$$\tau_f = c + h' \gamma_{sub} \cdot \tan\phi$$
$$= c + 0.6\, h \cdot \gamma_{sub} \cdot \tan\phi \quad ...(20.33)$$

F.S. at the point of the maximum shear

$$= \frac{\tau_f}{\tau_{max}} \quad ...(20.34)$$

It should be more than 1.

20.13.6. Stability of d/s slope under steady seepage from the considerations of horizontal shear at base under the d/s slope of the dam. The stability of the d/s slope under steady seepage is generally tested with Swedish slip circle method. However, the F.S. against the horizontal shear forces can be evaluated on the same principles as was done for the d/s slope in the previous article.

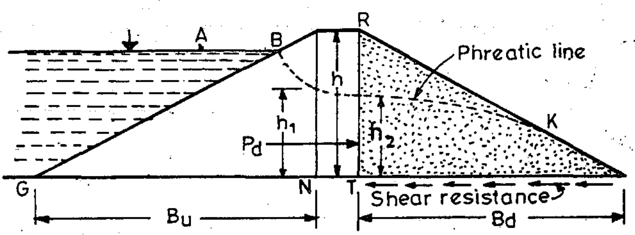


Fig. 20.27

With reference to Fig. 20.27.

The horizontal shear force P_d, in this case, is given by

$$P_d = \left[\frac{\gamma_2 \cdot h^2}{2}\tan^2\left(45 - \frac{\phi}{2}\right) + \frac{\gamma_w \cdot h_2^{\,2}}{\cdot 2}\right] \quad ...(20.35)$$

where γ_2 is the weighted density at the centre of the triangular shoulder downstream, given by

$$\gamma_2 = \frac{\gamma_{sub} \cdot h_2 + \gamma_{dry}(h - h_2)}{h} \quad ...(20.36)$$

Shear resistance R_d of d/s slope portion of dam, developed at base is given by

$$R_d = C + W \tan\phi,$$

where W = The weight of the d/s slope portion of dam. (*i.e.* $\Delta\, RTS$)

$C = c \times (B_d \times 1)$, where c is the unit cohesion.

The triangular profile RTS of the d/s slope portion of dam has an area say A_1 of dry soil above the seepage line and the area of submerged soil say A_2 below the seepage lines. These areas can be measured by a planimeter and then

$$W = [\gamma_{dry} A_1 + \gamma_{sub} \cdot A_2] \times 1$$

or

$$R_d = c \cdot B_d + \left[\gamma_{dry} A_1 + \gamma_{sub} \cdot A_2\right] \tan\phi \quad ...(20.37)$$

If the measuring of the areas is to be avoided, the entire weight W may be calculated on the basis of submerged soil, as it will be on a still safer side. In that case,

$$W = \gamma_{sub} \cdot (\tfrac{1}{2} \cdot B_d \cdot h)$$

Knowing P_d and R_d, the factor of safety against shear can be easily determined as

$$\text{F.S.} = \frac{R_d}{P_d} \quad \text{...(20.38)}$$

The factor of safety at the point of maximum shear can also be determined in the same manner as was explained for the u/s slope portion.

20.13.7. Stability of the foundation against Shear. The generally available silt and clay foundations below the base of an earth dam, are sufficiently impervious, and there is generally no necessity of providing any treatment for under-seepage and piping for such foundations. But these foundations are weak in shear and must be investigated. In order to keep the shear stress developed at the foundations, within limits, the embankment-slopes may have to be flattened or berms on either side may be provided. If the available foundations are of plastic or unconsolidated clays, their shear strength will be very less and the matter should be seriously and thoroughly investigated.

The method given below, for determining the factor of safety against the foundation shear, is an approximate method and is based on the assumption that a soil has an equivalent liquid unit weight which would produce the same shear stress as the soil itself.

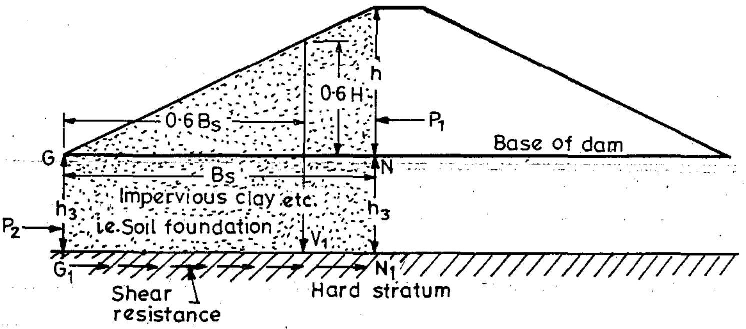


Fig. 20.28

The total horizontal shear force (P), under a slope of the dam is equal to the difference between the lateral thrust on a vertical through the top shoulder of the slope and a vertical through the toe of the slope.

or $$P = (P_1 - P_2) = \frac{\gamma_{eq}\,(h + h_3)^2}{2} \tan^2\left(45 - \frac{\phi_1}{2}\right) - \frac{\gamma_{eq} \cdot h_3^{\,2}}{2} \times \tan^2\left(45 - \frac{\phi_1}{2}\right)$$

or $$P = \gamma_{eq}\left[\frac{(h + h_3)^2 - h_3^{\,2}}{2}\right] \times \left[\tan^2\left(45° - \frac{\phi_1}{2}\right)\right] \text{ (kN)} \quad \text{...(20.39)}$$

where h_3 = the depth of foundation soil below the dam base, overlying the hard stratum below it.

γ_{eq} = the equivalent unit weight of dry soil in foundation and dam. It is given by

$$\gamma_{eq} = \frac{\gamma_{dry} \text{ for dam material} \times h + \gamma_{dry} \text{ for foundation material} \times h_3}{h + h_3} \quad ...(20.40)$$

where ϕ_1 = the equivalent angle of internal friction and is given by

$$\phi_1 = \tan^{-1}\left[\frac{c_f + \gamma_{eq}\,(h + h_3)\tan\phi_f}{\gamma_{eq}\cdot(h + h_3)}\right] \quad ...(20.41)$$

where c_f and ϕ_f are the values of unit cohesion and angle of internal friction for the soil in the foundation.

The term, $\left[\gamma_{eq}\cdot\tan^2\left(45 - \frac{\phi_1}{2}\right)\right]$ is known as **equivalent liquid unit weight.**

Now the average shear stress at the base of the slope

$$(\tau_{av}) = \frac{P}{B_s} \quad ...(20.42)$$

where B_s is the base width below the slope.

The value of B_s will be equal to B_u for u/s slope and B_d for d/s slope. The minimum value will generate maximum stresses and hence, that particular slope should be considered which gives the minimum value of B_s, *i.e.* the slope which is less flat and is, therefore, the worst slope.

Maximum stress has been found by photoelastic studies to be 1.4 times the average stress and it occurs at a distance of 0.6 B_s from the toe of the slope.

or τ_{max} = maximum shear stress = 1.4 $\tau_{av.}$

The unit shear resistance of the foundation soil below the toe at point G_1

$$= \tau_{f_1} = \left[c_f + \gamma_f\cdot h_3\tan\phi_f\right] \quad ...(20.43)$$

γ_f is the unit weight of foundation soil and if the average value is given for impervious soils, that value may be used in the equation (20.43). But if there is a possibility of foundation soil getting submerged due to large scale seepage that may take place through the foundation soil, then the submerged density may be used in equation (20.43).

Similarly, the unit shear resistance of the soil vertically below the upper point of the considered slope (say at point N_1) is given by τ_{f_2}.

$$\tau_{f_2} = c_f + \gamma_3\,(h + h_3)\tan\phi_f \quad ...(20.44)$$

where γ_3 = the equivalent unit weight of soil in the dam and foundation at the point N_1 and is given by

$$\gamma_3 = \frac{\gamma_f \times h_3 + \gamma_{dam}\times h}{h + h_3} \quad ...(20.45)$$

The values of γ_f and γ_{dam} may be taken as their dry densities or submerged densities depending upon the possibilities.

The average shear resistance

$$= \tau_f = \frac{\tau_{f_1} + \tau_{f_2}}{2} \quad ...(20.46)$$

Hence, overall factor of safety $= \dfrac{\tau_f}{\tau_{(av)}}$ ...(20.47)

This should be greater than 1.5.

The factor of safety at the point of maximum shear (*i.e.* the point V_1) must also be calculated as below :

The unit shear resistance at this point V_1

$$= \tau_{f(max)} = c_f + \gamma_4 (h_3 + 0.6h) \tan \phi_f$$

where γ_4 is the equivalent weight of soil in dam and foundation and is given by

$$\gamma_4 = \frac{\gamma_f \times h_3 + \gamma_{dam} \times (0.6\,h)}{(h_3 + 0.6h)} \quad ...(20.48)$$

The dry or submerged unit weights may be used as explained earlier, in the above equation.

F.S. = Factor of safety at the point of maximum shear

$$= \frac{\tau_{f\,(max)}}{\tau_{max}} \quad ...(20.49)$$

This should be greater than unity.

Example 20.4. *An earthen dam made of homogeneous material has the following data :*

Level of the top of the dam	*= 200.00 m*
Level of deepest river bed	*= 178.0*
H.F.L. of reservoir	*= 197.5 m*
Width of top of dam	*= 4.5 m*
Upstream slope	*= 3 : 1*
Downstream slope	*= 2 : 1*
Length of the horizontal filter from d/s toe, inwards	*= 25 m*
Cohesion of soil of dam	*= 24 kN/m²*
Cohesion of soil of foundation	*= 54 kN/m²*
Angle of internal friction of soil in the dam	*= 25°*
Angle of internal friction of soil in the foundation	*= 12°*
Dry weight of the soil in the dam	*= 18 kN/m³*
Submerged weight of the soil in the dam	*= 12 kN/m³*
Dry unit weight of the foundation soil	*= 18.3 kN/m³*
Coefficient of permeability of soil in the dam	*= 5×10^{-6} m/sec.*

The foundation soil consists of 8 m thick layer of clay, having negligible coefficient of permeability. Check the stability of the dam and its foundations.

Solution.

(1) **Overall stability of the dam section as a whole**

We will consider 1 m length of the dam. The section of the dam and the phreatic line is first of all drawn, as given in example 20.3 and shown in Fig. 20.29 (*a*). The dam section, etc. is generally drawn on a graph sheet so as to facilitate in measuring the areas above and below the seepage line, if planimeter is not available.

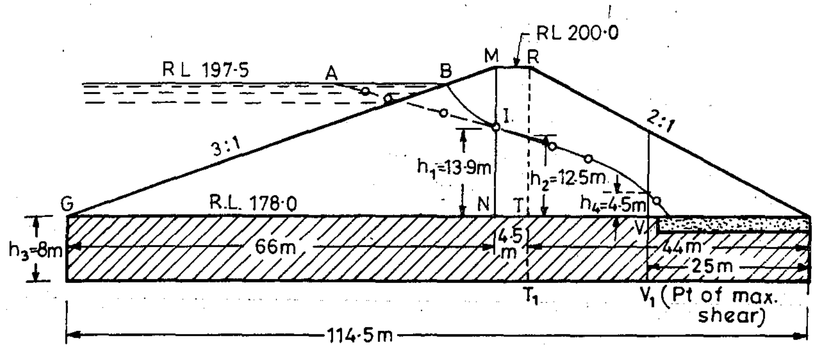


Fig. 20.29 (*a*)

The total area of dam section $= (114.5 + 4.5)\,\dfrac{22}{2} = 1{,}409$ sq. m

The area above the seepage line is measured and is approximately found to be 380 m^2. (In the absence of a planimeter, graph can be used).

$\therefore$ Area below the seepage line $= 1{,}409 - 380 = 1{,}029$ sq. m

Now

Weight of the dry portion of the dam section

$$= (380\ \text{m}^2 \times 1\ \text{m} \times 18\ \text{kN/m}^3 = 6830\ \text{kN}.$$

Weight of the submerged portion of the dam section

$$= 1029\ \text{m}^2 \times 1\ \text{m} \times 12\ \text{kN/m}^3 = 12{,}350\ \text{kN}$$

Total weight of dam (called average weight)

$$= 6{,}830 + 12{,}350 = 19{,}180\ \text{kN}$$

Shear resistance of the dam at the base

$$= C + W \tan \phi$$

where C = Total cohesive strength of the soil at the base

$= c \times B \times 1 = (24 \times 114.5 \times 1)$ kN

B = Total base width = 114.5 m

$W \tan \phi = 19{,}180 \tan 25°$

$\therefore$ Shear resistance at base,

$$R = 24 \times 114.5 \times 1 + 19180 \tan 25° = 11690\ \text{kN}$$

Horizontal force = Horizontal pressure of water.

$$= P = \tfrac{1}{2}\gamma_w h^2 = \tfrac{1}{2} \cdot 9.81\,(19.5)^2 = 1865\ \text{kN}$$

Factor of safety against failure due to horizontal shear at base

$$= \frac{11690}{1865} = 6.27 > 1.3\ (\therefore \text{Safe})$$

(2) **Stability of the u/s slope portion of dam (Under sudden drawdown)** *horizontal shear along the base under the u/s slope of dam*

Draw a vertical through the u/s extremity of the top width of dam [*i.e.* point *M*, Fig. 20.29 (*a*)] so as to cut the base of the dam at point *N*. This vertical *MN* cuts the seepage line at a point, the height of which is measured as $h_1 = 13.6$ m above the base of the dam.

Horizontal force (P_u) acting on the ΔGMN is given by equation (20.27) as :

$$P_u = \left[\frac{\gamma_1 h^2}{2} \tan^2\left(45 - \frac{\phi}{2}\right) + \gamma_w \cdot \frac{h_1^2}{2}\right]$$

where γ_1 = the weighted density at the centre of triangular shoulder upstream (ΔGMN) and is given by equation (20.28) as :

$$\gamma_1 = \frac{\gamma_{sub} \cdot h_1 + \gamma_{dry}(h - h_1)}{h}$$

$$= \frac{12 \times 13.9 + 18\,(22.0 - 13.9)}{22.0}$$

$$= 14.7 \text{ kN/m}^3$$

$$\therefore \quad P_u = \frac{14.7 \times (22.0)^2}{2} \tan^2\left(45° - \frac{25°}{2}\right) + 9.81 \times \frac{(13.9)^2}{2} = 2391 \text{ kN}$$

Shear resistance R_u of the u/s slope portion of dam developed at the base GN is given by equation (20.29) as :

$$R_u = C + W \tan \phi$$

$$= c\,(B_u \times 1) + (\gamma_{sub} \tfrac{1}{2} B_u\, h) \tan \phi\text{ ;}$$ neglecting the small dry soil area BMI, as it is very small and this neglection is on a safer side.

$$B_u = 66 \text{ m}$$

$$\therefore \quad R_u = 24 \times 66 + (12 \cdot \tfrac{1}{2} \cdot 66 \cdot 22.0) \tan 25°$$

$$= 1584 + 4062 = 5646 \text{ kN}$$

Factor of safety against horizontal shear along base under u/s slope

$$= \frac{R_u}{P_u} = \frac{5646}{2391} = 2.36 > 2.0 \qquad (\therefore \text{ safe})$$

Horizontal shear stress induced in the u/s slope portion of dam at base.

$$\tau_{av} = \frac{P_u}{B_u \times 1} = \frac{2391}{66} \text{ kN/m}^2 = 36.23 \text{ kN/m}^2$$

$$\tau_{max} = \text{Maximum shear}$$

$$= 1.4\,\tau_{av} = 1.4 \times 36.23 = 50.72 \text{ kN/m}^2$$

The maximum shear is developed at a point 0.6 B_u

$$= 0.6 \times 66 = 39.6 \text{ m away from point } G$$

The unit shear resistance developed at this point

$$\tau_f = c + 0.6\,\gamma_{sub} \tan \phi$$

$$= 24 + 0.6 \times 22.0 \times 12 \tan 25° = 97.9 \text{ kN/m}^2$$

$$\therefore \quad \text{F.S.} = \frac{\tau_f}{\tau_{max}} = \frac{97.9}{50.72} = 1.93 > 1 \qquad (\therefore \text{ safe}).$$

(3) **Stability of d/s portion of dam.** *Horizontal shear along base under the d/s slope of dam.*

Draw a vertical through the d/s extremity of the top width of dam (*i.e.* point R) to cut the base at point T [Fig. 20.29 (*a*)]. Let this vertical cut the seepage line in a point, the height of which from the base is measured as $h_2 = 12.5\ m$.

Horizontal force P_d acting on the portion of downstream dam (RTS) during steady seepage is given by equation (20.35) as :

$$P_d = \left[\frac{\gamma_2 h^2}{2} \tan^2\left(45° - \frac{\phi}{2}\right) + \gamma_w \frac{h_2^2}{2}\right]$$

where γ_2 is the weighted density at the centre of the triangular shoulder RTS and given by equation (20.36) as :

$$\gamma_2 = \frac{\gamma_{sub}\, h_2 + \gamma_{dry}\,(h - h_2)}{h}$$

$$= \frac{12 \times 12.5 + 18 \times (22.0 - 12.5)}{22.0}$$

$$= 14.6\ \text{kN/m}^3$$

$$P_d = \frac{14.6\,(22)^2}{2} \tan^2\left(45° - \frac{25°}{2}\right) + 9.81 \cdot \frac{(12.5)^2}{2} = 2200\ \text{kN}$$

Shear resistance R_d of the d/s slope portion of dam developed at base TS is given as :

$$R_d = C + W \tan \phi$$

The area A_1 of the dry soil within the $\Delta\, RTS$ above the seepage line ≈ 300 sq. m (from graph or planimeter).

The total area of the $\Delta\, RTS = \frac{1}{2} \times 44 \times 22 = 484\ \text{m}^2$

∴ Area of submerged soil

$$A_2 = 484 - 300 = 184\ \text{sq. m}$$

$$R_d = cB_d + \left[\gamma_{dry} A_1 + \gamma_{sub} A_2\right] \tan \phi$$

$$= 24 \times 44.0 + [18 \times 300 + 12 \times 184] \tan 25° = 4604\ \text{kN.}$$

F.S. against horizontal shear along base under d/s slope

$$= \frac{R_d}{P_d} = \frac{4604}{2200} = 2.09 > 2\ (\therefore\ \text{Safe})$$

Average shear induced at base

$$= \frac{P_d}{B_d} = \frac{2200}{44} = 50\ \text{kN/m}^2$$

Maximum shear stress induced

$$\tau_{max} = 1.4 \times 50 = 70\ \text{kN/m}^2$$

The maximum shear stress is developed at a point $0.6\ B_d$

$$= 0.6 \times 44 = 26.4\ \text{m away from toe}$$

This unit shear resistance developed at this point

$$\tau_f = c + 0.6h\, \gamma_{sub} \tan \phi$$

(assuming the entire height as submerged as it will give safer results)

$$= 24 + 0.6 \times 22 \times 12 \tan 25° = 97.9 \text{ kN/m}^2$$

$$\therefore \quad \text{F.S.} = \frac{\tau_f}{\tau_{max}} \frac{97.9}{70} = 1.40 > 1 \qquad (\therefore \text{ Safe})$$

(4) **Stability of the foundation soil**

Average compressive stress on foundation soil

$$= \frac{\text{Weight of dam}}{\text{Base area on which it acts}}$$

Since the compressive stress is maximum when the entire dam soil is dry, therefore, we will first calculate the dry weight of the dam.

Area of section of dam

= 1,409 sq. m (calculated earlier)

Dry weight of dam section

$$= 18 \times 1{,}409 = 25{,}362 \text{ kN}$$

Average compressive stress at base

$$= \frac{25362}{114.5} = 221.5 \text{ kN/m}^2$$

Shear stress induced at base

The total horizontal shear force (P) under the d/s slope of the dam (which is the worst case, *i.e.* the steepest slope) is given by equation (20.39) as :

$$P = \gamma_{eq}\left[\frac{(h+h_3)^2 - h_3^{\,2}}{2}\right]\left[\tan^2\left(45° - \frac{\phi_1}{2}\right)\right]$$

where γ_{eq} is the equivalent weight of dry soil in foundation and dam

$$\gamma_{eq} = \frac{18h + 18.3\, h_3}{h + h_3}$$

[$\because$ Unit wt. of foundation soil of thickness $h_3 = 18.3 \text{ kN/m}^3$]

where $h = 22$ m

$h_3 = 8$ m.

$$\therefore \quad \gamma_{eq} = \frac{18 \times 22 + 18.3 \times 8}{22 + 8} = 18.1 \text{ kN/m}^3$$

ϕ_1 is given by equation (20.41) as :

$$\gamma_{eq}(h + h_3)\tan\phi_1 = c_f + \gamma_{eq}(h + h_3)\tan\phi_f$$

or $$18.1\,(22+8)\tan\phi_1 = 54 + 18.1\,(22+8)\tan 12°$$

or $$\tan\phi_1 = 0.312$$

or $$\phi_1 = 17.3°$$

$$\therefore \quad P = 18.1\left[\frac{(22+8)^2 - (8)^2}{2}\right][\tan^2(45° - 8.65°)]$$

$$= \frac{18.1}{2} [900 - 64] [(0.737)^2] = 4100 \text{ kN}$$

Average shear stress induced at base of d/s slope

$$\tau_{av} = \frac{4100}{44} = 93.2 \text{ kN/m}^2$$

Maximum shear stress induced at $0.6 \times 44 = 26.4$ m away from the d/s toe inwards at point V_1 is given by

$$= \tau_{max} = 1.4 \times 93.2 = 130.4 \text{ kN/m}^2$$

Shear resistance of the foundation soil below the d/s slope portion of dam

Unit shear resistance τ_{f_1} below the toe at point S_1

$$= [c_f + \gamma_f \times h_3 \tan \phi_f]$$

$$= 54 + 18.3 \times 8 \times \tan 12°$$

$$\therefore \quad = 85.1 \text{ kN/m}^2$$

Unit shear resistance τ_{f_2} at point T_1

$$= c_f + \gamma_3 (h + h_3) \tan \phi_f$$

where $$\gamma_3 = \frac{\gamma_{sub\,for\,dam} \times h_2 + \gamma_{dry\,for\,dam} \times (h - h_2) + \gamma_f h_3}{h + h_3}$$

$$= \frac{12 \times 12.5 + 18 \times 9.5 + 18.3 \times 8}{30} = 15.6 \text{ kN/m}^3$$

$$\therefore \quad \tau_{f_2} = 54 + 15.6 (22 + 8) \times \tan 12° = 153.5 \text{ kN/m}^2$$

The average unit shear resistance developed at foundation level in a length equal to $T_1S_1 = 44$ m, is given by

$$\tau_f = \frac{\tau_{f_1} + \tau_{f_2}}{2} = \frac{85.1 + 153.5}{2} = 119.3 \text{ kN/m}^2$$

Over all F.S. against shear

$$= \frac{\tau_f}{\tau_{av}} = \frac{119.3}{93.2} = 1.28 < 1.5$$ **(Hence, unsafe)**

The foundation soil is thus weaker to carry the load and hence the d/s slope will have to be flattened.

Shear resistance at the point of maximum shear, *i.e.* at point V_1 is given as :

$$(\tau_f)_{max} = c_f + (0.6h + h_3)\, \gamma_4 \tan \phi_f$$

$$\gamma_4 = \frac{12 \times 4.5 + 18\,(0.6 \times 22 - 4.5) + 18.3 \times 8}{0.6 \times 22 + 8} = 16.8 \text{ kN/m}^3$$

$$(\tau_f)_{max} = 54 + [0.6 \times 22 + 8]\ 16.8 \tan 12° = 129.8 \text{ kN/m}^2$$

$$\text{F.S.} = \frac{[\tau_f]_{max}}{\tau_{max}} = \frac{129.8}{130.4} = 0.995 < 1.0 \qquad \text{(Hence, Unsafe)}$$

The foundation shear and F.S. can also be calculated below the u/s portion of dam soil, in the same manner as has been done for d/s slope portion, if required.

Example 20.5. *Check the stability of the d/s slope of the earthen dam section, given in example 20.4, on a possible slip circle. Determine the factor of safety available* *against such a slip. Also determine the net factor of safety that will be available, if by some how, the soil in the d/s triangular shoulder gets fully submerged. Also compare this net factor of safety with the factor of safety that can be obtained for similar conditions if the analysis was done by shear force determination at the base of the d/s slope.*

Solution. The dam section and phreatic line is first of all drawn as shown in Fig. 20.29 (*b*) and as given in previous example. The points *P* and *Q* are located for Fellenius construction such as explained earlier ; the line *PQ* represents the locus of the critical slip circle. A possible slip circular arc passing through the toe of the dam is then drawn with centre at O_1. The soil mass is then divided into slices of 5 metre width. It is found

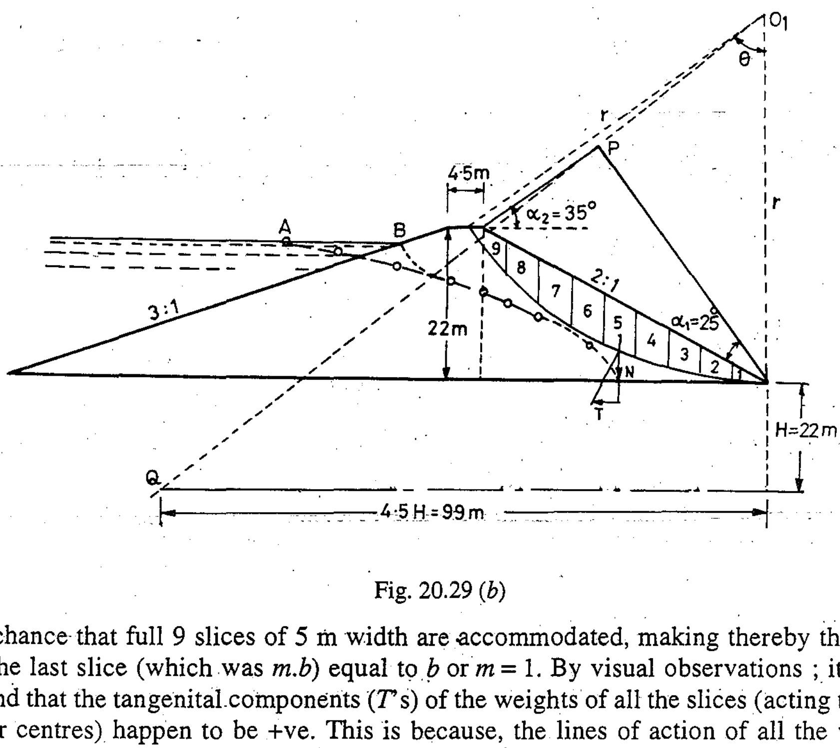


Fig. 20.29 (*b*)

by chance that full 9 slices of 5 m width are accommodated, making thereby the width of the last slice (which was *m.b*) equal to *b* or $m = 1$. By visual observations ; it is also found that the tangenital components (*T*'s) of the weights of all the slices (acting through their centres) happen to be +ve. This is because, the lines of action of all the weights are on one side (*i.e.* left side) of the centre O_1.

The end ordinates of all the slices say $y_1, y_2, y_3, \ldots, y_{n-1}$ or y_8 are measured and found to be 2.5 m, 4.5 m, 6 m, 7 m, 7.5 m, 6.0 m, and 4.5 m respectively.

The *N*-diagram is then drawn as shown in Fig. 20.30 (*a*). The dry density is used because the entire soil in the slices is dry.

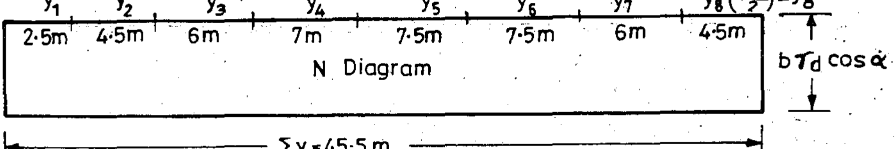


Fig. 20.30 (*a*). *N*-diagram.

Area of N-Diagram $= 45.5 \times 5 \times 18 \times \dfrac{2}{\sqrt{5}} = 3663$ kN

Similarly, T-Diagram is drawn as shown in Fig. 20.30 (b).

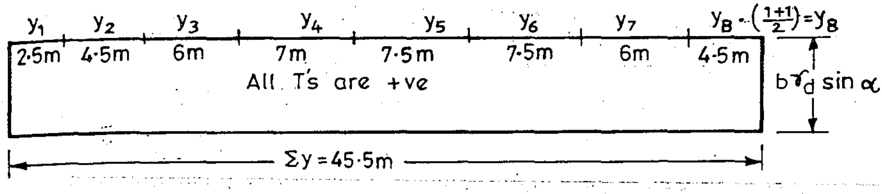


Fig. 20.30 (b). T-diagram.

Area of T-diagram $= 45.5 \times 5 \times 18 \times \dfrac{1}{\sqrt{5}} = 1831$ kN

Since the slip-circle-arc happens to fall completely above the phreatic line, the pore pressure area is zero ; or area of U- diagram is zero.

$$\therefore \quad \Sigma N - \Sigma U = \Sigma N = 3663 \text{ kN}.$$

Now
$$\text{F.S.} = \frac{c \cdot \widehat{AB} + (\Sigma N - \Sigma U) \tan \phi}{\Sigma T}$$

$$\text{where } \widehat{AB} = \frac{2\pi \cdot r}{360°} \times \theta°$$

The angle AO_1B (θ) is measured and found to be 58°. The radius r of the curve AB is also measured and found to be 53.5 m.

$$\therefore \quad \widehat{AB} = \frac{2\pi \times 53.5}{360°} \times 58° = 54 \text{ m}$$

$$c = 24 \text{ kN/m}^2 \qquad \text{(given in previous example)}$$

Hence
$$\text{F.S.} = \frac{24 \times 54 + 3663 \times \tan 25°}{1831} = 1.64 > 1.5 \qquad \textbf{(Hence, Safe)}$$

The corresponding F.S. obtained in example 20.4 from shear analysis was 2.08. Hence, F.S. should be more than 1.5 in slip circle analysis and more than 2 in shear analysis for safe designs.

Note. The F.S. obtained above by slip circle method cannot be called as the critical F.S. or minimum F.S.; because the analysis has been carried out for only one circle. In fact, various other circles should be drawn by taking the centres on line PQ, somewhere near P or away from it. If the circle happens to pass through the foundation soil, then the values of c_f and ϕ_f (*i.e.* c and ϕ for foundation soil) should be used in evaluating W and $c\,\Delta L$ for those particular slices. A tabular form solution, as explained earlier would then be better. When the slices are crossing the phreatic line, U-diagram has to be drawn and evaluated as explained earlier.

Case (b). When soil in the slices get fully submerged, then N's will be calculated on the basis of submerged weights and Ts will be calculated on the basis of saturated weights.

$$\therefore \quad \Sigma N = 45.5 \times 5 \times \gamma_{sub} \times \cos \alpha$$

$$= 45.5 \times 5 \times 12 \times \frac{2}{\sqrt{5}} = 244\text{[illegible]}$$

$$\Sigma T = 45.5 \times 5 \times \gamma_{sat} \sin \alpha$$
$$= 45.5 \times 5 \times 22 \times \sin 25° = 2115 \text{ kN}$$

Net $$\text{F.S.} = \frac{c \cdot AB + \Sigma N \tan \phi}{\Sigma T}$$
$$= \frac{24 \times 54 + 2442 \tan 25°}{2115} = 1.15$$

Net F.S. = 1.15 Ans. < 1.5 ($\therefore$ Unsafe)

Hence, if such a possibility of submergence occurs, the dam becomes unsafe.

Horizontal shear at base of d/s slope under complete submergence condition.

$$P_d = \frac{\gamma_{sub} h^2}{2} \tan^2 \left(45° - \frac{\phi}{2}\right) + \gamma_w \frac{h^2}{2}$$
$$= \frac{12 \times (22)^2}{2} \tan^2 (45° - 12.5°) + \frac{9.81 \times (22)^2}{2} = 3553 \text{ kN}$$
$$R_d = c \cdot B_d + \gamma_{sub} \left(\tfrac{1}{2} \cdot B_d \cdot h\right) \tan \phi$$
$$= 24 \times 44.0 + 12 \times \tfrac{1}{2} \times 44 \times 22 \times \tan 25° = 3764 \text{ kN}$$
$$\text{F.S.} = \frac{3764}{3553} = 1.06 < 1.5$$ (Hence, Unsafe)

$$\left.\begin{array}{ll} \text{F.S. with slip circle analysis} & = 1.09 \\ \text{F.S. with horizontal shear analysis} & = 1.06 \end{array}\right] \textbf{Ans.}$$

Conclusion. The dam becomes unsafe as soon as the soil in the downstream shoulder gets submerged.

SEEPAGE CONTROL IN EARTH DAMS

The water seeping through the body of the earthen dam or through the foundation of the earthen dam, may prove harmful to the stability of the dam by causing softening and sloughing of the slopes due to development of pore pressures. It may also cause piping either through the body or through the foundation, and thus resulting in the failure of the dam.

20.14. Seepage Control Through Embankments

Drainage filters called 'Drains' are generally provided in the form of (*a*) *rock toe* (*b*) *horizontal blanket* (*c*) *chimney drain*, etc. in order to control the seepage water. The provision of such filters reduces the pore pressure in the downstream portion of the dam and thus increases the stability of the dam, permitting steep slopes and thus affecting economy in construction. It also checks piping by migration of particles. These drains, consist of graded coarse material in which the seepage is collected and moved to a point where it can be safely discharged. In order to prevent movement of the fine material from the dam into the drain, the drain or filter material is graded from relatively fine on the periphery of the drain to coarse near the centre. A multi-layered filter, generally called **inverted filter** or *reverse filter* is provided as per the criteria suggested by Terzaghi for the design of such filters.

The various kinds of drains, which are commonly used are shown and described below :

20.14.1. Rock Toe or Toe Filter [Fig. 20.31 (*a*)]. The 'rock toe' consists of stones of size usually varying from 15 to 20 cm. A toe filter (graded in layers) is provided as a transition zone, between the homogeneous embankment fill and rock toe. Toe filter

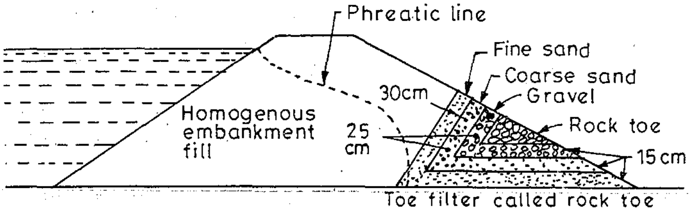


Fig. 20.31 (*a*). Rock Toe.

generally consists of three layers of fine sand, coarse sand, and gravel ; as per the filter criteria requirements. The height of the rock toe is generally kept between 25 to 35% of reservoir head. The top of the rock toe must be sufficiently higher than the tail water depth, so as to prevent the wave action of the tail water.

20.14.2. Horizontal Blanket or Horizontal Filter. [Fig. 20.31 (*b*) and (*c*)]. The horizontal filter extends from the toe (d/s end) of the dam, inwards, upto a distance varying from 25 to 100% of the distance of the toe from the centre line of the dam. Generally, a length equal to three times the height of the dam is sufficient. The blanket should be properly designed as per the filter criteria, and should be sufficiently pervious to drain off effectively.

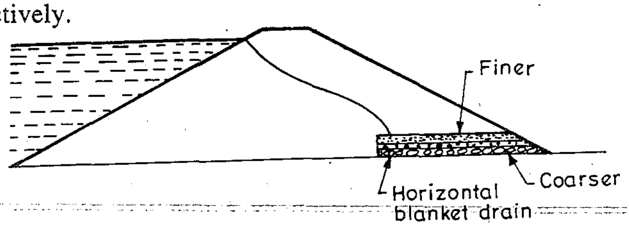


Fig. 20.31 (*b*). Horizontal Filter.

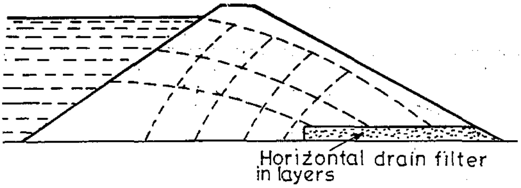


Fig. 20.31 (*c*). Inefficient 'Horizontal drain' in stratified embankments.

20.14.3. Chimney Drain. [Fig. 20.31 (*d*)]. The horizontal filter, not only helps in bringing the phreatic line down in the body of the dam but also provides drainage of the foundation and helps in rapid consolidation. But, the horizontal filter tries to make the soil more pervious in the horizontal direction and thus causes stratification. When large scale stratification occurs, such a filter becomes inefficient as shown in Fig. 20.30 (*c*). In such a possible case, a vertical filter (or inclined u/s or d/s) is placed along with the horizontal filter, so as to intercept the seep-

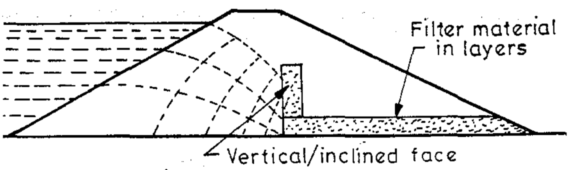


Fig. 20.31 (*d*). 'Chimney Drain' in Stratified Embankments.

ing water effectively, as shown in Fig. 20.31 (*d*). Such an arrangement is termed as *chimney drain*. Sometimes a horizontal filter is combined and placed along with a rock toe, as shown in Fig. 20.31 (*e*).

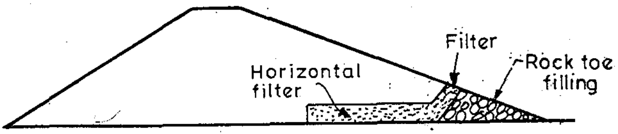


Fig. 20.31 (*e*). Horizontal filter combined with rock toe.

20.15. Seepage Control Through Foundations

The amount of water entering the pervious foundations, can be controlled by adopting the following measures :

20.15.1. Impervious Cutoffs. Vertical impervious cutoffs made of concrete or sheet piles may be provided at the upstream end (*i.e.* at heel) of the earthen dam (Fig 20.32). These cutoffs should, generally, extend through the entire depth of the pervious foundation, so as to achieve effective control on the seeping water. When the depth of the pervious foundation strata is very large, a cutoff, up to a lesser depth, called a *partial cutoff* may be provided. Such a cutoff reduces the seepage discharge by a smaller amount. So much so, that a 50% depth reduces the discharge by 25%, and 90% depth reduces the discharge by 65% or so.

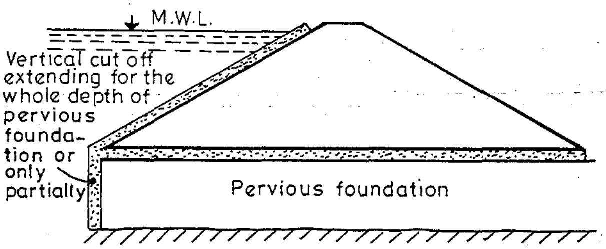


Fig. 20.32

20.15.2. Relief Wells and Drain Trenches. When large scale seepage takes place through the pervious foundation, overlain by a thin less pervious layer, there is a possibility that the water may boil up near the toe of the dam, as shown in Fig. 20.33 (*a*).

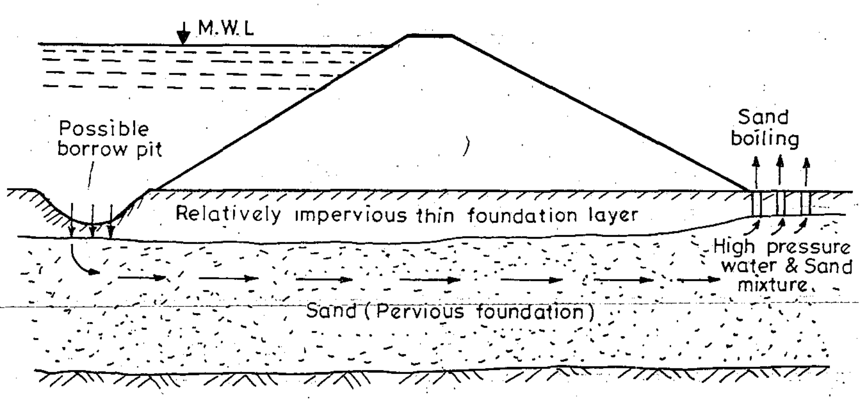


Fig. 20.33 (*a*). Sand Boiling Phenomenon.

Such a possibility, can be controlled by constructing relief wells or drain trenches through the upper impervious layer, as shown in Fig. 20.33 (*b*) and (*c*), so as to permit escape of seeping water. The possibility of sand boiling may also be controlled by

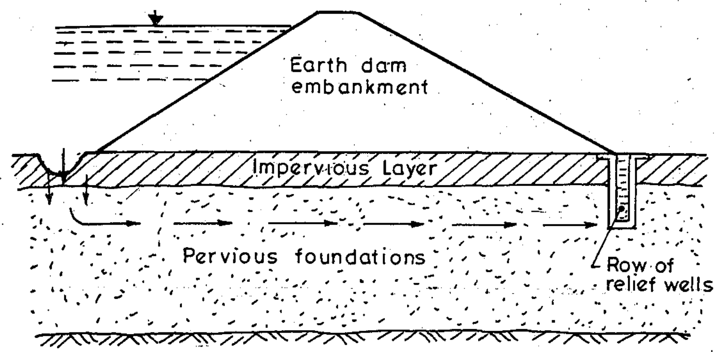


Fig. 20.33 (*b*). Provision of Relief Wells.

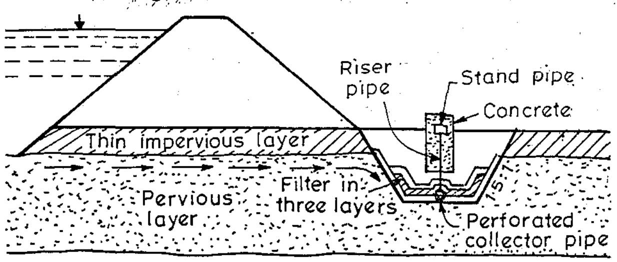


Fig. 20.33 (*c*). Enlarged View of Drain Trench.

providing d/s berms beyond the toe of the dam as shown in Fig. 20.33 (*d*). The weight of the overlying material, in such a case, is sufficient to resist the upward pressure and thus preventing the possibility of sand boiling. The provision of such berms, also protects the d/s toe from possible sloughing due to seepage.

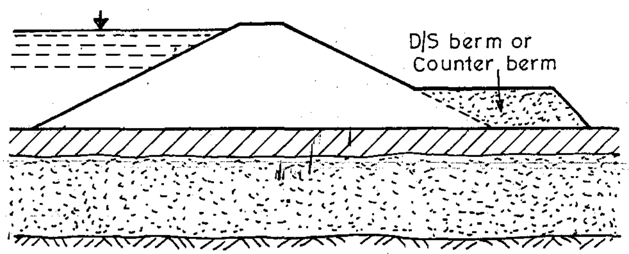


Fig. 20.33 (*d*). Provision of d/s Berms.

20.16. Design of Filters

The drainage filters must be designed in such a way that neither the embankment nor the foundation material can penetrate and clog the filters. The permeability or size of filter material should also be sufficient to carry the anticipated flow with an ample margin of safety. A rational approach to the design of filters has been provided by Terzaghi. According to him, the following filter criteria should be satisfied.

$$\frac{D_{15} \text{ of filter}}{D_{85} \text{ of base materials}} < 4 \text{ to } 5 < \frac{D_{15} \text{ of filter}}{D_{15} \text{ of base material}} \qquad ...(20.50)$$

The embankment soil or the foundation soil surrounding the filter, is known as base material.

When the ratio of D_{15} of filter to D_{85} of base material does not exceed 4 to 5, base material is prevented from passing through the pores of the filter. Similarly, when the ratio of D_{15} of filter to D_{15} of base material is more than 5 (between 5 to 40), the seepage forces within the filter are controlled up to permissible small magnitudes.

Multilayered filters (generally 3 layers) consisting of materials of increasing permeabilities from the bottom to top are, many a times, provided and are known as **inverted filters.** These filters are costly and should be avoided where possible. The minimum total thickness of filter is 1 m. However, if sufficient quantities of filter material are available at reasonable costs, thicker layers of filter may be provided. The thicker the layer, the greater the permissible deviation from the filter requirements.

20.17. Slope Protection

20.17.1. Protection of Upstream Slope. The upstream slope of the earth dam is protected against the erosive action of waves by stone pitching or by stone dumping, as shown in Fig. 20.34. The thickness of the dumped rock should be about I metre and should be placed over a gravel filter of about 0.3 m thickness. The filter prevents the washing of fines from the dam into the rip-rap. The provision of such a dumped rip-rap has been found to be most effective and has been found to fail only in 5% cases.

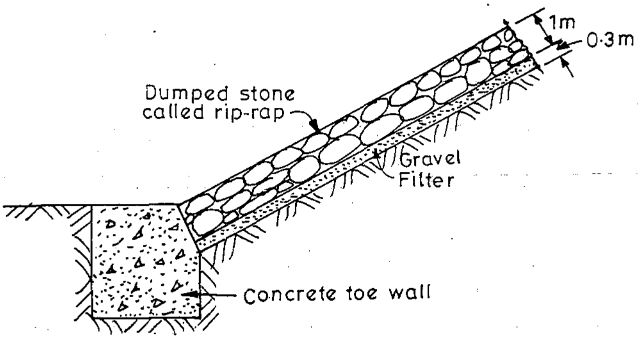


Fig. 20.34

The stone pitching, *i.e.* the hand packed rip rap requires a lesser thickness and may prove more economical if suitable rock is available only in limited quantity. However, when provided in smaller thickness (*i.e.* single layer), it is more susceptible to damage and has been founded to fail in about 30% of cases.

Concrete slabs may also be laid over the u/s slope of the earth dam. When such slabs are constructed, they must be laid over a filter and weep holes should be provided so as to permit escape of water when the reservoir is drawn down. If the filter is not provided, the fines from the embankment may get washed away from the joints creating hollows beneath the slab and causing the consequent cracking and failure of the slab under its own weight. Concrete slab protections have been found to fail in about 36% cases, mainly because of non-providing of filter below them.

20.17.2. Protection of Downstream Slope. The downstream slope of the earthen dam is protected against the erosive action of waves upto and slightly above the water depth, in a similar manner as is explained above for u/s slope.

Moreover, the d/s slope should be protected against the erosive action of rain and its run-off by providing horizontal berms at suitable intervals say about 15 m or so (Fig. 20.5) so as to intercept the rain water and discharge it safely. Attempts should also be made so as to grass and plant the d/s slopes, soon after their construction.

20.18. Rockfill Dams

Rockfill dams have characteristics lying somewhere between the characteristics of gravity dams and those of earthen dams. In other words, they are less flexible than earthen dams and more flexible, then gravity dams. Their foundation requirements are not as strict and rigid as are required for gravity dams. But the foundation requirements are more rigid than those for earthen dams which can be constructed almost on any types of foundations. The steeper slopes are used in rockfill dams and hence, the base width is quite less. The smaller base width and the possibility of large scale seepage restricts the foundation requirements of such dams.

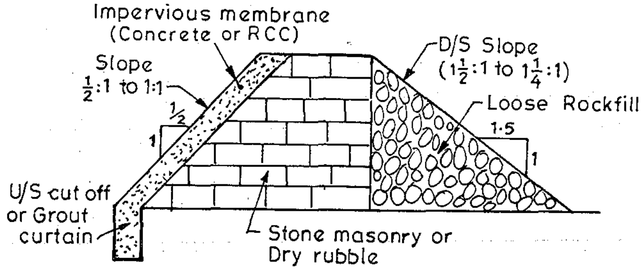


Fig. 20.35. Section of a rockfill dam.

A typical section of a rockfill dams is shown in Fig. 20.35. It essentially consists of an impervious membrane and embankment supporting the membrane. The embankment is divided into two portions. The u/s portion is made of stone masonry or dry rubble, and the d/s portion is made of loose rockfill. The u/s portion embankment supports the membrane and the water load : while the d/s portion supports the u/s embankment, membrane and waterload.

The impervious membrane is usually of concrete, with expansion joints at suitable intervals (say 10 m or so). The expansion joints may be filled with suitable bitumen filler to minimise leakage. Sometimes, R.C.C. membranes are provided with sufficient horizontal and vertical reinforcements, without any expansion joints. The slab thickness is generally less at the top and is more near the dam bottom. The usual slab thickness is between 15 to 50 cm, depending upon the design.

The upstream face containing the membrance, is sloped at a slope varying from say $\frac{1}{2} : 1$ ($\frac{1}{2} H : 1 V$) in low dams up to height of about 60 m or so, to $1 : 1$ or $1\frac{1}{2} : 1$ in high dams. The d/s slope of all the rockfill dams is kept at about $1.3 : 1$, which represents the angle of repose of the rockfill.

Rockfill dams are generally cheaper than concrete dams and can be constructed rapidly if proper rock is available. The rock must be strong enough to withstand high intensity leadings even when wet. The size of loose rock may vary from small stones to 3 m or so. Rockfill dams are very useful in seismic regions, as they provide high resistance to seismic forces because of their flexible character. However, rockfill dams are liable to large settlements, which may lead to cracking of concrete membrane. Repairs for the membrane are, therefore, undertaken from time to time, as and when the necessity arises.

PROBLEMS

1. (*a*) What are 'earthen dams' and under what circumstances are they preferred ?

(*b*) Enumerate the different types of earthen dams, and draw neat sketches showing each type.

[Hint : See article 20.2]

(*c*) Enumerate the two different methods which are adopted for constructing earthen dams. Which of these methods would you prefer and why ? **[Hint** : See article 20.3, preferring rolled fill method]

2. (*a*) What is meant by 'pore water pressure' : and what is its significance in the design of earthen dams?

(*b*) What are 'rockfill dams' and what are their advantages over earthen dams. Draw a neat sketch showing the cross section of a rockfill dam.

3. (*a*) Explain in detail the various forces causing instability in a gravity dam.

(*Madras University, 1975*)

(*b*) Draw a section of an earth dam of 20 m height indicating the various parts of the dam.

(*Madras University, 1975*)

4. (*a*) What are the different types of earth dams that are usually adopted. State where each type is adopted ?

(*b*) What are the causes of failures of earth dam ? (*Madras University, 1974*)

5. What are the precautions that you would take while constructing an earth dam ?

Explain the Swedish slip circle method of analysing the stability of an earth dam slopes.

(*Madras University, 1976*)

6. (*a*) Draw a neat section of an earth dam for the following site :

(*b*) Both silty clay and coarse sand are available at site. Hard stratum is available at about 5 m below the natural ground.

(*c*) Explain briefly how the stability of earthen slopes are checked by the slip circle method.

(*Madras University, 1973*)

7. (*a*) Differentiate between 'Rigid dams' and 'Non-rigid dams' giving examples of each type.

(*b*) Explain with neat sketches how you would carry out the stability analysis of an earth dam.

(*U.P.S.C., 1974*)

8. (*a*) Differentiate between 'horizontal' and 'vertical' piping in earth dams. Suggest permanent measures to check vertical piping.

(*b*) Show with the help of simple outline sketches suitable broad designs of earth dams for different available materials and the governing geology of the site.

Note. At least four different designs should be given.

9. (*a*) Give a suitable design for a 50 m high dam for site where both clay silt and sand gravel are available in plenty and where foundation is pervious to a depth of 10 m. Assume suitable data. Give reasons favouring the suggested design.

10. (*a*) Briefly discuss the checks that are required to be made to investigate the stability of an earthen dam. (*U.P.S.C., 1975*)

(*b*) An earthen dam has to be constructed to store a maximum depth of 12 m of water over river bed consisting of coarse sand and gravel up to a depth of 3 m below river bed followed thereafter by bard and sound rock. Clayee soil is available in plenty in the vicinity of the river. Draw and detail a suitable section of the dam at the river bed.

11. Explain how the following parameters affect design of an earth dam :

(*i*) optimum moisture content ;

(*ii*) C and ϕ value of soil ; permeability of soil

(*iii*) sudden draw-down of the reservoir.

Illustrate with neat sketches the following parts of an earthen dam and state their functions briefly:

(*i*) Rock toe ;

(*ii*) horizontal drainage blanket ;

(*iii*) cut-off ;

(*iv*) Rip-rap.

12. Enumerate and explain by neat sketches the different ways by which the earthen dams may fail. Also suggest suitable precautions that should be undertaken to avoid each type of failure.

13. (*a*) Explain the meaning and importance of equipotential lines' and 'stream lines' in connection with seepage analysis of earthen dams.

(*b*) Derive an expression for calculating the seepage discharge through earthen dam bodies made of isotropic soils. What correction factor is required to be multiplied with this expression, if the dam soil is non-isotropic.

14. (*a*) Define and explain the term 'phreatic line' in earthen dams.

(*b*) How would you procced to determine the phreatic line through homogeneous earthen dams provided :

(*i*) with a horizontal filter ;

(*ii*) Without a horizontal filter.

15. (*a*) Explain and elaborate the importance of 'seepage' through earthen dams.

(*b*) What precautions and remedial measures would you undertake to control the 'seepage' through

(*i*) earthen dam body ;

(*ii*) through the dam foundation.

16. Write short notes on any five of the followings :

(*i*) Rock toe

(*ii*) Climney drain

(*iii*) Relief wells

(*iv*) Slope protection in earthen dams

(*v*) Rock fill dams

(*vi*) Consolidation of earthen dams

(*vii*) Pore pressure and its significance in relation to earthen dam construction

(*viii*) 'Seepage failures' of earthen dams

(*ix*) Rational design of drainage filters for earthen dams.

17. A section of a homogeneous earth dam is shown in Fig. 20.36. Calculate the seepage discharge per metre length, through the body of the dam. The coefficient of permeability of the dam material may be taken as 8×10^{-5} m/sec.

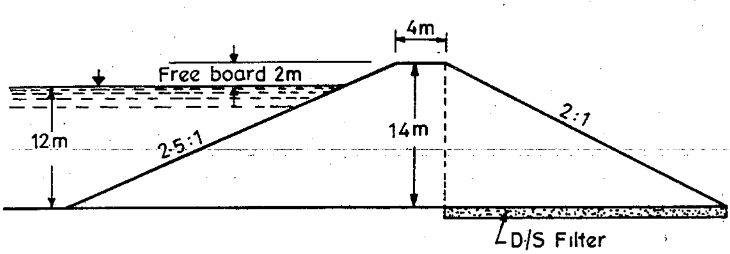


Fig. 20.36

[Ans. 2.88×10^{-1} cumecs/m]

18. An earthen dam made of homogeneous material has the following data :

(A) Hydraulic data of dam

Level of top of dam	= 210 m.
Level of deepest river bed	= 192 m.
HFL of reservoir	= 208 m.
Width of top of dam	= 10 m.
Upstream slope	= 3 : 1
Downstream slope	= 2.5 : 1
Length of horizontal filter from d/s toe inwards	= 16 m.

(B) Properties of the material of the dam

Dry density	= 18 kN/m^3
Saturation density	= 21 kN/m^3
Average angle of friction	= 30°
Average cohesion	= 16 kN/m^2.

(C) The foundation soil consists of 4 m thick layer of soil having the following properties :

Average unit weight $= 17\ kN/m^3$

Average cohesion $= 54\ kN/m^2$.

Average angle of internal friction $= 7°$

Check the dam section for the following :

(*i*) Sloughing of u/s slope during sudden drawdown.

(*ii*) Stability of foundation against shear.

[**Hint.** Follow example 20.4]

19. In order to determine the factor of safety of the d/s slope during steady seepage, the section of a homogeneous earth dam was drawn to scale of 1 cm = 10 m ; and the following results were obtained on a trial slip circle.

Area of N-diagram = 12.15 sq. cm.

Area of T-diagram = 6.50 sq cm.

Area of U-diagram = 4.02 sq cm.

Length of arc = 11.60 cm.

The dam material has the following properties :

Effective angle of internal friction $= 26°$

Unit of cohesion $= 19.62\ m^2$

Unit weight of soil $= 19.62\ m^2$

Determine the factor of safety of the slope. [**Ans.** F.S. 1.21]