10

Hydraulic Jump and its Usefulness in the Design of Irrigation Structures

10.1. General

Hydraulic jump is the jump of water that takes place when a super-critical flow changes into a sub-critical flow. When a stream of water moving with a high velocity and low depth (*i.e.* supercritical flow) strikes another stream of water moving with a low velocity and high depth (*i.e.* sub-critical flow), a sudden rise in the surface of the former takes place. This phenomenon is called *Hydraulic jump*, and is generally accompanied by a large scale turbulence, dissipating most of the kinetic energy of super-critical flow. Such a phenomenon may occur in a canal below a regulating sluice, at the bottom of a spillway, or at a place where a steep channel slope suddenly turns flat.

When water falls over a spillway or a vertical or a glacis fall, it acquires a lot of momentum and velocity. This high velocity, if allowed to persist, shall cause large scale erosion and scouring of the downstream soil of the work, and hence, it must be checked or controlled. In such situations, the phenomenon of hydraulic jump can be used with great advantage for dissipating the kinetic energy of the water.

If the jump is low, *i.e.* the change in the depth is small, the water shall not rise abruptly but will pass through a number of undulations. Such a low jump is called *an undular jump.* In this case, the energy dissipation shall be low and incomplete. But on the other hand, if the jump is high, *i.e.* when the change in depth is large, the jump shall occur abruptly and is called a *direct jump.* Such a jump involves large scale dissipation of energy and is of considerable importance in the design of hydraulic-cum-irrigation structures.

It may be noted that the depth before the jump is always less than the depth after the jump. The depth before the jump is called the **initial depth** (y_1) and the depth after the jump is called the **sequent depth** (y_2). These depths are shown on specific energy curve (Fig. 10.1) and must be differentiated from *alternate depths* y_1 and y_2'. y_2' is the depth that shall occur in a sub-critical flow if there was no loss of energy in the jump formation ; while y_2 is the actual depth that occurs after the jump, involving the energy loss H_L.

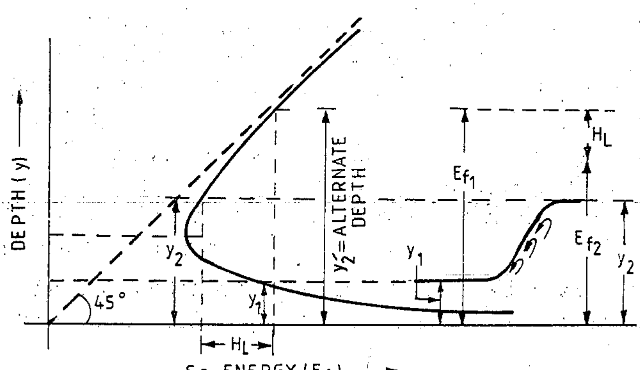


Fig. 10.1. Hydraulic jump interpreted by specific energy curve.

10.2. Types of Jump

Depending upon the incoming Froude No. F_1, the jump on a horizontal floor can be classified, as follows :

(*i*) For $F_1 = 1$, the flow is critical and hence, no jump can form.

(*ii*) For $F_1 = 1$ to 1.7, the water surface shows undulations and the jump is called *undular jump*.

(*iii*) For $F_1 = 1.7$ to 2.5, a series of small rollers develop on the surface of the jump, but the down-stream water surface remains smooth. The velocity throughout is fairly uniform and the energy loss is low. This jump is called a *weak jump*.

(*iv*) For $F_1 = 2.5$ to 4.5, there is an oscillating jet entering the jump bottom to surface and back again with no periodicity. Each oscillation produces a large wave of irregular period, which, very commonly in canals, can travel for miles, doing unlimited damage. This jump is called an *oscillating jump*. The *value of incoming Froude number for barrages and canal regulators generally lies in this zone* or in the previous zone *i.e.* zone of weak jump. The energy dissipators and stilling basins for such conditions need careful design. Different States and Organisations have recommended standard design criteria for the design of stilling basins. A description of these standard basins is available in I.S : 4997—1968. A little description of these stilling basins is given in chapter 20.

(*v*) For $F_1 = 4.5$ to 9.0, the jump is well balanced and its performance is at its best. The energy dissipation ranges from 45 to 70%. The jump is called a *steady jump*.

(*vi*) For $F_1 = 9.0$ and larger, it is a *strong jump* and the energy dissipation may reach up to 85%.

The ranges of Froude number given above, are not definite and may overlap to a certain extent depending on local conditions.

In the subsequent pages, we shall explain the mathematical treatment of hydraulic jump phenomenon on a level surface as well as on a sloping glacis. The methods of plotting the hydraulic jump profiles are also given.

10.3. Momentum Formula

In Fig. 10.2, a stream of water passing through a hydraulic jump has its velocity, depth and energy of flow as V_1, y_1 and E_{f_1} respectively before the jump, and V_2, y_2 and E_{f_2} after the jump. Let H_L be the loss of energy in the jump.

Then $H_L = E_{f_1} - E_{f_2}$

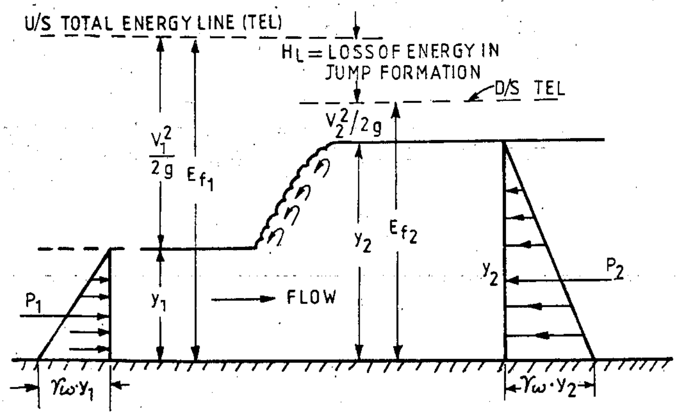
By equation of continuity, we can say that the discharge passing per unit width of the water-way (q) is given as :

$$q = V_1 \cdot y_1 = V_2 \cdot y_2$$

or
$$V_1 = \frac{q}{y_1} \qquad \text{...(10.1)}$$

and
$$V_2 = \frac{q}{y_2} \qquad \text{...(10.2)}$$

According to Newton's second law of motion, the rate of change of momentum must be equal to the applied force.



U/s
Hydrostatic Pressure
Distribution at section u/s
$P_1 = \frac{1}{2}\gamma_w \cdot y_1^2$

D/s
Hydrostatic Pressure
Distribution at section d/s
$P_2 = \frac{1}{2}\gamma_w \cdot y_2^2$

Fig. 10.2.

Mass of water flowing per second

$$= \frac{\gamma_w q}{g}, \qquad \text{where } \gamma_w \text{ is the unit wt. of water.}$$

Change of momentum per second

$$= \frac{\gamma_w q}{g}\left[V_1 - V_2\right] \qquad \text{...(10.3)}$$

Difference in the total pressure on the two sides of the jump

$$= P_2 - P_1 = \frac{1}{2}\gamma_w y_2^2 - \frac{1}{2}\gamma_w y_1^2$$

$$= \frac{1}{2}\gamma_w (y_2^2 - y_1^2) \qquad \text{...(10.4)}$$

Equating (10.3) and (10.4), we get

$$\frac{\gamma_w q}{g}\left[V_1 - V_2\right] = \frac{\gamma_w}{2}\left[y_2^2 - y_1^2\right] \qquad \text{or} \qquad y_2^2 - y_1^2 = \frac{2q}{g}\left[V_1 - V_2\right]$$

Now substituting $V_1 = \frac{q}{y_1}$ and $V_2 = \frac{q}{y_2}$, we get

$$y_2^2 - y_1^2 = \frac{2q}{g}\left[\frac{q}{y_1} - \frac{q}{y_2}\right] = \frac{2q^2}{g}\left[\frac{y_2 - y_1}{y_1 y_2}\right]$$

or $$y_1 y_2 (y_2 + y_1) = \frac{2q^2}{g} \qquad \text{...(10.5)}$$

or $$y_1 y_2^2 + y_1^2 y_2 = \frac{2q^2}{g} \qquad \text{or} \qquad y_2^2 + y_1 y_2 - \frac{2q^2}{g y_1} = 0$$

or $$y_2 = \frac{-y_1 \pm \sqrt{y_1^2 + \frac{8q^2}{g y_1}}}{2}$$

Neglecting unfeasible – ve sign, we get

$$y_2 = \frac{-y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{gy_1}} \quad ...(10.6)$$

or

$$y_2 = \frac{y_1}{2}\left[-1 + \sqrt{1 + \frac{8q^2}{gy_1^3}}\right] = \frac{y_1}{2}\left[-1 + \sqrt{1 + \frac{8V_1^2 y_1^2}{gy_1^3}}\right]$$

$$= \frac{y_1}{2}\left[-1 + \sqrt{1 + 8\frac{V_1^2}{gy_1}}\right]$$

or

$$y_2 = \frac{y_1}{2}\left[-1 + \sqrt{1 + 8F_1^2}\right] \quad ...(10.7)$$

where F_1 = Incoming Froude number $= \frac{V_1}{\sqrt{gy_1}}$

Similarly, we can show that

$$y_1 = \frac{y_2}{2}\left[-1 + \sqrt{1 + 8F_2^2}\right] \quad ...(10.8)$$

where F_2 = Outgoing Froude number $= \frac{V_2}{\sqrt{gy_2}}$

The assumptions made in the derivation of the above formula are :

(*i*) The jump takes place abruptly.

(*ii*) Flow remains streamlined throughout.

(*iii*) Friction is negligible.

(*iv*) Bed is horizontal.

However, in the case of a sloping glacis, the bed does no longer remain horizontal, and the horizontal component of the weight of flowing water will also be involved besides P_1 and P_2 in evaluating the applied force, as shown in Fig. 10.3.

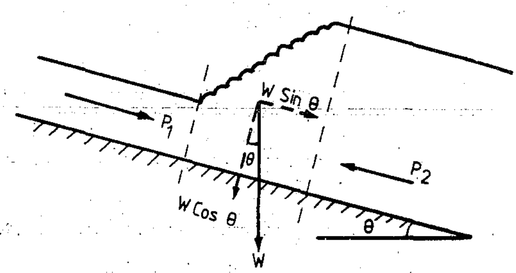


Fig. 10.3

In this case, applied force shall be given by $[P_2 - (P_1 + W \sin \theta)]$ in place of $P_2 - P_1$ in equation (10.2) and hence, the equation derived above for y_2 and y_1 shall not hold good precisely. But if the slope of the glacis is not too pronounced, the gravity component $W \sin \theta$ can be neglected on the grounds that the error so introduced is practically cancelled by the errors involved in other assumptions. Hence, the equations derived for a horizontal bed may be applied to a sloping glacis also, within the limits of accuracy.

10.3.1. Loss of Energy (H_L) in the Standing Wave

$$H_L = E_{f_1} - E_{f_2}$$

$$\text{where } E_{f_1} = y_1 + \frac{V_1^2}{2g} \quad ...(10.9)$$

$$E_{f_2} = y_2 + \frac{V_2^2}{2g} \quad ...(10.10)$$

$$\therefore \qquad H_L = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$

$$= (y_1 - y_2) + \frac{1}{2g}(V_1^2 - V_2^2) = (y_1 - y_2) + \frac{1}{2g}\left[\frac{q^2}{y_1^2} - \frac{q^2}{y_2^2}\right]$$

or
$$H_L = (y_1 - y_2) + \frac{q^2}{2g}\left[\frac{y_2^2 - y_1^2}{y_1^2 y_2^2}\right] \qquad ...(10.11)$$

From equation (10.5), we get

$$y_1 y_2 (y_2 + y_1) = \frac{2q^2}{g} \quad \text{or} \quad \frac{q^2}{2g} = \frac{y_1 y_2}{4}(y_2 + y_1)$$

Substituting this value in equation (10.11) we get

$$H_L = (y_1 - y_2) + \frac{y_1 y_2}{4}(y_2 + y_1)\left(\frac{y_2^2 - y_1^2}{y_1^2 y_2^2}\right) = (y_2 - y_1)\left[\frac{(y_2 + y_1)^2}{4 y_1 y_2} - 1\right]$$

$$= (y_2 - y_1)\left[\frac{(y_2 + y_1)^2 - 4 y_1 y_2}{4 y_1 y_2}\right] = (y_2 - y_1)\left[\frac{(y_2 - y_1)^2}{4 y_1 y_2}\right]$$

or
$$\boxed{H_L = \frac{[y_2 - y_1]^3}{4 y_1 y_2}} \qquad ...(10.12)$$

Usually, in any hydraulic jump, the following eight variables are involved : E_{f_1}, V_1, y_1, E_{f_2}, V_2, y_2, q and H_L. These variables are related by six independent equations, as given below :

$$y_1 y_2 (y_1 + y_2) = \frac{2q^2}{g} \qquad ...(10.5)$$

$$H_L = \frac{(y_2 - y_1)^3}{4 y_1 y_2} \qquad ...(10.12)$$

$$E_{f_1} = y_1 + \frac{V_1^2}{2g} \qquad ...(10.9)$$

$$E_{f_2} = y_2 + \frac{V_2^2}{2g} \qquad ...(10.10)$$

$$V_1 = \frac{q}{y_1} \qquad ...(10.1)$$

$$V_2 = \frac{q}{y_2} \qquad ...(10.2)$$

Hence, if any two variables are known, the remaining six can be worked out by using these six equations, mathematically. The mathematical solution is complicate and to avoid large scale calculations, *Blench* has given some curves by taking q and H_L as known variables (as in actual problem, the discharge intensity *q and the drop in the total energy level H_L are generally known*).

Blench had given curves, relating H_L and E_f for different values of q (Plate 10.1). These curves are very useful in determining the location of the jump on a sloping glacis, as explained below.

When Blench curves are not available, the following mathematical equations can be used to evaluate y_1 and y_2 by known values of q and H_L :

Compute y_c = critical depth = $\sqrt[3]{\dfrac{q^2}{g}}$ ...(10.3)

Now express $\dfrac{y_1}{y_c} = X$ (say) ...(10.4)

$\dfrac{y_2}{y_c} = Y$ (say) ...(10.5)

$\dfrac{H_L}{y_c} = Z$ (say) ...(10.6)

where H_L and y_c are known, hence Z is known.

Relations between Z and X ; and Z and Y are worked out as :

$$Z = \frac{-X^6 + 20X^3 + 8 - (X^4 + 8X)^{3/2}}{16\,X^2} \quad ...(10.7)$$

$$Z = \frac{-Y^6 - 20\,Y^3 - 8 - (Y^4 + 8Y)^{3/2}}{16\,Y^2} \quad ...(10.8)$$

For different values of X and Y, values of Z can be tabulated, and curves $X - Z$ and $Y - Z$ plotted, to read value of X and Y for known value of Z.

To obtain more direct solution, Swamee C.P, has obtained an approximate solution $Y - Z$ curves, and related Y with Z by the eqn. :

$$Y = 1 + 0.93556\,Z^{0.368} \text{ for } Z < 1 \quad ...(10.9)$$

$$Y = 1 + 0.93556\,Z^{0.240} \text{ for } Z > 1 \quad ...(10.10)$$

From the known value of Z, value of Y *i.e.* y_2/y_c can be calculated to finally compute value of y_2.

With known values of Z and X, value of X *i.e.* y_1/y_c can also be computed by using eqn :

$$Z = \frac{(Y - X)^3}{4\,X\,Y} \quad ...(10.11)$$

This computed value of X *i.e.* y_1/y_c can be used to compute y_1.

Values of E_{f_1} and E_{f_2} can also be computed by using the values of X and Y, as follows :

$$\frac{E_{f_1}}{y_c} = \varepsilon = X + \frac{1}{2X^2} \quad ...(10.12)$$

$$\frac{E_{f_2}}{y_c} = \eta = Y + \frac{1}{2Y^2} \quad ...(10.13)$$

Location and Profile of the Jump on a Sloping Glacis

10.4.1. Position of the Jump. Let water be passing over the crest of a work (such as a ... etc.) with a certain head. Then discharge per unit width = $C_d\,H^{3/2}$, where H is the head ...ured from the total energy line to the crest and C_d is the coefficient of discharge. Knowing ...e level of u/s TEL is known. For the given discharge, the depth of water on the d/s is ...n from gauge discharge curves of the channel. This fixes TEL on the d/s. The difference ... levels of u/s TEL and d/s TEL gives H_L.

Knowing q and H_L, E_{f_2} can be obtained from Blench Curves (Plate 10.1). Subtracting ...om d/s TEL, the level at which the jump will form can be easily obtained and hence the

position o... This is hov... jump on tl... obtained.

Know... be calcu... $E_{f_1} - E_{f_2} = $... ponding va... the kno... E_{f_1} and E_{f_2} read from

10.4.2 jump form... as follows... u/s TEL m... different p... on increasi... be tabulate... reducing ti... hence, wat...

10.4.3. point (P), i...

Knowi... different val... P along the ... of x and he... be tabulate... these values... F_1, different... read out f... Hence, diffe... are known, ... in the direct... as origin, a... can be easi...

The wa... in Plate 10.3... value of F_1 ... point of jum... of x, corresp...

position of point P is fixed. This is how the position of the jump on the sloping glacis is obtained.

Knowing E_{f_2}, E_{f_1} can also be calculated by using, $E_{f_1} - E_{f_2} = H_L$. The corresponding values of y_1 and y_2 for the known values of E_{f_1} and E_{f_2} can be directly read from the 'Energy of Flow Curves' given by Montague (Plate 10.2).

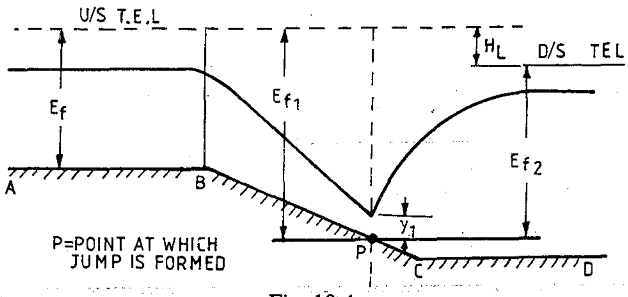


Fig. 10.4

10.4.2. Profile Before the Jump. The water surface profile before the point of jump formation (P) can be easily plotted with the help of Montague's curves (Plate 10.2) as follows : For different points on the glacis from B to P, E_{f_1} will vary, being equal to u/s TEL minus glacis level at the considered point. Since glacis level is different at different points in the length BP, the E_{f_1} will be different at different points and shall go on increasing. For these different values of E_{f_1}, different corresponding values of y_1 can be tabulated from Montague's Curves (Plate 10.2). These values of y_1 shall go on reducing till the point P is reached. These values can be plotted over the glacis, and hence, water surface profile before the jump can be plotted easily.

10.4.3. Profile after the Jump. To plot the water surface profile after the jump point (P), it is necessary to know the Incoming Froude No. F_1.

$$F_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{q}{\sqrt{gy_1^3}}$$

Knowing q and y_1, F_1 can be determined. Graphs are available between $\left(\frac{x}{y_1}\right)$ Vs $\left(\frac{y}{y_1}\right)$ for different values of Froude No. F_1, as shown in Plate 10.3 (a). Taking different points beyond P along the profile, different values of x and hence that of x/y, can be tabulated. Corresponding to these values of x/y_1 for a fixed F_1, different values of y/y_1, can be read out from Plate 10.3 (a). Hence, different values of x and y are known, where (x, y) is any pt. in the direction of flow w.r.t. the point P (i.e. glacis level at the point of jump formation) as origin, as shown in Fig. 10.5. Hence, the water surface profile after the jump point can be easily plotted.

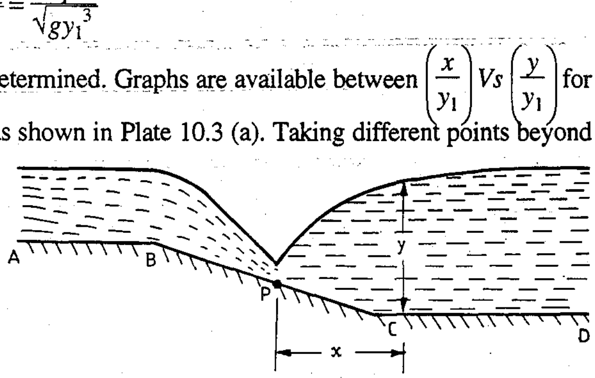


Fig. 10.5

The water surface profile after the jump can also be plotted with the help of curves shown in Plate 10.3 (b). Values of $\frac{y}{y_2 - y_1}$ for known values of $\frac{x}{y_2 - y_1}$ can be read out for a given value of F_1 ; where (x, y) in this case are the ordinates of any point on the profile w.r.t. the point of jump formation (P') as origin, as shown in Fig. 10.6. Hence, for any assumed values of x, corresponding values of y can be worked out and profile plotted easily.

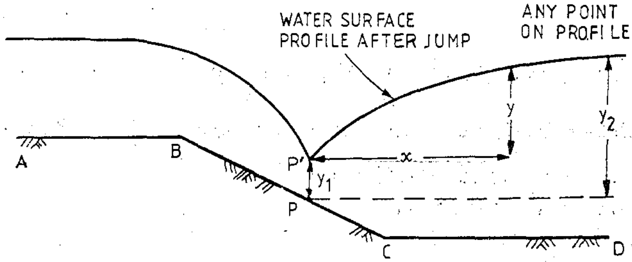


Fig. 10.6

10.5. Hydraulic Jump on a Sloping Glacis as Energy Dissipator

Much has already been said about the use of hydraulic jump phenomenon as energy dissipation device in the design of hydraulic and irrigation structures. The use of 'sloping glacis' for bringing out hydraulic jump to occur is of utmost importance because of the fact : *that the position of the hydraulic jump on a sloping glacis is definite and can be predicted, while on a level floor the position of the jump is unstable. However, on a 'sloping glacis', the energy dissipation is less efficient because* of the vertical component of the velocity remaining intact. In a jump on a glacis, it is only the horizontal component of velocity which takes part in the impact and vertical component remains uneffected.

The length of the jump, *i.e.* the region in which heavy turbulence is created is generally found to be five times the height. Moreover, the start point of the jump is fairly definite but the lower end is indefinite. This point is, however, taken as the place where water surface becomes sensibly level.

Due to heavy turbulence created in the region of the jump. It is necessary to provide a pucca hard floor in this region. A jump if allowed to form on a level surface, cannot be confined precisely to this definite region, because the position of the jump varies through a wide range with a slight change in the discharge. Hence the jump, if formed on a level surface, may not confine itself to the pucca platform and may travel to downstream protection or natural erodible bed of the channel causing deep scours and sometimes even failure of the structure. *Hence, a sloping glacis is always preferred to a horizontal bed for affecting hydraulic jump phenomenon, because the position of the jump on a 'glacis' is always definite although less efficient.* Hence, a sloping glacis having a slope of 2 : 1 to 5 : 1 is generally provided, and hydraulic jump is made to occur on the glacis itself, and in no case lower than the toe of the glacis.

PROBLEMS

1. (*a*) What is meant by 'Hydraulic jump' and how does it helps in designing irrigation structures?

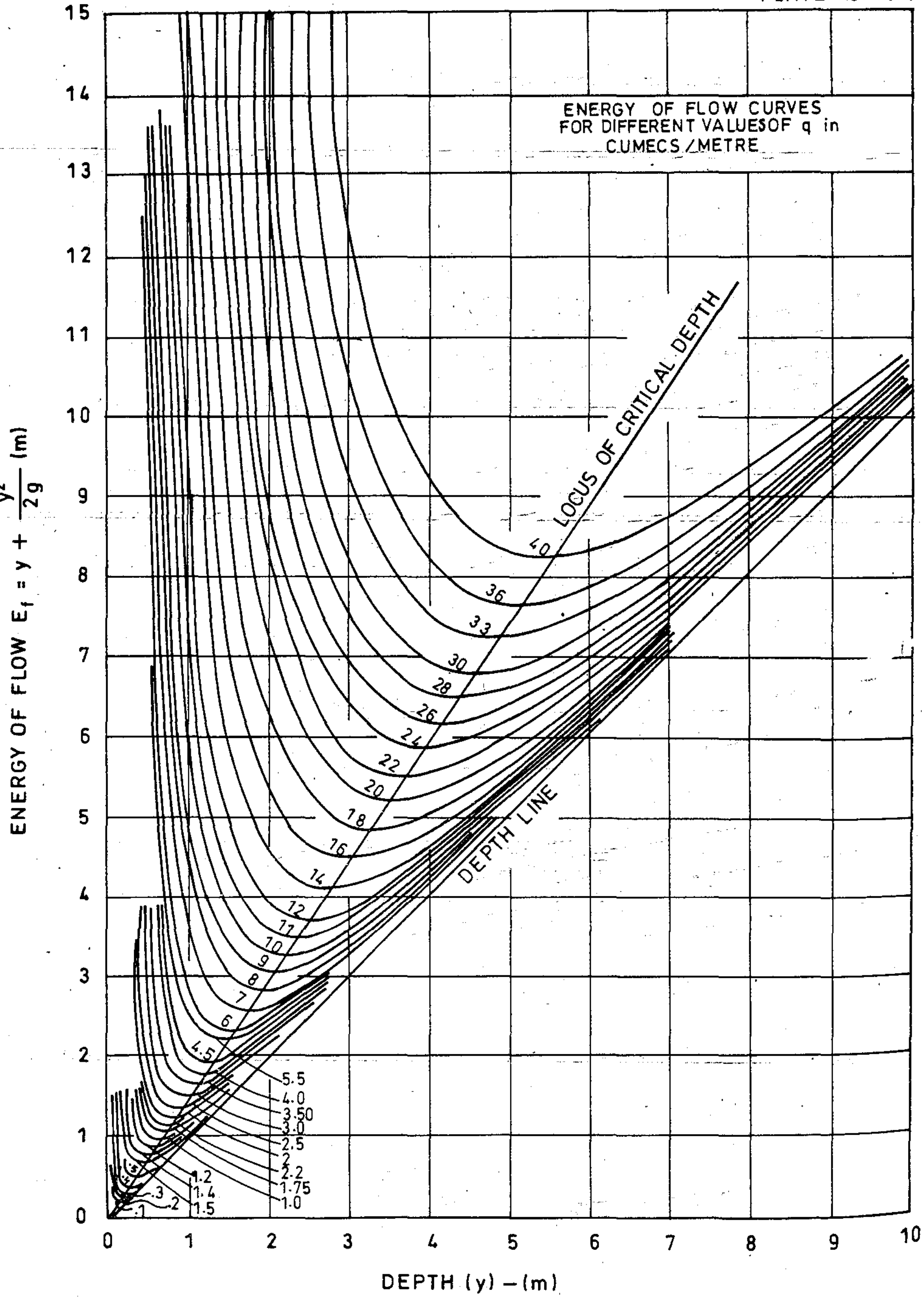
(*b*) Differentiate between 'Sequent depth' and 'Alternate depth'.

(*c*) Derive an expression for expressing energy dissipation obtained by a jump, in terms of initial depth (y_1) and sequent depth (y_2).

2. (*a*) What is a hydraulic jump ? How does it help in dissipating the energy of the water falling over a weir or a dam. What would happen if this energy is not properly dissipated ?

(*b*) What is the importance of "Incoming Froude number", and how does it help in indicating the success of a jump formation ?

(*c*) How would you fix the jump portion when water is flowing over a sloping glacis. Also explain as to how the pre-jump as well as post jump profile can be plotted and what useful purpose will be served by such plottings ?


ENERGY OF FLOW CURVES FOR DIFFERENT VALUESOF q in CUMECS /METRE
LOCUS OF CRITICAL DEPTH
DEPTH LINE
ENERGY OF FLOW $E_f = y + \frac{v^2}{2g}$ (m)
DEPTH (y) – (m)
0
1
2
3
4
5
6
7
8
9
10
11
12
13
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6
4.5
5.5
4.0
3.50
3.0
2.5
2
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1.75
1.0
1.2
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.3
.2
.1

PLATE NO. 10.1

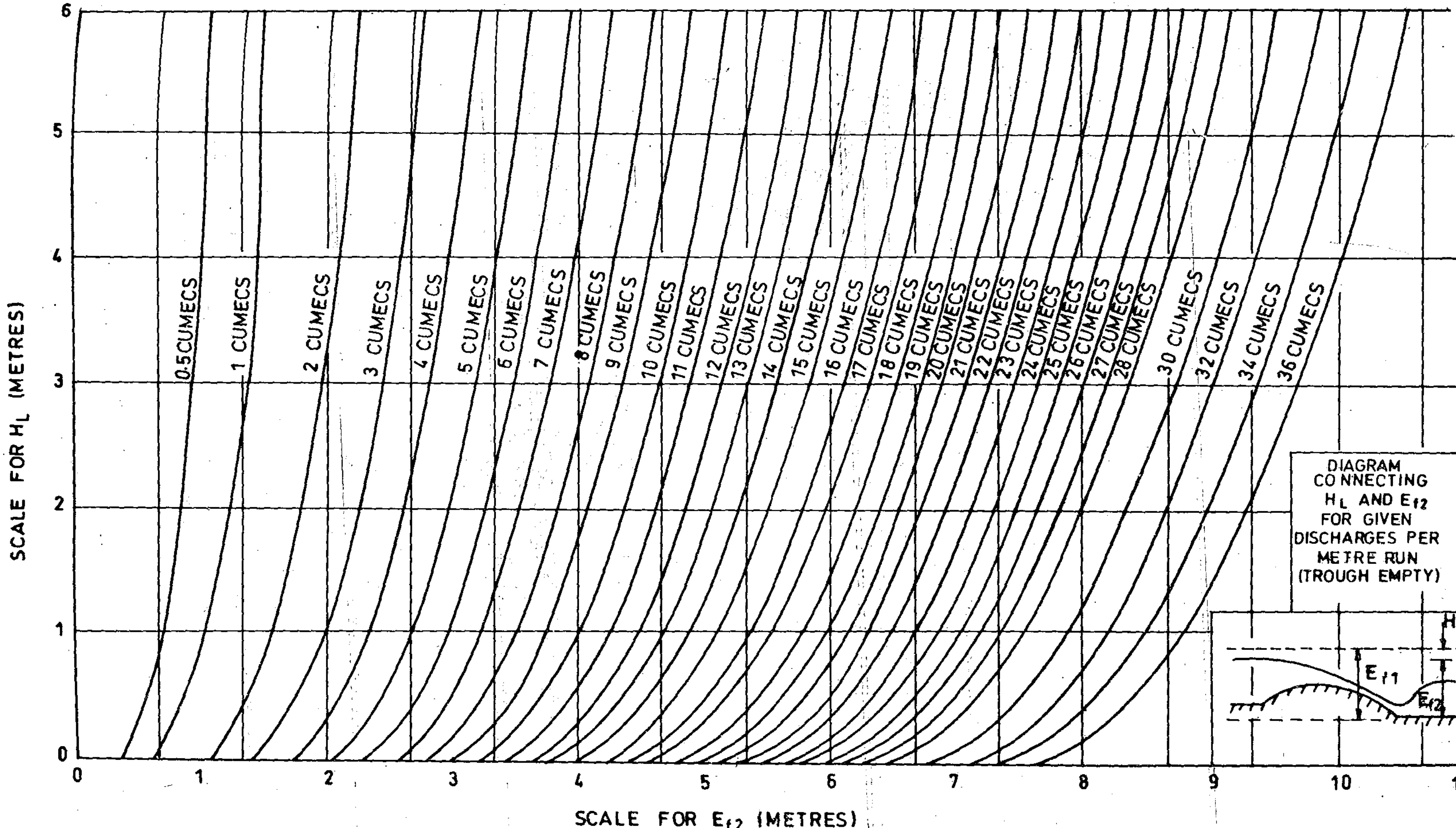
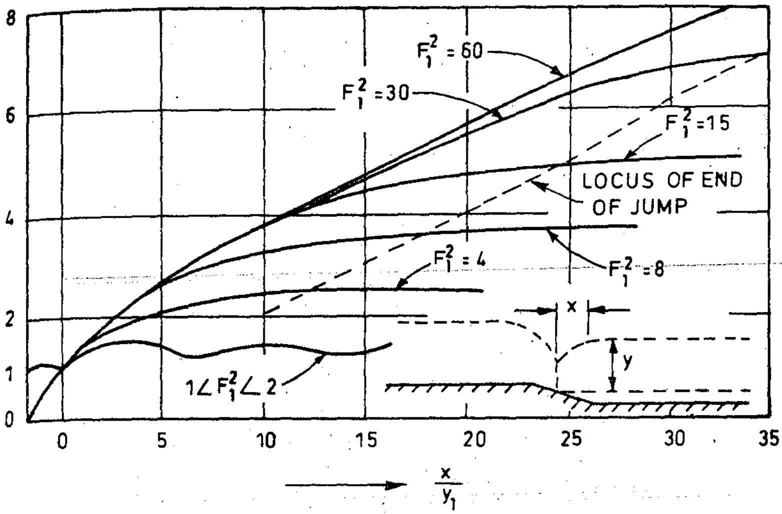


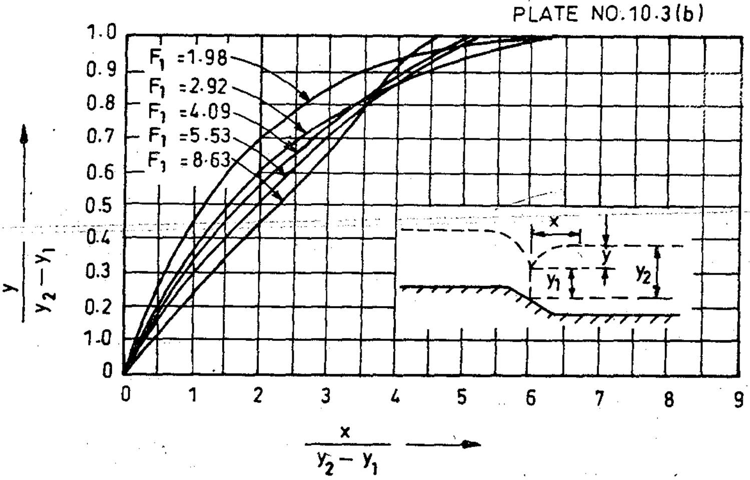
DIAGRAM CONNECTING H_L AND E_{f2} FOR GIVEN DISCHARGES PER METRE RUN (TROUGH EMPTY)

PLATE NO. 10.3 (a)



CURVE FOR PLOTTING POST JUMP PROFILE

PLATE NO. 10.3 (b)



ALTERNATE CURVE FOR PLOTTING POST JUMP PROFILE