Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC - Ist Semester–Examination – May/June- 2012

Subject code: X10001

Subject Name: Mathematics-I

Date:29/05/2012 **Time: 10:30 am – 01:30 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Solve $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$ **03**

(ii) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$ **04**

(b) (i)If $\theta = t^n e^{-r^2/4t}$,what value of n will make $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$? **04**

(ii) Trace the curve $y^2(2a - x) = x^3$ **03**

Q.2 **(a)** (i)Using Gauss Jordan method, find the inverse of the matrix **03**

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

(ii) Find the Eigen values and Eigen vectors of the matrix **04**

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

(b) Solve the following differential equations:

(i) $\left(1 + e^{x/y}\right)dx + e^{x/y}\left(1 - x/y\right)dy = 0.$ **04**

(ii) $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0.$ **03**

OR

(b) Solve the following differential equations:

(i) $(1 + y^2)\, dx = (\tan^{-1} y - x)\, dy$ **04**

(ii) $x\frac{dy}{dx} + y \log y = xy e^x$ **03**

Q.3 Attempt the following:

(a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$,show that **05**

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x + y + z)^2}$$

(b) If $u = \sin^{-1}\left[\frac{x + y}{\sqrt{x} + \sqrt{y}}\right]$,prove that **05**

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{-\sin u \cos 2u}{4\cos^3 u}$$

(c) If $x = r\cos\theta$ and $y = r\sin\theta$; evaluate $\frac{\partial(x, y)}{\partial(r, \theta)}$ & $\frac{\partial(r, \theta)}{\partial(x, y)}$ **04**

OR

Q.3 Attempt the following:

(a) If $u = f(r)$, where $r^2 = x^2 + y^2$, **05**

prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

(b) If $z = x f\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$; **05**

show that $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0$

(c) Examine the function $f(x, y) = x^3 + y^3 - 3axy$ for maxima & minima. **04**

Q.4 Attempt the following:

(a) Evaluate $\iint_R y\,dx\,dy$ where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. **05**

(b) Change the order of integration & evaluate $\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2 + y^2}$ **05**

(c) Using double integration ,find area lying between the parabola $y = 4x - x^2$ and the line $y = x$. **04**

OR

Q.4 Attempt the following:

(a) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$ by changing to polar coordinates. **05**

(b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\,dy\,dz}{\sqrt{(1 - x^2 - y^2 - z^2)}}$ **05**

(c) Calculate the volume of the solid bounded by the planes $x = 0,\ y = 0,\ x + y + z = 1$ and $z = 0$. **04**

Q.5 Attempt the following:

(a) A particle moves along the curve $x = t^3 + 1,\ y = t^2,\ z = 2t + 3$ where t is the time. Find the components of its velocity and acceleration at $t = 1$ in the direction $i + j + 3k$. **05**

(b) Find div $\vec{F}$ and curl $\vec{F}$ where $\vec{F} = grad\,(x^3 + y^3 + z^3 - 3xyz)$ at the point (1, 2, 3) **05**

(c) The rate at which body cools is proportional to the difference between the temperature of the body and that of the surrounding air. If body in air at $25^\circ C$ will cool from 100° *to* 75° in one minute, find its temperature at the end of three minutes. **04**

OR

Q.5 Attempt the following:

(a) Find the directional derivative of the scalar function $f(x, y, z) = x^2 + xy + z^2$ at the point $A(1, -1, -1)$ in the direction of the line AB where B has coordinates (3, 2, 1) **05**

(b) Use Green's theorem to evaluate $\int_c (x^2 + xy)dx + (x^2 + y^2)dy$ where C is the square formed by the lines $y = \pm 1,\ x = \pm 1$. **05**

(c) Find the orthogonal trajectories of the family of curves $x^2 - y^2 = c$. **04**

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Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC - SEMESTER – I • EXAMINATION – WINTER 2012

Subject code: X 10001 **Date: 11/01/2013**

Subject Name: Mathematics - I

Time: 10.30 am - 01.30 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i)Define the rank of Matrix. Determine the rank of Matrix A=$\begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 2 & 3 & 1 \end{bmatrix}$ **03**

(ii)Is the Matrix $\begin{bmatrix} 1 & -5 & 4 & 5 \\ 0 & 1 & 2 & 3 \end{bmatrix}$ in row echelon form or reduced row echelon form? **02**

(iii) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. **02**

(b) (i)Solve the following system of equations:
$x + y + z = 6, x - 2y - 3z = 4, x + 4y - 9z = 6.$ by Gauss-elimination and back substitution method. **04**

(ii) Using Gauss-Jordan method find the inverse of the matrix **03**

A=$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$.

Q.2 **(a)** (i) Find the Eigen values and Eigen vectors of the matrix A=$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. **04**

(ii) State the Euler's theorem for homogeneous function and if $u = \sin^{-1}\left(\dfrac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{5}} + y^{\frac{1}{5}}}\right)$ then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{1}{20}\tan u$. **03**

(b) (i) If $y = f(x+2t) - g(x - 2t)$, prove that $\dfrac{\partial^2 y}{\partial t^2} = 4\dfrac{\partial^2 y}{\partial x^2}$. **04**

(ii) If $x = r\cos\theta, y = r\sin\theta$ then find $\dfrac{\partial(x,y)}{\partial(r,\theta)}$. **03**

OR

(b) (i) Discuss the maxima and minima of $x^2 + y^2 + 6x + 12$. **04**

(ii) Find the equation of tangent plane at the point (1,1,1) on the surface $x^2 + y^2 + z^2 = 3$. **03**

Q.3 **(a)** Solve the following differential equations:

(i) $(x^2 - y^2)dx = (2xy)dy$. **04**

03

(ii) $\frac{dx}{dy}+x=y$.

(b) (i) Solve the exact differential equation **04**

$(x^2+y^2-a^2)xdx \quad (x^2 \quad +y^2 \quad b^2)ydy \quad 0. - \qquad =$

(ii) Find the orthogonal trajectories of the family of the curve $x^2-y^2=c$. **03**

OR

Q.3 **(a)** Solve the following differential equations:

(i) $\frac{dy}{dx}-\frac{y}{x+1}=e^x(x \quad 1), \qquad +$ **04**

(ii) $(e^y+1)\cos xdx+e^y\sin xdy \quad 0. \qquad =$ **03**

(b) (i) Solve $\cos(x+y)dy=dx$. **04**

(ii) Find the orthogonal trajectories of the family of the curve **03**

$r^2=c\sin(2\theta)$.

Q.4 **(a)** Trace the curves: **10**

(i) $r=a(1+\cos\theta), a \quad 0.$ (ii) $xy^2=9a^2(2a-x)$.

(b) Evaluate $\int_0^2\int_0^2(x^2+y^2)dxdy$. **04**

OR

Q.4 **(a)** Trace the curves: **10**

(i) $y^2(a-x)=x^3$, (ii) $r^2=a^2\cos 2\theta$.

(b) Change the order of integration $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}dydx$ and evaluate it. **04**

Q.5 **(a)** (i) Evaluate $\int_0^1\int_0^{\sqrt{(1-y^2)}}(x^2+y^2)dxdy$ by changing into polar coordinates. **04**

(ii) Evaluate $\int_0^a\int_0^{a-x}\int_0^{a-x-y}xdzdydx$. **03**

(b) (i) Find the constants a,b,c so that vector **03**

$(x+y+az)\hat{i} \quad (bx+ \quad 2y \quad z)\hat{j} \quad (-x \quad cy \quad +2z)\hat{k}$ is irrotational.

(ii) Obtain the area of an Ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$,by using Green's theorem. **04**

OR

Q.5 **(a)** (i) Change the order of integration $\int_0^{\frac{1}{\sqrt{2}}}\int_x^{\sqrt{1-x^2}} y^2dA$ and evaluate it. **04**

(ii) Evaluate $\int_0^2\int_0^x\int_0^{\sqrt{x+y}} zdxdydz$. **03**

(b) (i) Is the vector $\vec{v}=(x-3y)\hat{i} \quad (y \quad 2z)\hat{j} \quad (x \quad 3z)\hat{k}$ solenoidal ? **03**

(ii) Find $\int_c \vec{F}d\vec{r}$,where $\vec{F}=\frac{y\hat{i}-x\hat{j}}{x^2+y^2}$ and c is the circle $x^2+y^2=4$ traversed counter clockwise. **04**

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Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC - SEMESTER–I • EXAMINATION – SUMMER 2013

Subject Code: X10001 **Date: 03-06-2013**

Subject Name: Mathematics - I

Time: 02.30 pm - 05.30 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1

(a) Reduce the matrix $A=\begin{bmatrix}1 & 4 & 5\\ 2 & 6 & 8\\ 3 & 7 & 22\end{bmatrix}$ into row echelon form and find its rank. **05**

(b) Solve following linear system by Gauss – Jordan elimination method. **05**

$3x+3y+2z=1$

$x+2y=4$

$10y+3z=-2$

$2x-3y-z=5$

(c) Solve $(x^2-yx^2)\dfrac{dy}{dx}+(y^2+xy^2)=0.$ **04**

Q.2

(a) Find the eigenvalues and eigenvectors of the matrix $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$. **07**

(b) 1) Solve $(x^2-y^2)dx-xydy=0$. **04**

2) Solve $(x^2-ay)dx=(ax-y^2)dy$ **03**

OR

(b) Trace the curve $y^2(a+x)=x^2(a-x)$ **07**

Q.3

(a) If $u=\log(x^3+y^3+z^3-3xyz)$ show that $\left(\dfrac{\partial}{\partial x}+\dfrac{\partial}{\partial y}+\dfrac{\partial}{\partial z}\right)^2 u=-\dfrac{9}{(x+y+z)^2}$ **05**

(b) If $x^2=au+bv$ and $y^2=au-bv$ then show that $\left(\dfrac{\partial u}{\partial x}\right)_y\left(\dfrac{\partial x}{\partial u}\right)_v=\dfrac{1}{2}$. **05**

(c) Show that $grad\left(\dfrac{1}{r}\right)=-\dfrac{\bar{r}}{r^3}$. **04**

OR

Q.3 **(a)** Find the maximum and minimum value of the function $f(x,y)=x^4+y^4-2x^2+4xy-2y^2$. **07**

(b) Evaluate the line integral $\int_C[(5xy-6x^2)\bar{i}+(2y-4x)\bar{j}]$ where C is the curve $y=x^3$ from the point $(1,1)$ to $(2,8)$. **07**

Q.4

(a) If $u=\tan^{-1}\left(\dfrac{y^2}{x}\right)$ prove that $x^2\dfrac{\partial^2 u}{\partial x^2}+2xy\dfrac{\partial^2 u}{\partial x\partial y}+y^2\dfrac{\partial^2 u}{\partial y^2}=-\sin^2 u\sin 2u$. **07**

(b) 1) Find curl $\bar{F}$ at the point $(1,2,3)$, where $\bar{F}=x^2yz\bar{i}+xy^2z\bar{j}+xyz^2\bar{k}$. **04**

2) Show that $\bar{F}=(-x^2+yz)\bar{i}+(4y-z^2x)\bar{j}+(2xz-4z)\bar{k}$ is solenoidal. **03**

OR

Q.4 **(a)** If $x = r\cos\theta,\ \ y = r\sin\theta,$ prove that $JJ' = 1.$ **07**

(b) Verify Green's theorem for $\int_C [(xy + y^2)dx + x^2 dy]$, where C is bounded by $y = x$ and $y = x^2$. **07**

Q.5

(a) Change the order of integration in $I = \int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dydx$ and hence evaluate. **05**

(b) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2)dxdy.$ **05**

(c) Find the orthogonal trajectories of the family of the parabolas $y = ax^2$. **04**

OR

Q.5

(a) Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2)dydx$ by changing to polar coordinates. **05**

(b) Evaluate $\int_0^a \int_0^b \int_0^c (x^2 + y^2 + z^2)dxdydz$. **05**

(c) Find the directional derivatives of $f(x, y, z) = xy^2 + yz^3$ at the point $(2,-1,1)$ in the direction of the vector $\bar{i} + 2\bar{j} + 2\bar{k}$. **04**

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